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Tax Evasion and Dynamic Inefficiency

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Abstract

I show within a two-period overlapping generations model with income tax evasion that when the penalty rate set by the government is su¢ ciently small, it is theoretically possible for the capital stock to exceed the golden-rule level on the balanced-growth path. However, such a dynamic inefficiency cannot be guaranteed when the probability of evasion detection is nil.

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1 Introduction

It is well-known from the standard overlapping-generations (OLG) model *à la* Diamond (1965) that there exists a possibility of capital overaccumulation in a decentralized perfectly competitive market economy. That is, the capital stock on the balanced growth path might exceed the golden-rule level, reflecting the equilibrium which is Pareto-dominated. The reason for such a dynamic inefficiency is that OLG models imply an infinite number of households, as generations continually enter and exit the model upon birth and death. The infinity of generations is precisely what violates the conditions of the welfare theorems, implying that a benevolent social planner can improve upon the performance of the competitive market if the latter chooses to hold too much capital.

There is a rich set of theoretical literature which deals with the problem of capital overaccumulation in various forms of Diamond's framework.[1] Empirical studies concerning modern economies (many of which maintain historically small savings rate) find that the problem of capital overaccumulation is likely to be irrelevant (see, e.g., Abel *et al*. 1989).

In this note I consider the standard Diamond economy with income tax evasion phenomenon in an attempt to outline some additional conditions which might cause the equilibrium capital stock exceed the golden-rule level in a decentralized environment. The importance of analyzing income tax evasion behavior arises from the fact that it is a chronic problem pertinent to virtually every economy,[2] albeit lesser developed economies are the ones where tax noncompliance tends to get more serious (see, e.g., Alm *et al*. 1993, Chen 2003, Gupta 2004). Often many of those countries also happen to display less efficient compliance enforcement, and the public sector with more corrupt tax collectors, who are prone to turn a blind eye on tax evasion (possibly for some reward). An important question, which to the best of my knowledge has not been yet addressed, whether a widespread tax evasion caused by a weak enforcement mechanism is capable of generating too much ill-gotten capital, exceeding the golden-rule level. I show in this study that the answer is *yes*, in principle. However, if the weak enforcement is the reason to blame, then the only sufficient condition for dynamic inefficiency is a small enough penalty rate imposed on the taxes concealed, *not* a low success rate of the audit process per se. Furthermore, this paper demonstrates that an extreme weakness of the enforcement parameters alone is still unlikely to cause capital overaccumulation in the economy with tax noncompliance as that would require everyone in the economy to report negative wages, and such a blatant dishonesty is unrealistic at least at the aggregate. Consequently, a small value of capital's share in total value added remains as the only viable cause of dynamic inefficiency (which is a standard result in an OLG environment with constant returns to scale (CRS) technology and logarithmic preferences). With that said, I now turn to the model.

2 The model

Time is discrete an indexed by $t \in [0, +\infty)$.[3] Economy begins at $t = 0$ with initial predetermined conditions. Economic agents live for two periods. In the first period, while young, the agents

[1] See Decreuse and Thibault (2001), Gutiérrez (2008) and many references therein.

[2] Refer, for instance, to Slemrod (2007) for a review of the problem in the context of the U.S. economy.

[3] For alternative or similar two-period tax evasion setups refer, for example, to Sengupta's (1998) model with capital income taxation only, or to Caballé and Panadés (2000) model with no population or technology growth but with productive government.

supply labor inelastically and earn corresponding wages. They optimally decide how much of their first-period earnings to consume, and how much to reveal to the tax collector. Wage earnings are taxed at a flat rate $\theta \in (0,1)$. In the first period a fraction of taxpayers is caught cheating and consequently fined, and the remaining fraction escapes the audit. The agents' remaining savings finance the retirement consumption in the second period. For simplicity, I rule out any capital income taxes.

People's preferences are represented by the von Neuman-Morgenstern expected utility function, which is time additively separable in consumption. The taxpayer's problem is to

$$\underset{\{c_{1t},x_{1t}\}}{Max} E_t[u] = u(c_{1t}) + \frac{1}{1+\rho}\left[qu\left(c_{2t+1}^{c}\right) + (1-q)u\left(c_{2t+1}^{nc}\right)\right], \tag{1}$$

where $\frac{1}{1+\rho}$ is the psychological discount factor and $\rho > 0$. Utility function, $u(\bullet)$, is strictly increasing, strictly concave, and twice-continuously differentiable. The amount of wages reported to the government is x_{1t}. Consumption of the agent when young in period t is c_{1t}, and that of the agent when old in the next period is c_{2t+1}. Superscripts "c" and "nc" in the second-period denote the consumption of a caught and not-caught agent, respectively. The probability of detection and punishment is $q \in (0,1)$. If the agent is caught cheating, he faces a fine rate, $\phi > 1$, which will be imposed on the amount of concealed tax liabilities, as in most countries.

On the production side of the economy there is a competitive producer with CRS production function reflecting Harrod-neutral technological progress: $F(K_t, A_tL_t)$, with A_t describing the effectiveness of labor (L_t). The aggregate capital stock at time t is K_t. It is assumed $A_{t+1} = (1+g)A_t$ and $L_{t+1} = (1+n)L_t \; \forall t \in [0,+\infty)$, and $g > 0$, $n > 0$. The production function in the intensive form can be stated as $f(k_t)$, where $k_t \equiv \frac{K_t}{A_tL_t}$, and f is strictly increasing, strictly concave, and twice-continuously differentiable. The Inada conditions are satisfied: $\lim_{k_t \to 0} f'(k_t) = +\infty$ and $\lim_{k_t \to +\infty} f'(k_t) = 0$.

Let w_{1t} stand for the labor's wage per unit of effective worker. Assuming factor demands are determined by marginal principles, it is simple to show that $w_{1t} = f(k_t) - k_t f'(k_t)$. Similarly, the gross rate of return on capital (net of depreciation, $\delta \in [0,1]$), R_t, can be presented as $f'(k_t) + 1 - \delta$.

Therefore, the household's budget constraints can be fully described as

$$c_{2t+1}^{c} = R_{t+1}\left(w_{1t}A_t - \theta x_{1t} - \phi\theta\left(w_{1t}A_t - x_{1t}\right) - c_{1t}\right), \tag{2}$$

and

$$c_{2t+1}^{nc} = R_{t+1}\left(w_{1t}A_t - \theta x_{1t} - c_{1t}\right). \tag{3}$$

By specifying the functional forms, it is possible now to fully solve the model analytically.

2.1 Solving the model

Assume a standard constant-relative-risk-aversion utility and Cobb-Douglas technology. To obtain the analytical solutions, I set the coefficient of relative risk aversion to unity.[4] Production function

[4]Setting the elasticity of intertemporal substitution of consumption close to 1 is consistent with the large body of micro studies (see Attanasio 1999).

is $F(K_t, A_tL_t) = K_t^{\alpha}(A_tL_t)^{1-\alpha}$, where $\alpha \in (0,1)$ is the share of the capital. The intensive production function becomes then $f(k_t) = k_t^{\alpha}$.

Marginal relations on the production side of the economy lead to

$$w_{1t} = (1-\alpha)\, k_t^{\alpha}, \tag{4}$$

and

$$R_t = \alpha k_t^{\alpha-1} + 1 - \delta. \tag{5}$$

It is straightforward to show (see Appendix) that the taxpayer's maximization (1) with (2) and (3) leads to

$$c_{1t} = \frac{(1+\rho)(1-\theta)}{2+\rho} w_{1t}A_t, \tag{6}$$

and

$$x_{1t} = \frac{\phi\theta(2-q) + \rho\theta(\phi-1) + q\phi - \theta - 1}{\theta(2+\rho)(\phi-1)} w_{1t}A_t. \tag{7}$$

To ensure an interior solution for evasion it is necessary that $q\phi < 1$ holds, which precisely is Yitzhaki's (1974) condition. This can easily be deduced from (7) by requiring the numerator of $x_{1t}/w_{1t}A_t$ be less than the denominator.

Now, to fully describe the equilibrium state of the economy in terms of the capital per unit of effective worker, it is useful to state the following proposition.

Proposition 1 *The equilibrium value of the capital stock per unit of effective worker is given by*

$$k^* = \left[\frac{\phi(1-\alpha)(1-\theta)(1-2q+q^2\phi)}{(\phi-1)(2+\rho)(1+n)(1+g)}\right]^{\frac{1}{1-\alpha}}, \tag{8}$$

and is positive, unique and globally stable as long as some evasion takes place.

Proof. Denote the total savings of the group of not caught and caught taxpayers as S_t^{nc} and S_t^{c}, respectively. Clearly,

$$S_t^{nc} = L_t(1-q)(w_{1t}A_t - \theta x_{1t} - c_{1t}), \tag{9}$$

and

$$S_t^{c} = L_t q(w_{1t}A_t - \theta x_{1t} - \phi\theta(w_{1t}A_t - x_{1t}) - c_{1t}). \tag{10}$$

Since overall savings generates the capital stock in the next period, using (6) and (7) in (9) and (10), leads to

$$S_t^{nc} + S_t^{c} = K_{t+1} = \frac{\phi(1-\theta)(1-2q+q^2\phi)}{(\phi-1)(2+\rho)} L_t w_{1t}A_t. \tag{11}$$

At the steady state $\frac{K_{t+1}}{A_{t+1}L_{t+1}} \equiv k_{t+1} = k_t = k^*$, which, recalling (4) and using (11), uniquely results in (8).

Now, note (8) is positive if $1-2q+q^2\phi > 0$. The latter is equivalent to $q(2-q\phi) < 1$. Since for some tax evasion to take place $q\phi$ must be less than 1, it is clear that the left hand-side of the latter inequality is always positive. The question is whether it can be at least 1. For it to equal 1, there must be some feasible range for q such that $q^2\phi - 2q + 1 = 0\ \forall\phi > 1$, which is not possible.

To ensure that the monotonic function $q\,(2 - q\phi) < 1\ \forall q \in (0, 1)$ and $\phi > 1$ it is sufficient now to show that $\lim_{q\to 0} [q\,(2 - q\phi)] < 1$, which is indeed true. ■

Finally, the sufficient conditions for the capital stock on the balanced growth path to exceed the golden-rule level in the economy with income tax evasion is summarized in the following proposition.

Proposition 2 *If the penalty rate, ϕ, and/or capital's share, α, are sufficiently small, the economy with income tax evasion will be dynamically inefficient. However, a small enough value of ϕ would violate a condition for an interior solution.*

Proof. In an economy with positive depreciation, technological and population growth, the golden-rule equilibrium capital stock is defined by $f'(k^*_{GR}) = n + g + \delta$. Using $f(k_t) = k_t^\alpha$, it is easy to find from (8)

$$f'(k^*) = \frac{\alpha\,(\phi - 1)\,(2 + \rho)\,(1 + n)\,(1 + g)}{\phi\,(1 - \alpha)\,(1 - \theta)\,(1 - 2q + q^2\phi)}. \tag{12}$$

Clearly, $\lim_{\alpha\to 0} f'(k^*) = \lim_{\phi\to 1} f'(k^*) = 0 < f'(k^*_{GR})$. This implies $k^* > k^*_{GR}$ as $\alpha \to 0$ and/or $\phi \to 1$.

However, note from the numerator of (7) that $\lim_{\phi\to 1} [\phi\theta\,(2 - q) + \rho\theta\,(\phi - 1) + q\phi - \theta - 1] = -1 + q + \theta - q\theta = (\theta - 1)\,(1 - q) < 0$, which would violate $x_{1t}/w_{1t}A_t > 0$.[5] ■

An important implication of the latter proposition is that even if the likelihood of getting caught cheating is extremely small in the economy with tax noncompliance (meaning that many taxpayers will successfully multiply their ill-gotten savings since as can be seen from (12) $[1 - 2q + q^2\phi]'_q = -2 + 2q\phi < 0$ as $q\phi < 1$), this alone does *not* necessarily lead to the capital overaccumulation. On the other hand, even if the likelihood of getting caught is high (causing little incentives to evade), but the surcharge rate imposed on the top of hidden taxes is still small enough (or almost never enforced due to a variety of institutional reasons), the economy's capital stock on the balanced growth path will exceed the golden-rule level. However, a low enough value of the fine rate would mean that everyone in the economy reports *negative* wages (and thus gets subsidized by the government), which is not a viable scenario. That is, as long as in the economy even with an extremely rampant tax evasion at least some positive income declaration takes place due to the fines paid when detected, the capital is unlikely to rise beyond the golden-rule level.

3 Conclusion

I have considered a standard OLG model with income tax evasion to show that there is an additional sufficient condition for the dynamic inefficiency. Namely, a sufficiently small value of the fine rate imposed on the amount of concealed tax liability is capable of causing the economy to overaccumulate the capital. On the other side of the enforcement mechanism, the success rate of the tax audit plays little or no role in driving the economy to a Pareto-dominated level of the capital stock. However, since a small enough fine rate would violate one of the conditions for an interior solution, such a scenario can be ruled out as unrealistic.

[5] When $\phi \to 1$, the numerator of (7) approaches a negative value faster than the denominator of (7) approaches zero.

Appendix

Assuming logarithmic preferences, problem (1) with (2) and (3) generates the following first-order conditions after some simplifications:

$$\begin{cases} \frac{1}{c_{1t}} = \frac{1}{1+\rho}\left[\frac{1-q}{w_{1t}A_t-\theta x_{1t}-c_{1t}} + \frac{q}{w_{1t}A_t-\theta x_{1t}-\phi\theta(w_{1t}A_t-x_{1t})-c_{1t}}\right] \\ 0 = \frac{1-q}{w_{1t}A_t-\theta x_{1t}-c_{1t}} + \frac{q(1-\phi)}{w_{1t}A_t-\theta x_{1t}-\phi\theta(w_{1t}A_t-x_{1t})-c_{1t}} \end{cases}. \tag{A.1}$$

Using the second equation in (A.1) permits to re-write the first expression in (A.1) as

$$\begin{aligned} \frac{1+\rho}{c_{1t}} &= \frac{q(\phi-1)}{w_{1t}A_t-\theta x_{1t}-\phi\theta(w_{1t}A_t-x_{1t})-c_{1t}} + \frac{q}{w_{1t}A_t-\theta x_{1t}-\phi\theta(w_{1t}A_t-x_{1t})-c_{1t}} \\ &= \frac{q\phi}{w_{1t}A_t-\theta x_{1t}-\phi\theta(w_{1t}A_t-x_{1t})-c_{1t}}. \end{aligned} \tag{A.2}$$

Using the very first and the very last term in (A.2), cross-multiplying and collecting terms, leads to

$$(q\phi+1+\rho)\,c_{1t}+\theta(1+\rho)(1-\phi)\,x_{1t}=(1+\rho)(1-\phi\theta)\,w_{1t}A_t. \tag{A.3}$$

Similarly, we can restate the second equation in (A.1) as

$$(q\phi-1)\,c_{1t}+\theta(\phi-1)\,x_{1t}=(q\phi(1-\theta)+\phi\theta-1)\,w_{1t}A_t. \tag{A.4}$$

Hence, we have the new linear system of equations:

$$\begin{cases} (q\phi+1+\rho)\,c_{1t}+\theta(1+\rho)(1-\phi)\,x_{1t}=(1+\rho)(1-\phi\theta)\,w_{1t}A_t \\ (q\phi-1)\,c_{1t}+\theta(\phi-1)\,x_{1t}=(q\phi(1-\theta)+\phi\theta-1)\,w_{1t}A_t \end{cases}. \tag{A.5}$$

Now define

$$|B| \equiv \begin{vmatrix} q\phi+1+\rho & \theta(1+\rho)(1-\phi) \\ q\phi-1 & \theta(\phi-1) \end{vmatrix}, \tag{A.6}$$

$$|B_1| \equiv \begin{vmatrix} (1+\rho)(1-\phi\theta)\,w_{1t}A_t & \theta(1+\rho)(1-\phi) \\ (q\phi(1-\theta)+\phi\theta-1)\,w_{1t}A_t & \theta(\phi-1) \end{vmatrix}, \tag{A.7}$$

$$|B_2| \equiv \begin{vmatrix} q\phi+1+\rho & (1+\rho)(1-\phi\theta)\,w_{1t}A_t \\ q\phi-1 & (q\phi(1-\theta)+\phi\theta-1)\,w_{1t}A_t \end{vmatrix}, \tag{A.8}$$

apply *Cramer's rule* and confirm that indeed $|B_1|\,/\,|B|$ and $|B_2|\,/\,|B|$ result in (6) and (7), respectively.

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