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## The allocation of time to crime: A simple diagrammatical exposition

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### Abstract

In his seminal article on the allocation of time to crime, Isaac Ehrlich (1973) derives five interesting theoretical results. He uses a state-preference diagram to derive one result, retreating to mathematics for deriving the remaining four results. This note shows that all five results can easily be derived from an alternative and simpler diagrammatical exposition that involves intersection of curves rather than tangency between curves.

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## 1. Introduction

Most theoretical models of criminal behavior consider the decision to commit a crime as binary: the individual may either participate in criminal activity or in legitimate work [e.g., Becker (1968), Glaeser et al (1996), Polinsky and Shavell (2000)]. An exception is Isaac Ehrlich's (1973) seminal article on the allocation of time to crime, which allows the individual to divide his time between criminal activities and legitimate work. Two decades later, Richard Freeman (1996) supported empirically Ehrlich's approach, arguing that

> "*Many youths combine crime and work or shift between them readily. Because most criminals are self-employed, and because the U.S. job market has considerable flux, crime and legitimate work are not dichotomous choices for most young men. Joe holds a job, robs someone he meets on a dark empty street and sells drugs on the weekend.*"(p. 34).

Freeman's evidence renews interest in Ehrlich's approach and provides motivation for revisiting his work.

Ehrlich derives five theoretical results: (a) a sufficient condition for entry into crime – regardless of the attitude toward risk – is that the marginal differential return from crime exceeds the marginal expected penalty at the position where no time is devoted to crime; (b) a risk-neutral offender will spend more time in crime relative to a risk avoider, whereas a risk preferrer will spend more time in crime relative to both; (c) an increase in the probability of apprehension will reduce the time spent in crime - regardless of the offender's attitude toward risk; (d) an increase in the marginal penalty will reduce the time spent in crime of a risk avoider and a risk-neutral offender, but might increase the time spent in crime of a risk preferrer; (e) a 1 percent increase in the probability of apprehension accompanied by a 1 percent decrease in the marginal penalty will not affect the time spent in crime of a risk-neutral offender, but will decrease the time spent in crime of a risk preferrer and will increase the time spent in crime of a risk avoider. The last three results relate to offenders who are only partially engaged in crime rather than fully specializing in it. Otherwise they may not respond at all.

Ehrlich uses a state-preference diagram to demonstrate the optimal solution of a risk avoider as a point of tangency between a convex indifference curve and a concave opportunity boundary. He also uses the diagram to derive result (a) above. However, he retreats to mathematics for deriving all other results. The purpose of the present note is to show that all five results can easily be derived from an alternative diagrammatical exposition. The proposed exposition is also simpler than the state-preference diagram, as it involves intersection of curves rather than tangency between curves. Consequently, the optimal solution of a risk preferrer, which in a state-preference diagram necessitates a cumbersome tangency between two concave curves, becomes clearer and graphically comparable with the optimal solutions of the other risk-attitude types.

## 2. Ehrlich's model

Ehrlich considers an individual who, at the beginning of a given period, may allocate a given amount of time, $T$, between criminal activity, $S$, and legitimate work, $T - S$. The monetary value of his returns from crime is $m(S)$, where $m'(S) > 0$ and $m''(S) < 0$. The monetary value of his returns from legitimate work is $w(T-S)$, where $w'(T-S) > 0$ and $w''(T-S) < 0$. With probability $p$ the individual will be apprehended at the end of the period and punished. The monetary value of the punishment is assumed to be $F(S)$, where $F'(S) > 0$ and $F''(S) \geq 0$. Denoting initial assets by $W_0$, the individual's net assets at the end of the period will be

$$W^{+} = W_0 + m(S) + w(T-S) \tag{1}$$

if he is not apprehended, and

$$W^{-} = W_0 + m(S) + w(T-S) - F(S) \tag{2}$$

if he is apprehended. The individual seeks the value of $S$ that maximizes his expected utility from the alternative net asset levels

$$EU(W) = (1-p)U(W^{+}) + pU(W^{-}). \tag{3}$$

Substituting (1) and (2) in (3) and differentiating with respect to $S$, the first-order condition for expected-utility maximization is given by

$$\frac{d'(S)}{F'(S) - d'(S)} = \frac{pU'(W^{-})}{(1-p)U'(W^{+})} \quad , \tag{4}$$

where $d'(S) \equiv m'(S) - w'(T-S)$ is the marginal differential return from crime. An interior solution requires that the left-hand side of (4) is positive, or that $d'(S) > 0$ and $F'(S) - d'(S) > 0$ for some $0 < S < T$.[1] Ehrlich uses a state-preference diagram to demonstrate the solution of condition (4) for a risk avoider, interpreting the left-hand side of (4) as the slope of an opportunity boundary and its right-hand side as the slope of an indifference curve which holds expected utility constant. He also uses the diagram to derive result (a) above. All other results are derived mathematically from the first-order condition with no reference to the diagram.

## 3. An alternative diagrammatical exposition

Let the first-order condition be rewritten as

$$\frac{(1-p)d'(S)}{pF'(S) - pd'(S)} = \frac{U'(W^{-})}{U'(W^{+})} \quad . \tag{4'}$$

Denote the left- and right-hand side of (4') by $\alpha(S)$ and $\beta(S)$, respectively. Figure 1 depicts

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[1] A prerequisite for this is that $d'(0) > 0$ (otherwise the individual will never engage in crime) and $F'(T) - d'(T) > 0$ (otherwise the individual will always specialize in crime).

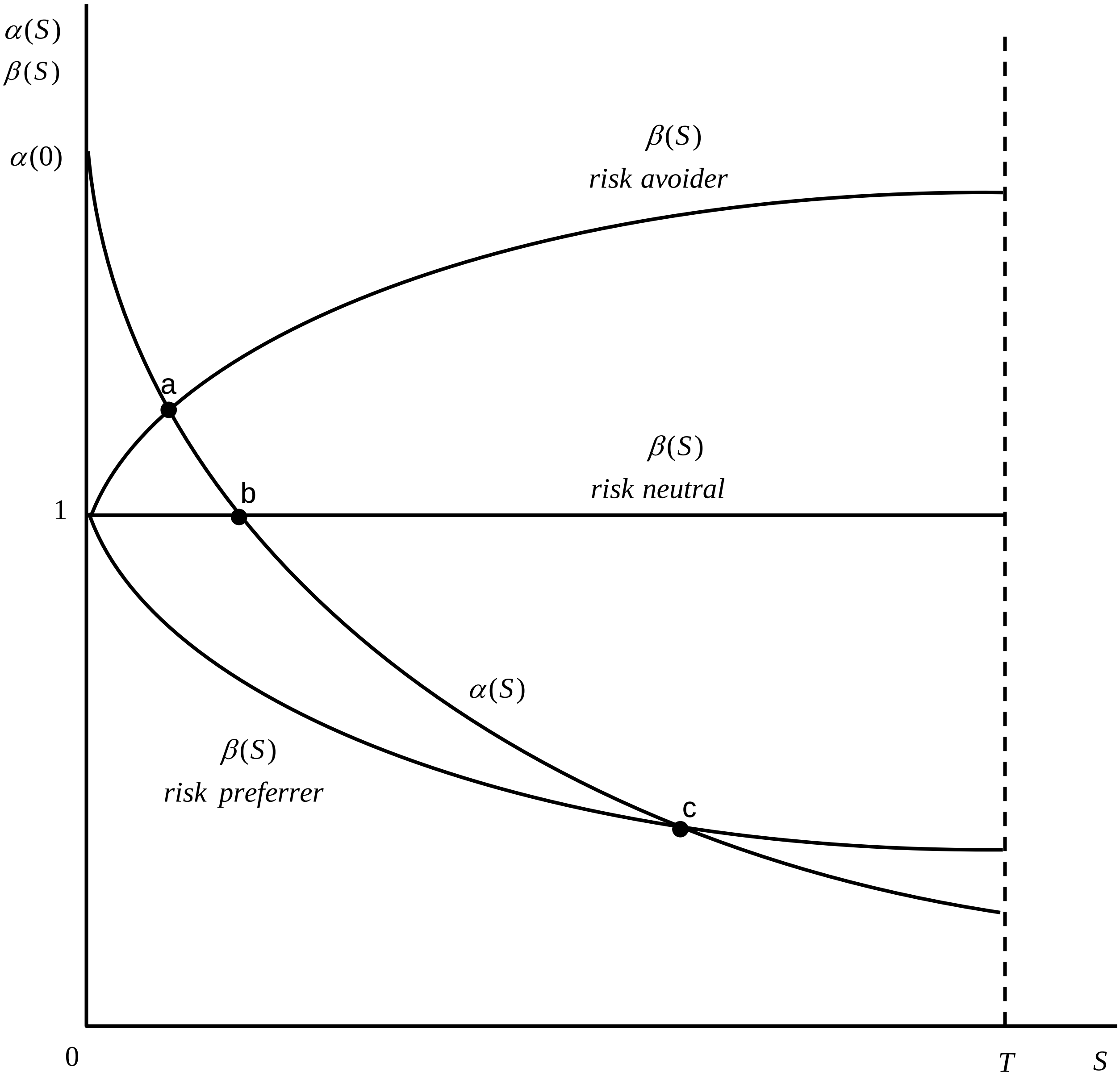


Figure 1: The allocation of time to crime

$\alpha(S)$ and $\beta(S)$ as a function of $S$. The assumptions on the return and penalty functions imply that $\alpha'(S) < 0$. Hence, the $\alpha(S)$ curve declines from left to right. For any $S > 0$, the value of $\beta(S)$ and the sign of $\beta'(S)$ depend on the individual's attitude toward risk: for a risk-neutral individual, $\beta(S) = 1$ and $\beta'(S) = 0$; for a risk avoider, $\beta(S) > 1$ and $\beta'(S) > 0$; for a risk preferrer, $\beta(S) < 1$ and $\beta'(S) < 0$. Hence, the $\beta(S)$ curve is a horizontal line at 1 for a risk neutral, rises from left to right above 1 for a risk avoider, and declines from left to right below 1 for a risk preferrer.[2] When $S = 0$, $\beta(S) = 1$ for all individuals, regardless of the attitude towards risk. Equilibrium is obtained at the point of intersection between the $\alpha(S)$ and $\beta(S)$ curves.

A glance at Figure 1 immediately confirms result (b): the equilibrium level of $S$ for a risk avoider (point a) is lower than that of a risk neutral (point b), which is lower than that of a risk preferrer (point c). A sufficient condition for obtaining equilibrium at some S > 0 (rather than at $S = 0$) is that the $\alpha(0)$ point lies above 1. This implies that $d\,'(0) > pF\,'(0)$, which confirms result (a). An increase in $p$ will shift the entire $\alpha(S)$ curve downward, generating lower equilibrium levels for all risk-attitude types (points a', b', and c' in Figure 2). This confirms result (c). An increase in $F'(S)$ (for a given level of $S$) will generate a similar shift in the $\alpha(S)$ curve, but it may affect the $\beta(S)$ curve as well, because it raises $F(S)$ and reduces $W^{-}$. For a risk avoider, the $\beta(S)$ curve will shift upward, strengthening the former effect in reducing $S$ (point a" in Figure 2). For a risk preferrer the $\alpha(S)$ curve will shift downward, acting to *increase* $S$. The combined effects of the shift in the $\alpha(S)$ and $\beta(S)$ curves might lead to a higher equilibrium level for $S$ (point c" in Figure 2), which confirms result (d).[3] Finally, a 1 percent increase in $p$ accompanied by a 1 percent decrease in $F'(S)$ will leave the product $pF'(S)$ intact. Differentiating $\alpha(S)$ with respect to $p$, holding $pF\,'(S)$ constant, yields

$$\frac{\partial[\alpha(S)]}{\partial p} = \frac{d\,'(S)[d\,'(S) - pF\,'(S)]}{[pF\,'(S) - pd\,'(S)]^2}, \tag{5}$$

the sign of which is determined by the sign of $d\,'(S) - pF\,'(S)$. The latter expression is positive as long as $\alpha(S) > 1$, zero when $\alpha(S) = 1$, and negative if $\alpha(S) < 1$. It thus follows that the $\alpha(S)$ curve will shift upward in its segment above 1 and downward in its segment below 1 (dotted curve in Figure 3). This will act to raise the equilibrium level of $S$ for a risk avoider (point a'), but to lower it for a risk preferrer (point c'). In addition, the fall in $F(S)$ resulting from the decrease in $F'(S)$ will shift the $\beta(S)$ curve downward for a risk avoider and upward

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[2] For convenience, the $\beta(S)$ curve is drawn as if $\beta''(S) < 0$ for a risk avoider and $\beta''(S) > 0$ for a risk preferrer. This, however, must not be the case. The results would not be affected if the sign of $\beta''(S)$ was the opposite to the one assumed in the diagram.

[3] The effect of a change in the marginal penalty on the time spent in crime is particularly difficult to demonstrate in a state-preference diagram as it involves a change in slopes of both the opportunity boundary and the indifference curves. In a review of Ehrlich's model, Pyle (1982) attempts to depict it, holding, however, the individual's utility level constant. Furthermore, he considers the response of a risk avoider only, for whom the slope of the indifference curve increases with $S$ while the slope of the boundary opportunity decreases. For a risk preferrer, the slope of the indifference curve decreases as well, making it practically impossible to demonstrate his response diagrammatically. Similarly, it is practically impossible to demonstrate result (e) in a state-preference diagram.

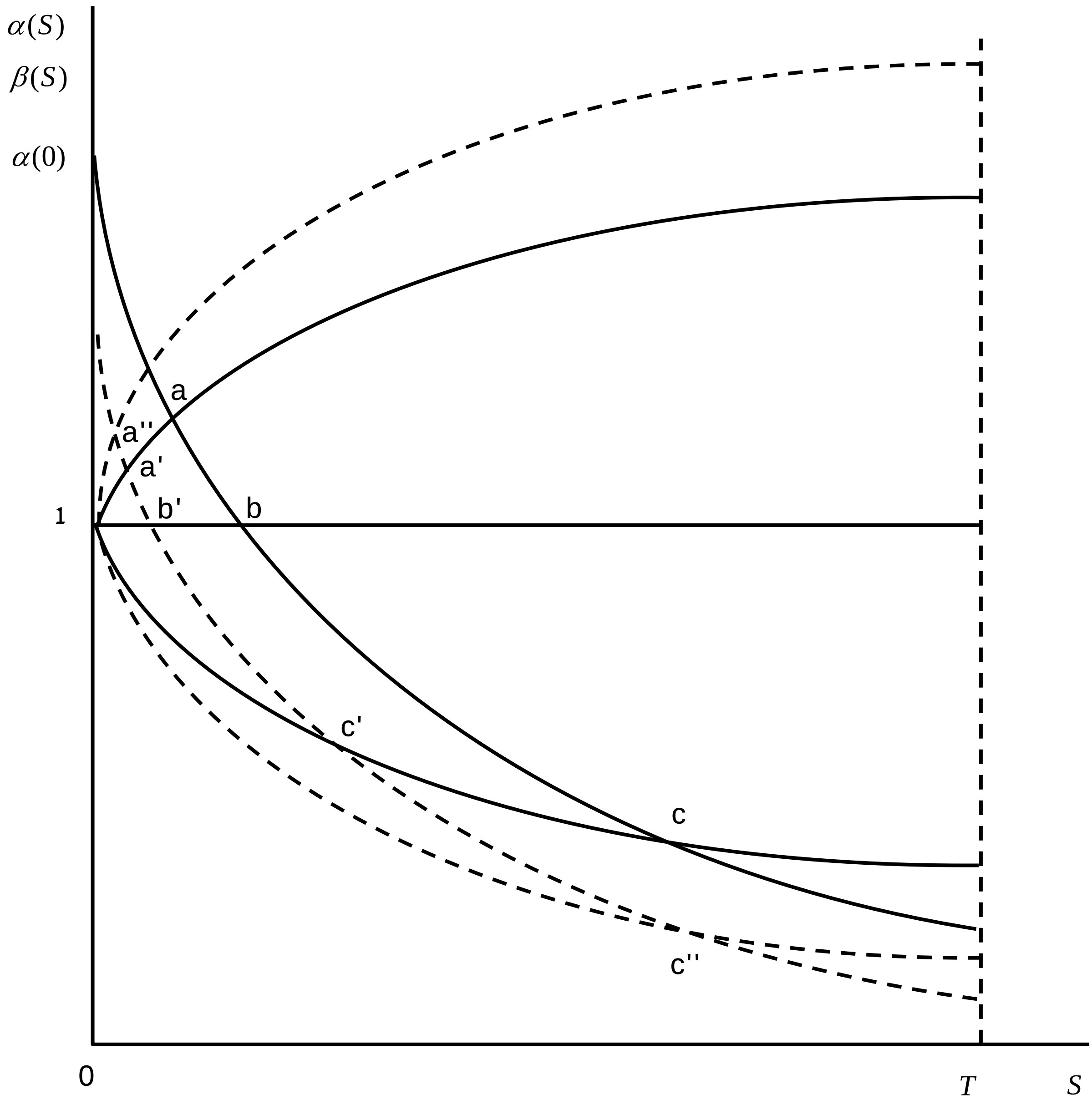


Figure 2 : The individual's response to an increase in the probability of apprehension or the marginal penalty

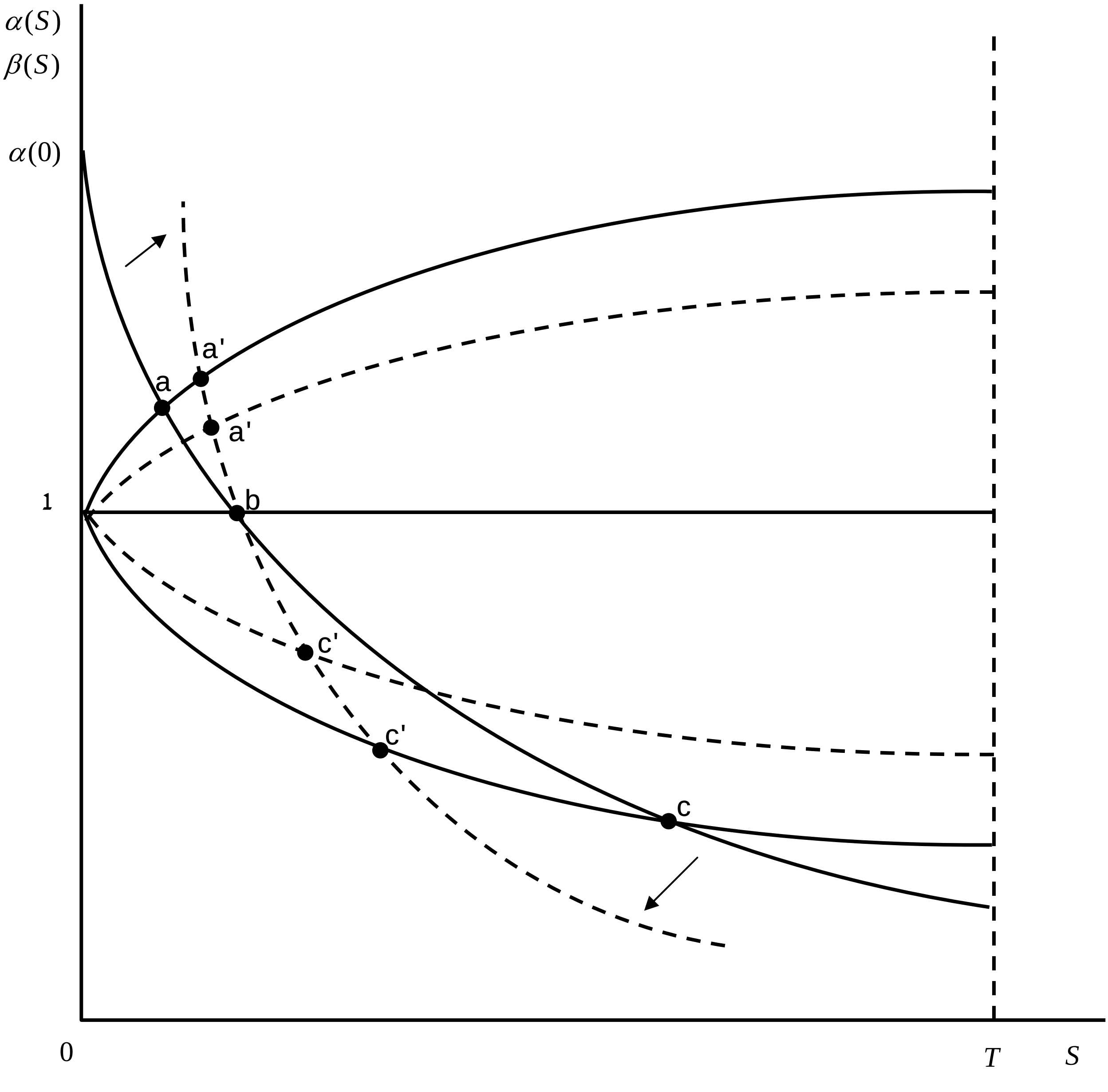


Figure 3: The individual’s response to a 1 percent increase in the probability of apprehension accompanied by a 1 percent decrease in the marginal penalty

for a risk preferrer, strengthening the effect on $S$ of the clockwise shift in the $\alpha(S)$ curve (points a' and c', respectively). Result (e) is thus confirmed as well.

## 4. Concluding remarks

In his seminal article on participation in illegitimate activities, Ehrlich (1973) derives five theoretical results which link the allocation of time to crime to the individual's attitude toward risk. He uses a state-preference diagram, which involves indifference curves defined over the individual's final wealth when apprehension takes place and when it does not, to demonstrate one result and mathematics for deriving the rest. This diagrammatical exposition, probably more conventional for models at that time, is interesting but complicates the graphical analysis and hides the intuition behind the results. The present paper transforms Ehrlich's first-order condition into equality with two terms, graphically presented as a function of the time devoted to crime. One term consists of the ratio between the marginal utilities when apprehension takes place and when it does not, which is fully determined by the individual's attitude toward risk. The other term consists of the ratio between the (expected) marginal differential return from crime when apprehension does not take place and when it does, which is determined by the assumptions on the returns from legitimate work and criminal activity as well as by the monetary value of the punishment. This exposition is considerably helpful for understanding the driving force behind the results, because they derive from intersections of curves rather than from tangency between curves. There are also opportunities for further research based on this approach, such as inquiring how the results would change if individuals are risk loving in term of losses but risk abiding in terms of gains, as argued by prospect theory.

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