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### **Lender learning and entry under demand uncertainty**

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#### **Abstract**

This paper examines the effect of demand uncertainty on the properties of the first period contract between a lender and the incumbent, when there is a threat of entry. The main findings are that unlike the cost uncertainty case, entry has no effect on the incumbent's incentives and it leads the lender to learn less.

# 1 Introduction

The purpose of this paper is to consider the impact of demand uncertainty rather than cost uncertainty on the results of Jain, Jeitschko and Mirman 2002, henceforth JJM. They examine the interaction between the contracting relationship between an incumbent and a lender and the classic entry-deterrence game (see Milgrom and Roberts, 1983), when the incumbent's marginal cost is its private information. In their case, the agency problem is that the incumbent wants to signal high cost to the lender (to reduce repayment) but low cost to the entrant (to show the entrant that entry is unprofitable). In contrast, when the demand intercept is only known to the incumbent, it has the incentive to signal low intercept to the lender as well as to the entrant. In this paper, I examine the impact of this change in incentives on the probability of entry, learning and the properties of the equilibrium contract between the incumbent and the lender.<sup>1</sup>

I find several interesting results that differ from JJM, focusing on the case that is analogous to theirs, namely, when entry occurs if and only if demand is high<sup>2</sup>. First, the analysis shows that in contrast to JJM, the threat of entry does not benefit the lender because it does not weaken the incumbent's incentives to misrepresent demand to the lender in the first period. Further, with demand uncertainty, in contrast to JJM, probability of entry increases in learning and thus, the lender chooses to learn less. Thus, the effect of entry on learning and incumbent incentives depends crucially on the source of uncertainty.

## 2 Model without Entry

Following JJM, the inverse demand for the good is given by,

$$p = a - q + \epsilon,$$

where  $a$  is the source of private information. That is, the true intercept value is only known to the incumbent. The lender believes that  $a \in \{\underline{a}, \bar{a}\}$ ,  $\underline{a} < \bar{a}$ , with probability of  $a = \bar{a}$  being  $\rho$ . The slope is normalized to be 1 for simplicity. I similarly use upper bars and lower bars on prices and quantities to denote these variables under high demand and low demand respectively. The demand shock  $\epsilon$  is a random, unobservable term that is distributed uniformly on the interval  $[-\eta, \eta]$ , where  $\eta > 0$ . There are two time periods with identical inverse demand functions. Further, the random component of demand is assumed to be i.i.d. over the two time periods. Costs are assumed to be zero, for simplicity.

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<sup>1</sup>Jain and Mirman, 2002, also study asymmetric information about demand, rather than costs. However, they only study a static model and thus, their focus is on the effects of different market structures and informational assumptions on contracts, rather than learning and entry.

<sup>2</sup>In their case, entry occurs if and only if the incumbent's cost is not low.

The incumbent chooses quantity  $q$  and thereafter  $\epsilon$  and  $p$  are realized. The price  $p$  is publicly observable, but the quantity,  $q$ , is the private information of the producing firm(s) and unverifiable to others. The incumbent requires outside funding in the amount of  $F$  in each period. Thus, contracts are assumed to be short term. A financial intermediary, i.e., a bank, provides these funds in exchange for later repayment of  $R$ . For simplicity, and without loss of generality,  $F$  is normalized to be zero. The contract between the bank and the incumbent takes the form of a repayment schedule that maps the observed market price  $p$  into an amount  $R(p)$  that the incumbent must pay to the bank. As in JJM, I only analyze the expected repayment, however.

The parameters of the model satisfy the following assumptions:<sup>3</sup>

*Assumption 1:* Belief  $\rho$  is such that the incumbent produces a strictly positive output in each state of demand.

*Assumption 2:* For any given output, the range of possible prices is large, that is,  $\eta$  is large enough to generate full learning as well as no learning.

## 2.1 The Benchmark Second Period

After observing the first period price and updating its beliefs about demand, there are only two possible scenarios for the lender to consider because of the uniform distribution of the demand shock: either it learns the true demand intercept or learns nothing. I consider the two cases in turn.

### 2.1.1 The Bank is Sure about demand

In this case, given full information, the lender simply calculates the “first best” level of profit for each state of demand, and sets this amount as repayment in return for lending funds  $F$  in the second period. Letting  $u_b$  denote the bank’s second period expected payoff, I obtain,

$$u_b = \frac{a^2}{4}. \tag{1}$$

for  $a \in \{\underline{a}, \bar{a}\}$ . If the bank’s beliefs are correct, and in equilibrium they are correct, the payoff of the firm is its reservation level of utility, namely 0. It is assumed that this is enforced through a forcing contract in the second period with substantial penalties for out-of-equilibrium price observations. If the bank incorrectly believes demand to be low when it actually is high, the incumbent’s expected payoff is,

$$\underline{u} = (\bar{a} - \underline{a})(\underline{a} - \underline{q}^*). \tag{2}$$

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<sup>3</sup>These assumptions are standard to ensure a non-trivial analysis. See Jeitschko, Mirman and Salgueiro, 2002.

That is, the firm's payoff is its demand advantage over the low demand type multiplied by the first-best level of price of a low demand firm,  $\underline{p}^* = \underline{a} - \underline{q}$ .<sup>4</sup>

### 2.1.2 The Bank is Unsure about demand

In this case, the bank maximizes its second period profit,  $(1 - \rho)\underline{R} + \rho\overline{R}$ , where  $\underline{R}$  is the expected repayment received when demand is low and  $\overline{R}$  when demand is high, subject to the incumbent's individual rationality and incentive compatibility constraints respectively:

$$\begin{aligned}\underline{R} &= (\underline{a} - \underline{q})\underline{q}, \\ \overline{R} &= (\overline{a} - \overline{q})\overline{q} - (\overline{a} - \underline{a})(\underline{a} - \underline{q})\end{aligned}\tag{3}$$

Thus, the lender chooses  $\underline{q}$  and  $\overline{q}$ , to maximize,

$$\rho((\overline{a} - \overline{q})\overline{q} - (\overline{a} - \underline{a})(\underline{a} - \underline{q})) + (1 - \rho)(\underline{a} - \underline{q})\underline{q}.$$

The first-order-conditions are sufficient and yield,

$$\begin{aligned}\underline{q} &= \frac{\underline{a}}{2} + \frac{\rho}{1 - \rho} \frac{\overline{a} - \underline{a}}{2}, \\ \overline{q} &= \frac{\overline{a}}{2}, \\ u_b &= \frac{\underline{a}^2}{4} + \frac{\rho}{1 - \rho} \frac{(\overline{a} - \underline{a})^2}{4},\end{aligned}\tag{4}$$

Thus, the contract yields the standard features. Further, a result similar to JJM holds:

$$\widehat{p} \equiv \rho\overline{p} + (1 - \rho)\underline{p} = \frac{\underline{a}}{2} \equiv \underline{p}^*.\tag{5}$$

This has implications for the entrant.

## 2.2 The Benchmark First Period

In the first period, the bank maximizes,

$$(1 - \rho)\underline{R} + \rho\overline{R} + Eu_b,\tag{6}$$

subject to the two individual rationality and incentive compatibility constraints of the first period<sup>5</sup>. In order to calculate expected profits, the following Lemma is needed.

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<sup>4</sup>Note that the high incumbent cannot target the low incumbent's target output because that could lead to severe penalties. Instead, it is the price target that is mimicked. In JJM, targeting outputs and prices is equivalent.

<sup>5</sup>For convenience, the same notation is used here for expected repayments as for the second period.

**Lemma 1** *The posterior belief  $\rho_2$  is given by,*

$$\rho_2(p_1|\bar{p}, \underline{p}) = \begin{cases} 0, & \text{if } p_1 \in (-\infty, \bar{p} - \eta), \\ \rho, & \text{if } p_1 \in [\bar{p} - \eta, \underline{p} + \eta], \\ 1, & \text{if } p_1 \in (\underline{p} + \eta, \infty). \end{cases}$$

$$\Pr\{\rho_2 = 0\} = (1 - \rho) \frac{(\bar{p} - \underline{p})}{2\eta},$$

$$\Pr\{\rho_2 = \rho\} = \frac{2\eta + \underline{p} - \bar{p}}{2\eta},$$

$$\Pr\{\rho_2 = 1\} = \rho \frac{(\bar{p} - \underline{p})}{2\eta}.$$

Thus, the expected value function of the lender is,

$$\begin{aligned} Eu_b &= \frac{a^2}{4}(1 - \rho) \frac{(\bar{a} - \underline{a}) - (\bar{q} - \underline{q})}{2\eta} + \frac{\bar{a}^2}{4} \rho \frac{(\bar{a} - \underline{a}) - (\bar{q} - \underline{q})}{2\eta} + \\ &+ \left( \frac{a^2}{4} + \frac{\rho}{1 - \rho} \frac{(\bar{a} - \underline{a})^2}{4} \right) \frac{2\eta - (\bar{a} - \underline{a}) + (\bar{q} - \underline{q})}{2\eta} \\ &= \rho \left( \frac{(\underline{a})^2}{4} - \frac{(\bar{a})^2}{4} + \frac{(\bar{a} - \underline{a})^2}{(1 - \rho)4} \right) \frac{(\bar{q} - \underline{q})}{2\eta} + A \end{aligned} \quad (7)$$

where  $A$  does not depend on  $\bar{p}$  or  $\underline{p}$ .

I now consider the constraints. Since the low demand firm's second period payoff is zero, regardless of the bank's beliefs, the individual rationality constraint of this type is as before and given by Equation 3.

Suppose now that demand is high. Since its individual rationality constraint is slack, one needs only consider the high-demand firm's incentive compatibility constraint. This constraint differs from the static second period constraint in that the high demand firm must be paid up-front its discounted potential gain from deception, equal to  $(\bar{a} - \underline{a})\underline{p}^* = (\bar{a} - \underline{a})\frac{a}{2}$ , see Equation 2. Thus,

$$\bar{R} = (\bar{a} - \bar{q})\bar{q} - (\bar{a} - \underline{a})(\underline{a} - \underline{q}) - (\bar{a} - \underline{a})\frac{a}{2} \frac{(\bar{a} - \underline{a}) - (\bar{q} - \underline{q})}{2\eta}. \quad (8)$$

Substituting the bank's future expected payoff (Equation 7), and the two binding constraints (Equations 3 and 8), into the bank's maximization problem (Equation 6), the first order conditions of the bank's problem yield,

$$\begin{aligned} \underline{q} &= \frac{a}{2} + \frac{\rho}{1 - \rho} \left( \frac{\bar{a} - \underline{a}}{2} + B \frac{1}{4\eta} \right), \\ \bar{q} &= \frac{\bar{a}}{2} - B \frac{1}{4\eta}, \end{aligned}$$

where  $B = \frac{(\bar{a})^2}{4} - \frac{(\bar{a}-a)^2}{(1-\rho)^4} - \frac{(\bar{a}-a)(a)}{2} - \frac{(a)^2}{4} < 0$ .

Since  $B < 0$ , the following result follows:

**Proposition 1** *The bank sets the first period outputs further apart (expected first period prices closer together) compared to the first period outputs in a static model.*

As in JJM, this result derives from two underlying factors: one is the ratchet effect payment (Equation 2) that the high-demand incumbent requires in order to reveal its type and the second is the experimentation effect. The probability of future gain for the high demand incumbent increases as learning increases. To reduce this gain, the lender sets the price targets closer together. On the other hand, the lender has an incentive to learn so that its future expected profits increase.<sup>6</sup> The Proposition shows that the experimentation effect is dominated by the ratchet effect.

Note that this experimentation result is different from the monopoly experimentation result of MSU where a monopolist is unable to learn when demand curves are parallel. There, the reason is that a change in  $q$  simply shifts both distributions equally, and thus reveals no information. Here, the contracting relationship makes it possible for the lender to learn because the lender can choose two different price targets and therefore, influence the distribution of posterior beliefs.

### 3 Model with Entry

The potential entrant in the second period has the same information as the lender and therefore, the same updated beliefs. Based on these beliefs, the entrant decides whether or not to enter the market. As in JJM, firms compete in quantities if entry occurs.

The entrant incurs a fixed cost of entry, denoted by  $K$ , that it can finance internally. There are no other costs, as is the case with the incumbent. The entrant is also risk-neutral and maximizes expected profits. The size of fixed cost  $K$  determines the entry rule. There are only two possibilities for entry even though there are three states. Equation 5 shows that the entry decision must be the same in the two states  $\{0, \rho\}$  because entrant's profits are  $\hat{p}^2$ . The expected price needs to be derived for the Cournot game. Since the analysis is similar to JJM and JM, I simply report the results:

$$\begin{aligned} q_e &= \frac{a}{3}, \bar{q} = \frac{\bar{a}}{2} - \frac{a}{6}, \underline{q} = \frac{a}{3} + \frac{\rho}{1-\rho} \frac{\bar{a}-a}{2}, \\ \hat{q} &= \frac{\hat{a}}{2} - \frac{a}{6} + \rho \frac{\bar{a}-a}{2}, \\ \hat{p} &= \hat{a} - \hat{q} - q_e = \frac{a}{3}. \end{aligned}$$

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<sup>6</sup>To see this, note that, by Assumption 1,  $\frac{dEu_b}{d(\bar{q}-\underline{q})} = \rho \left( \frac{(a)^2}{4} - \frac{(\bar{a})^2}{4} + \frac{(\bar{a}-a)^2}{(1-\rho)^4} \right) \frac{1}{2\eta} < 0$ .

I limit the analysis to the case when entry occurs if and only if demand is high, so that probability of entry is influenced by first period decisions.

**Proposition 2** *The probability of entry increases in learning.*

The proof is straightforward from Lemma 1.

This result is the opposite of JJM. Recall that there, entry occurs if and only if cost is high ( $\rho_2 = 0$ ). Since entry decision must be the same for  $\rho_2 \in \{0, \rho\}$ , probability of entry decreases in learning in their setting.

Since entry only occurs in the good state, the second period outcomes for  $\rho_2 \in \{0, \rho\}$  continue to be given by equations 1 and 4 in the benchmark model. When  $\rho_2 = 1$ , the expected payoff of the lender is the Cournot duopoly profit:

$$u_b = \frac{(\bar{a})^2}{9b}.$$

As in the benchmark model, I obtain,

$$Eu_b = \rho \left( \frac{(\underline{a})^2}{4} - \frac{(\bar{a})^2}{9} + \frac{(\bar{a} - \underline{a})^2}{(1 - \rho)4} \right) \frac{(\bar{q} - \underline{q})}{2\eta} + A', \quad (10)$$

where  $A'$  does not depend on  $\bar{q}$  or  $\underline{q}$ .

Consider now the constraints of the two types of incumbent. It turns out that unlike the cost uncertainty case, the incentive compatibility constraint of the incumbent firm with high demand does not change due to entry. This is because entry does not occur if the high-demand incumbent pretends to be low-demand. Thus, both binding constraints continue to be given by the benchmark model and the only change induced by potential entry is in the expected profits of the bank, given by Equation 10.

The first order conditions of the bank's two-period maximization problem are sufficient and yield the following outputs for the first period:

$$\begin{aligned} \underline{q} &= \frac{\underline{a}}{2} + \frac{\rho}{1 - \rho} \left( \frac{\bar{a} - \underline{a}}{2} + B' \frac{1}{4\eta} \right), \\ \bar{q} &= \frac{\bar{a}}{2} - B' \frac{1}{4\eta}, \end{aligned}$$

where  $B' = \frac{(\bar{a})^2}{9} - \frac{(\bar{a} - \underline{a})^2}{(1 - \rho)4} - \frac{(\bar{a} - \underline{a})(\underline{a})}{2} - \frac{(\underline{a})^2}{4} < B < 0$ .

The following result follows:

**Proposition 3** *The first period outputs are set further apart implying less learning in equilibrium with entry than without.*

This result is significant since it shows that the threat of entry leads the bank to learn less in contrast to JJM. The intuition is straightforward. The signal dampening effect is unchanged since low demand does not invite entry. At the same time, information is less valuable since the probability of entry increases as learning increases. Thus the overall effect of the threat of entry is to decrease learning and thus reduce the probability of entry.<sup>7</sup>

## 4 Conclusion

This paper shows that the source of asymmetric information matters in determining the effect of entry on learning by a lender. When it is the marginal cost parameter that is private information of the incumbent, the threat of entry benefits the lender by weakening the incentive problem. Further, probability of entry decreases in learning, leading the lender to learn more. When the demand intercept is private information of the incumbent, there is no benefit to the lender from potential entry and probability of entry increases in learning. Thus, the lender learns less.

## References

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<sup>7</sup>If fixed cost of entry  $K$  is such that entry occurs surely, a similar result obtains overall, that is the lender learns less.