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Decomposing equality of opportunity by income sources

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Abstract

An opportunity egalitarian view focus not only on equality of outcomes but essentially on the means or opportunities to realize such outcomes. In this paper we propose a methodology to decompose an opportunity egalitarian Atkinson index by income sources seeking for a measure of overall equality into an opportunity and a responsibility component.

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1 Introduction

Decomposition of inequality by income sources is quite interesting in order to capture whether total inequality is concentrated in specific income items. It may be useful to express how much of the total income inequality produced into the society is explained by an income inequality due to a precise source. Most of the studies on this issue have focused on the Gini and the Theil indices given that they have particular features for decomposing inequality by income sources. Shorrocks (1982) proposes one of the pioneering methodology to afford these kinds of decomposition. He proves that an infinite number of decompositions can be obtained for each kind of index without further restrictions. That's called "natural decomposition" property which is valid for all inequality indices. For example, as regards the Gini Index, Lerman and Yitzhaki (1985) (LY henceforth) propose a decomposition based on the covariance formula of the Gini index. That procedure is similar to the one used by Fei, Ranis and Kuo (1980) even if the interpretation is quite different. Particularly, LY propose that the overall Gini coefficient for the entire income distribution is equal to the sum of the Gini coefficients calculated by using the covariance between each income source and the cumulative distribution function of total income. In this way, they obtain the impact of the marginal change in a given income source on overall inequality. Even if it's a natural decomposition, it permits to indicate a clear measurement of the contribution of each source to income inequality.

In this paper, we decompose income sources in terms of equality of opportunity. Taking cue from LY (1985), we show how the opportunity egalitarian Atkinson measure can be decomposed by income sources. We define a society with N individuals. Each individual outcome y_i is generated by a function that assigns individual income to combinations of income sources derived by personal responsibility and by social circumstances beyond the individual control. Therefore, our aim is to decompose the Atkinson index of equality as a weighted average of the inequality produced by each income source in terms of effort or circumstances. We measure the equality effect of each income source showing the "ethically acceptable" and the "ethically offensive" factor components.

2 An example

For example, we can distinguish between different kinds of income. On one side, endowments or financial capitals K are considered as social condition-

ing which may reflect differences in social classes. On the other side, the labor income L may be interpreted as effort variable which gather individual responsibility, such that $y_i = (K_i, L_i)$. The total income doesn't differ between individuals.

Income sources	Individuals		
	Ind.1	Ind.2	Ind.3
K	5	10	9
L	12	7	8

Here, we proposes a society composed by three individuals (N). Each individual produces a wealth derived by personal endowments (K_i) and an amount of money due to labour income (L_i) i.e. we have two j -sources (m). On this perspective, six income units (S) are produced into the society. Each income source (j) has the same dimension (length) which is equal to three in our example. The dimension of each income source (j) perfectly corresponds to the number of individuals (N) into the society.

3 The model

We have a population of N individuals. Each individual income y_i is made of m income sources y_{ij} with $j = 1, \dots, m$, such that, for all $i = 1, \dots, N$, we have that $y_i = \sum_{j=1}^m y_{ij}$. A distribution of income is represented by $Y = \{y_1, \dots, y_i, \dots, y_N\} \in \mathfrak{R}_+^N$ and a distribution of the j -income source by $Y_j = \{y_{j,1}, \dots, y_{j,N}\} \in \mathfrak{R}_+^N$. Moreover, we can consider a generic income unit y_{ij} , which is the j component of the income of individual i . Let $S = N \times m$ be the number of income units in our society. Hence y_p , for all $p = 1, \dots, S$, is a generic income unit.

Let $e = (\mu, \dots, \mu, \dots, \mu) \in \mathfrak{R}_+^S$ represent the distribution where all income units are equal to:

$$\mu = \frac{1}{S} \sum_{p=1}^S y_p = \frac{1}{S} \sum_{j=1}^m \sum_{i=1}^N y_{ij} \quad (1)$$

We can define $e_j = (\mu_j, \dots, \mu_j, \dots, \mu_j) \in \mathfrak{R}_+^N$ as the distribution where each income unit in the distribution of income source j receives the same mean income

$$\mu_j = \sum_{i=1}^N y_{ij}$$

4 Atkinson index by income sources

Now, the social welfare function can then be expressed by the average utility of the S -income units in the society as:

$$W = \frac{1}{S} \sum_{p=1}^S U_p(y_p) \quad (2)$$

The function $U_p(y_p)$ refers to the utility function produced by each income unit. We note that in this case there's no space for the individual specific role. We only think to what contributions each income unit can provide in terms of social welfare. This can be expressed in the following form according to Atkinson (1970):

$$U_p(y_p) = \frac{1}{1-\epsilon} y_p^{1-\epsilon} \quad \text{if } \epsilon > 0 \quad \epsilon \neq 1 \quad (3)$$

$$U_p(y_p) = \log y_p \quad \text{if } \epsilon = 1$$

Given the income sources environment, the equally distributed equivalent (*ede*, hereafter) income y_e^1 is implicitly defined by:

$$W(y_1, \dots, y_p, \dots, y_S) = W(y_e, \dots, y_e, \dots, y_e)$$

Therefore, from expression (3), we get:

¹It is identified as the hypothetical level of income that each individual should receive in order to keep the society to the same level of social welfare stemmed from the actual income units.

$$U(y_e) = \frac{1}{1-\epsilon} (y_e)^{1-\epsilon} \quad (4)$$

The social welfare function in terms of income units can then be expressed as:

$$W = \frac{1}{S} \sum_{p=1}^S \frac{y_p^{1-\epsilon}}{1-\epsilon} \quad (5)$$

From (4) and (5), the functional form of the *ede* income y_e is now given by:

$$y_e = \left[\frac{1}{S} \sum_{p=1}^S y_p^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad \text{if } \epsilon > 0 \quad \epsilon \neq 1$$

$$y_e = \left[\prod_{p=1}^S y_p \right]^{\frac{1}{S}} \quad \text{if } \epsilon = 1 \quad (6)$$

While the Atkinson index of inequality I_A of the distribution Y according to the evaluation in terms of income units is:

$$I_A = 1 - \frac{y_e}{\mu} = 1 - \frac{\left[\frac{1}{S} \sum_{p=1}^S y_p^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\mu} \quad \text{if } \epsilon > 0 \quad \epsilon \neq 1$$

$$I_A = 1 - \frac{y_e}{\mu} = 1 - \frac{\left[\prod_{p=1}^S y_p \right]^{\frac{1}{S}}}{\mu} \quad \text{if } \epsilon = 1 \quad (7)$$

5 Opportunity egalitarian Atkinson index

We propose a social welfare function for income components which can be expressed as:

$$W = \frac{1}{S} \sum_{j=1}^m \sum_{i=1}^N U_{ij}(y_p) \quad (8)$$

Unlike to the the type or tranche approaches used in the population subgroup decomposition, we don't need to impose any further restrictions for the form of the utility function $U_{ij}(y_p)$. In this case, we only seek to identify the Atkinson's social welfare function expressing the opportunity egalitarian principle with respect to each income source. Further, a unique inequality aversion parameter is required. Therefore we define the form of the utility function² as:

$$U_{ij}(y_p) = \frac{1}{1-\epsilon} y_{ij}^{1-\epsilon} \quad \text{if } \epsilon > 0 \quad \epsilon \neq 1 \quad (9)$$

While, from (8) and (9), the opportunity egalitarian social welfare function can then be expressed as:

$$W = \frac{1}{S} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{1-\epsilon} y_{ij}^{1-\epsilon} \quad (10)$$

Let $(y_{e1}, \dots, y_{ej}, \dots, y_{em})$ be defined by:

$$\frac{1}{S} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{1-\epsilon} y_{ij}^{1-\epsilon} = \frac{1}{S} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{1-\epsilon} y_{ej}^{1-\epsilon} \quad (11)$$

Then, the *ede* income y_{ej} for each income source $j \in \{1, \dots, m\}$ is given by:

$$\sum_{i=1}^N \frac{1}{1-\epsilon} y_{ij}^{1-\epsilon} = N \frac{1}{1-\epsilon} y_{ej}^{1-\epsilon} \quad (12)$$

²Similar result can be provided for $\epsilon = 1$

which implies that:

$$y_{e_j} = \left[\frac{1}{N} \sum_{i=1}^N y_{ij}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (13)$$

From (11) and (13), a direct expression for the opportunity egalitarian *ede* income y_e in terms of income units can be defined by:

$$\begin{aligned} \frac{1}{S} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{1-\epsilon} y_{ij}^{1-\epsilon} &= \frac{1}{m} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{N} \frac{1}{1-\epsilon} y_{ij}^{1-\epsilon} = \\ &= \frac{1}{m} \sum_{j=1}^m \frac{1}{1-\epsilon} (y_{e_j})^{1-\epsilon} = \frac{1}{m} m \frac{(y_e)^{1-\epsilon}}{1-\epsilon} \end{aligned} \quad (14)$$

such that:

$$y_e = \left[\frac{1}{m} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{N} y_{ij}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (15)$$

Finally, we can express the opportunity egalitarian Atkinson index by income components as:

$$I_A = 1 - \frac{y_e}{\mu} = 1 - \frac{\left[\frac{1}{m} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{N} y_{ij}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\mu} \quad (16)$$

6 Measuring equality of opportunity

In a society completely described by income sources, we seek for a decomposition methodology in order to distinguish between an opportunity and a responsibility component. Each source is completely defined to be under responsibility or compensation spheres. Such objective may be easier to reach

using the opportunity egalitarian Atkinson index of equality which is given by $E_A = 1 - I_A$. It follows that:

$$E_A = \frac{\left[\frac{1}{m} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{N} (y_{ij})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\mu} \quad (17)$$

While, the opportunity egalitarian Atkinson index of equality for the j -income source E_j can be given by:

$$E_j = \frac{\left[\frac{1}{N} \sum_{i=1}^N (y_{ij})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\mu_j} \quad (18)$$

Let $q_j = \frac{\mu_j}{\mu}$ be the mean income share for the j -th source. It follows that we can decompose the Atkinson index of equality E_A by income sources in terms of equality of opportunity as:

$$\begin{aligned} E_A &= \frac{\left[\frac{1}{m} \sum_{j=1}^m \sum_{i=1}^N \frac{1}{N} (y_{ij})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\mu} = \frac{\left[\frac{1}{m} \sum_{j=1}^m \frac{\sum_{i=1}^N \frac{1}{N} (y_{ij})^{1-\epsilon} \mu_j^{1-\epsilon}}{\mu_j^{1-\epsilon}} \right]^{\frac{1}{1-\epsilon}}}{\mu} = \\ &= \frac{\left[\frac{1}{m} \sum_{j=1}^m E_j^{1-\epsilon} \mu_j^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\mu} = \left[\frac{1}{m} \sum_{j=1}^m \left(\frac{\mu_j}{\mu} \right)^{1-\epsilon} E_j^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} = \\ &= \left[\frac{1}{m} \sum_{j=1}^m (q_j E_j)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (19) \end{aligned}$$

The same analysis can be developed for $\epsilon = 1$ by appropriately substituting products for summations. Under (19), the contribution of each source to equality is equal to the product of the weighted income share and the Atkinson of equality for each source.

7 Concluding Remarks

Such decomposition provides results in terms of equality of opportunity. In the case of decomposition by population subgroups, we particularly focus on the between and the within component which refer to the opportunity and the responsibility factors dependent by which kind of approach is used. Rather different even if simpler than such decomposition, the opportunity egalitarian decomposition by income sources evaluates the opportunity components and the responsibility ones directly through the income sources produced in the society. For example, the opportunity component can be provided by the sum of the capital income and the endowments of individuals which practically expresses elements beyond the control of individuals, while labour income can be better justified as responsibility component i.e. as a factor within individual's control. In this case, we obtain current information about the impact on the overall equality of the marginal change produced by an income source. Moreover, different degrees of equality of opportunity and effort level as functions of income sources can be provided. This clearly represents a simplification of this model just to point out idea that the opportunity egalitarian principle can be compatible with a decomposition of Atkinson index by income sources. Some extensions may be left for future research.

References

- [1] Atkinson, A.B. (1970), "On the measurement of inequality", *Journal of Economic Theory*, **2**, 244–263
- [2] Blackorby, C., Donaldson, D. and Auersperg, M. (1981), "A new procedure for the measurement of inequality within and among population subgroups", *Canadian Journal of Economics*, **14**, 665–685
- [3] Cowell, F.A. (2000), "Measurement of inequality", In: Atkinson, A.B., Bourguignon, F. (eds.) *Handbook of Income Distribution*, 87-166. North-Holland, Amsterdam
- [4] Deutsch, J. and Silber J. (1999), "Inequality Decomposition by Population Subgroups and the Analysis of Interdistributional Inequality", in J. Silber, editor, *Handbook on Income Inequality Measurement*, Kluwer Academic Press, Dordrecht and Boston, 363-397

- [5] Ebert, U. (1999), "Dual Decomposable Inequality Measures", *The Canadian Journal of Economics*, **32**, 234-246.
- [6] Fei J., Ranis G. and Kuo W.Y. (1980) "Growth and the Family distribution of Income by factor components", *Quarterly Journal of Economics*, **92** (1), 451-473
- [7] Lerman, R. (1999), "How do income sources affect income inequality?", in J. Silber, editor, *Handbook on Income Inequality Measurement*, Kluwer Academic Press, Dordrecht and Boston, 341-362.
- [8] Lerman, R. and Yitzhaki, S. (1985), "Income inequality by income sources: a new approach and application to the United States", *Review of Economics and Statistics* LXVII , **1**, 151-156
- [9] Mookherjee, D. and Shorrocks, A. (1982), "A Decomposition Analysis of the Trend in UK Income Inequality", *The Economic Journal*, **92**, 886-902
- [10] Roemer, J.E. (1998), "*Equality of Opportunity*", Harvard University Press. Cambridge, Ma.
- [11] Shorrocks, A.F. (1982), "Inequality decomposition by factor components", *Econometrica*, **50**, 193-211
- [12] Shorrocks, A.F. (1984), "Inequality decomposition by population subgroups", *Econometrica*, **52**, 1369-1385
- [13] Silber, J. (1989), "Factors Components, Population Subgroups and the Computation of the Gini Index of Inequality", *The Review of Economics and Statistics*, LXXI, **2**, 107-115
- [14] Villar, A. (2005), "On the welfare evaluation of income and opportunity", *Contributions to Theoretical Economics*, Berkeley Electronic Press, **5**, 1129-1129
- [15] Weymark J. (2003), "Generalized Gini Indices of Equality of Opportunity", *Journal of Economic Inequality*, **1**(1), 5-24.