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Parliamentary voting rules and strategic candidacy

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Abstract

In this paper we study the vulnerability of parliamentary voting procedures to strategic candidacy. Candidates involved in an election are susceptible to influence the outcome by opting out or opting in. In the context of three-alternative elections and under the impartial anonymous culture assumption, we evaluate the frequencies of such strategic candidacy opportunities.

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1 Introduction

In social choice theory the main result on strategic voting is the Gibbard-Satterthwaite (Gibbard 1973, Sattethwaite 1975) theorem, which states that there is no voting rule selecting a unique outcome which is both nondictatorial and immune to individual misrepresentation of preferences. In this paper we are concerned with a related but different aspect of strategic behavior: a potential candidate in an election may have an incentive to change the outcome of the voting rule in his favour by his simple entry or exit, given the preferences of the voters. Such a behavior is known as strategic candidacy.

Strategic candidacy has been studied by several authors. For example, Dutta, Jackson and Le Breton (2000, 2001), Samejima (2007) provide many results on this topic.

Although each of the above mentioned papers studies incentives related to strategic candidacy or classes of voting rules immune to strategic candidacy, the focus of our paper is quite different. Our contribution in this paper is to evaluate the vulnerability of parliamentary voting procedures to strategic candidacy. Specifically, we examine the vulnerability to strategic candidacy of amendment and successive elimination voting procedures (see details in Section 2), which are extensively used throughout the world for voting on motions in parliaments (see for example Rasch, 2000) and are the topic of the article of Dutta, Jackson and Le Breton (2000). More precisely, under these rules and in the context of three-alternative elections, our aim is to determine how frequent opportunities of this phenomenon are. We do this for the two versions of strategic candidacy mentioned above: opting out, and opting in.

The paper is organized as follows: in section 2 we introduce notations and definitions; Section 3 is devoted to the evaluation of strategic candidacy opportunities and provides our results. Section 4 concludes the paper.

2 Notations and definitions

Consider an election in which $A = \{a_1, a_2, a_3\}$ is a finite set of three alternatives or potential candidates, and N is the set of n individuals or voters, whose preferences are aggregated in order to determine the elected candidate. We also assume that candidates are allowed to vote, as it is the case in many elections.

Individual preference relations are linear orders (complete, transitive and antisymmetric binary relations) over the set of A candidates.

We now define the voting procedures considered in this paper. We begin with the *amendment procedure*. It consists in organizing a succession of qualified majority (with quota α) contests between alternatives in the following way: using an *agenda* - a predetermined order, $a_1a_2a_3$ in this paper - between the alternatives, a_1 is taken against a_2, and then winner is taken against a_3. The winner of this last confrontation is declared elected. Next, as for the amendment rule, *successive elimination procedure* is based on an agenda, though in a slightly different way: at the first step a (possibly qualified) majority *yes-no* vote is organized on a_1, and if a_1 wins a majority, a_1 is elected and the procedure ends. If not, at the second step a similar vote is organized on a_2, and a_2 is elected if it collects a majority of votes. If neither a_1 nor a_2 collect a majority of votes, then a_3 is elected. Note that, when individuals vote on a_1, they must in fact decide whether they prefer alternative a_1 to the subset $\{a_2, a_3\}$ or $\{a_2, a_3\}$ to a_1. In other words, they compare subsets of alternatives of possibly more than one element. Then, we distinguish two possible types of behavior: *maximin* (a pessimistic behavior), or *maximax* (an optimistic behavior). Under maximin behavior, a voter does not vote for a candidate only when he ranks him (or her) at the last position in his preference order. Under maximax behavior, a voter votes for a candidate when he ranks him at the first position in his preference order.

Under either voting procedure, ties are broken in favor of the one with the greatest index.

Strategic candidacy occurs when some candidate can exit (or enter) the election and change the outcome in his favor. In order to give some formal definitions of these notions, we need additional notations. Let F be the voting procedure and $R^N = (R^1, ..., R^n)$ denote a profile of individual preferences, one preference relation R^i for each individual i. Let $R^N|A - \{a_h\} = (R^1|A - \{a_h\}, ..., R^n|A - \{a_h\})$ denote the restriction of every individual preferences to the subset $A - \{a_h\}$ of alternatives.

A voting procedure F is vulnerable to strategic candidacy at profile R^N by *opting out* if there exist $a_h \in A$ and some profile R^N such that $F(R^N|A - \{a_h\})R^{a_h}F(R^N)$.

A voting procedure F is vulnerable to strategic candidacy at profile R^N by *opting in* if there exist $a_h \in A$ and some profile R^N such that $F(R^N) \neq a_h$ and $F(R^N)R^{a_h}F(R^N|A - \{a_h\})$.

a_h is a potential candidate, and by opting out, his exiting leads to the election a candidate he prefers to the one who is elected with the whole set A of alternatives. Note that it is required, for opting in, that a_h be not not elected when he enters the election; this is so because it is always possible to construct a profile at which a_h is elected, for example by ranking it at the first position in all individual preferences.

3 Evaluation of strategic candidacy opportunities

In this section we characterize, for each of the voting procedures under consideration, all voting situations at which there exists an opportunity for strategic candidacy. We successively study opting out and opting in. We begin with opting out. Note that, as said in Section 2, we assume that candidates are allowed to vote. This implies that each alternative appears at least once at the first position in individual preferences; and this fact is taken into consideration in all the analysis below.

3.1 Opting out

Under the amendment procedure, one can easily check that where there is a Condorcet winner, that is an alternative that beats any other alternative in pairwise majority contests, there is no way for strategic candidacy. And since a_1 or a_2 are elected if and only if they are Condorcet winners, the only possibilities for strategic candidacy occur when a_3 is elected. Since a_3 has no reason to opt out when (s)he wins, there are only under two possibilities: (i) either a_1 opts out (see Example 1), or a_2 opts out (and the reader can easily construct a very similar example):

Example 1 *Consider the following profile and* $\alpha = \frac{1}{2}$

R^N				$R^N \vert \{a_2, a_3\}$		
1	*2*	*3*		*1*	*2*	*3*
a_1	a_2	a_3		a_2	a_2	a_3
a_2	a_3	a_1		a_3	a_3	a_2
a_3	a_1	a_2				

a_1 beats a_2, a_3 beats a_1; then a_3 wins. If a_1 opts out, a_2 beats a_3 and it follows that a_2 wins. We then conclude that a_1 is a strategic candidate.

All the observations above can more precisely be summarized in the proposition below.

Proposition 1 *With* $A = \{a_1, a_2, a_3\}$*, AmP is vulnerable to strategic candidacy at some profile* R^N *by opting out if there is an* $\alpha-$*majority cycle over* A *at* R^N*.*

In other words, Proposition 1 says the amendment rule is vulnerable to strategic candidacy opting out if at some profile R^N there exists some i such that $R^i = a_1a_2a_3$, a_1 beats a_2, a_3 beats a_1 and a_2 beats a_3 or $R^i = a_2a_1a_3$, a_2 beats a_1, a_3 beats a_2 and a_1 beats a_3.

We next study successive elimination under maximin and maximax behavior.

Under successive elimination with maximin behavior ($SE\min$), when a_2 is elected, there is no way for strategic candidacy by opting out. This so because a_1 is ranked last by a majority of winners and a_2 beats a_3. For exactly similar reasons the conclusion is the same when a_3 is elected. Then, the only possibilities appear when a_1 is elected, as illustrated in the example below.

Example 2 *Consider the following profile and* $\alpha = \frac{1}{2}$

R^N				$R^N \vert \{a_1, a_3\}$		
1	*2*	*3*		*1*	*2*	*3*
a_1	a_2	a_3		a_1	a_3	a_3
a_2	a_3	a_1		a_3	a_1	a_1
a_3	a_1	a_2				

a_1 beats $\{a_2a_3\}$, then a_1 wins. If a_2 opts out, a_3 beats a_1 and then a_3 wins. a_2 is a strategic candidate.

The following proposition describes all cases at which strategic candidacy by opting out is susceptible to occur.

Proposition 2 $SE\min$ *is vulnerable to strategic candidacy at profile* R^N *by opting out if there exists some* i *such that*

(i) $R^i = a_2a_3a_1$*,* a_1 *beats* $\{a_2, a_3\}$ *and* a_3 *beats* a_1 *or*

(ii) $R^i = a_3a_2a_1$*,* a_1 *beats* $\{a_2, a_3\}$ *and* a_2 *beats* a_1*.*

Under successive elimination with maximax behavior ($SE\max$), no strategic candidacy is possible when a_1 is elected since this requires that more than half the number of voters rank it first. But when a_2 or a_3 is elected, such situations become possible:

Example 3 *Consider the following profile and* $\alpha = \frac{1}{2}$

R^N			$R^N\|\{a_1, a_2\}$		
1	*2*	*3*	*1*	*2*	*3*
a_1	a_2	a_3	a_1	a_2	a_1
a_2	a_1	a_1	a_2	a_1	a_2
a_3	a_3	a_2			

$\{a_2 a_3\}$ beats a_1, a_2 beats a_3, then a_2 wins. If a_3 opts out, a_1 beats a_2 and a_1 wins. a_3 is a strategic candidate.

Proposition 3 $SE\max$ *is vulnerable to strategic candidacy at* R^N *by opting out if there exists some* i *such that*
(i) $R^i = a_3 a_1 a_2$, $\{a_2 a_3\}$ *beats* a_1, a_2 *beats* a_3 *and* a_1 *beats* a_2 *or*
(ii) $R^i = a_2 a_1 a_3$, $\{a_2 a_3\}$ *beats* a_1, a_3 *beats* a_2 *and* a_1 *beats* a_3.

3.2 Opting in

As above, we begin with amendment procedure. The reader can easily check that the only cases where strategic candidacy is susceptible to occur are when a_3 is elected at the unrestricted profile R^N. In some of those cases, it will be possible to restrict the profile to $R^N|A - \{a_2\}$ or to $R^N|A - \{a_1\}$, so that a_1 or a_2 is elected, as is illustrated in Example 4.

Example 4 *Consider the following profile and* $\alpha = \frac{1}{2}$

$R^N\|\{a_1, a_3\}$					R^N				
1	*2*	*3*	*4*	*5*	***1***	*2*	*3*	*4*	*5*
a_3	a_1	a_3	a_1	a_1	a_2	a_2	a_3	a_1	a_1
a_1	a_3	a_1	a_3	a_3	a_3	a_1	a_2	a_3	a_3
					a_1	a_3	a_1	a_2	a_2

a_1 beats a_3, then a_1 wins. If a_2 opts in, a_2 beats a_1, a_3 beats a_2, then a_3 wins. a_2 is a strategic candidate. And it appears that R^N is be a cycle. To put it in another way, beginning with $R^N|\{a_1, a_3\}$, the complete profile - after the entry of a_2 - is vulnerable to strategic candidacy if and only if a_2 enters in such a way that R^N leads to a cycle.

All such profiles are summarized in the proposition below.

Proposition 4 AmP *is vulnerable to strategic candidacy at profile* R^N *by opting in if there exists an* $\alpha-$*majority cycle over* A *at* R^N.

In other words, if there is an $\alpha-$majority at R^N, then one can always construct a restriction of R^N at which some candidate will find it profitable to enter the election.

Under successive elimination with maximin behavior, cases of strategic candidacy occur only when a_2 or a_3 is elected at the unrestricted profile, as shown in Example 5 and summarized by Proposition 5.

Example 5 *Consider the following profile and* $\alpha = \frac{1}{2}$

$R^N \mid \{a_1, a_2\}$						R^N				
1	*2*	*3*	*4*	*5*		*1*	*2*	*3*	*4*	*5*
a_1	a_1	a_2	a_2	a_2		a_3	a_1	a_2	a_2	a_2
a_2	a_2	a_1	a_1	a_1		a_1	a_2	a_1	a_1	a_3
						a_2	a_3	a_3	a_3	a_1

a_2 beats a_1 then a_2 wins. If a_3 opts in, a_1 beats $\{a_2, a_3\}$ then a_1 wins. a_3 is a strategic candidate.

Proposition 5 $SE\min$ *is vulnerable to strategic candidacy at* R^N *by opting in if there exists some* i *such that*
(i) $R^i = a_3a_1a_2$*,* a_2 *beats* a_1 *and* a_1 *beats* $\{a_2, a_3\}$ *or*
(ii) $R^i = a_2a_1a_3$*,* a_3 *beats* a_1 *and* a_1 *beats* $\{a_2, a_3\}$*.*

Under successive elimination with maximax behavior, strategic candidacy occurs only when a_1 is elected.

Example 6 *Consider the following profile and* $\alpha = \frac{1}{2}$

$R^N \mid \{a_1, a_3\}$						R^N				
1	*2*	*3*	*4*	*5*		*1*	*2*	*3*	*4*	*5*
a_1	a_1	a_3	a_1	a_3		a_2	a_1	a_3	a_1	a_2
a_3	a_3	a_1	a_3	a_1		a_1	a_3	a_2	a_3	a_3
						a_3	a_2	a_1	a_2	a_1

a_1 beats a_3 then a_1 wins. If a_2 opts in, $\{a_2a_3\}$ beats a_1, a_3 beats a_2, then a_3 wins. a_2 is a strategic candidate.

Proposition 6 $SE\max$ *is vulnerable to strategic candidacy at profile* R^N *by opting in if there exists some* i *such that*
(i) $R^i = a_2a_3a_1$*,* a_1 *beats* a_3*,* $\{a_2a_3\}$ *beats* a_1 *and* a_3 *beats* a_2 *or*
(ii) $R^i = a_3a_2a_1$*,* a_1 *beats* a_2*,* $\{a_2a_3\}$ *beats* a_1 *and* a_2 *beats* a_3*.*

3.3 Susceptibility to strategic candidacy

As said in the introduction of this paper, we are concerned with the quantitative evaluation of strategic candidacy opportunities. Our calculations will be based on a specific probabilistic model, known under the name of *impartial anonymous culture* (IAC); under IAC, voters are anonymous, in the sense that their identity does not matter: if we permute the preferences of two individuals, this will have no consequence on the outcome of the vote. Two profiles of preferences are considered as identical if in these two profiles the number of voters having a given type of preference relation is the same. With $A = \{a_1, a_2, a_3\}$, the preference relation R^i of a given voter i is one of the following six possible linear orders: $R_1 : a_1a_2a_3$; $R_2 : a_1a_3a_2$; $R_3 : a_2a_1a_3$; $R_4 : a_2a_3a_1$; $R_5 : a_3a_1a_2$; $R_6 : a_3a_2a_1$. Let n_j denote the number of voters whose preference relation is R_j $(j = 1, 2, ..., 6)$. Then,we must have $n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = n$. Consequently in the sequel, instead of profiles in the sense defined above, we consider *voting situations* or simply situations, defined as vectors of the form $s = (n_1, n_2, n_3, n_4, n_5, n_6)$. Strategic candidacy frequencies will be calculated on the basis of this following ratio:

$$\frac{\text{Number of voting situations at which strategic candidacy is possible}}{\text{Total number of voting situations}}$$

The method used to compute these frequencies is based on Gehrlein and Fishburn (1976) and is the same as the one in Mbih, Moyouwou and Picot (2008). All technical details are available from the authors upon simple request. Here, in order to illustrate, we simply provide an example of closed form formulae giving the frequencies for successive elimination with maximax and opting out when the quota α is equal to $\frac{1}{2}$; these formulae are obtained from systems of linear inequalities describing all voting situations at which strategic candidacy by opting out is susceptible to arise.

Proposition 7 *For all α such that $0 < \alpha \leq 1$, successive elimination with maximax is vulnerable to strategic candidacy by opting out if and only if*

$$\begin{cases} n_3 + n_4 + n_5 + n_6 \geq \alpha n \\ n_1 + n_3 + n_4 > n - \alpha n \\ n_1 + n_2 + n_5 > n - \alpha n \\ n_1 + n_2 \geq 1 \\ n_3 + n_4 \geq 1 \\ n_5 \geq 1 \\ n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = n \end{cases} \quad or \quad \begin{cases} n_3 + n_4 + n_5 + n_6 \geq \alpha n \\ n_2 + n_5 + n_6 \geq \alpha n \\ n_1 + n_2 + n_3 > n - \alpha n \\ n_1 + n_2 \geq 1 \\ n_5 + n_6 \geq 1 \\ n_3 \geq 1 \\ n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = n \end{cases}$$

Applying the Gehrlein-Fishburn technique, we then obtain the following result:

Proposition 8 *When $\alpha = \frac{1}{2}$ and $n \geq 3$, then the frequency of successive elimination with maximax to strategic candidacy by opting out is given by*

$$\begin{cases} \frac{20n+3n^2+33}{160n+16n^2-384} \text{ if } n \text{ is odd} \\ \frac{464n+172n^2+36n^3+3n^4}{944n^2-272n+272n^3+16n^4-960} \text{ if } n \text{ is even} \end{cases}$$

Table 1 in the appendix gives theoretical frequencies of strategic candidacy, with respect to the number of voters.

4 Concluding remarks

The main information brought by this work is how frequent parliamentary voting procedures, and specifically amendment and successive elimination voting rules, are vulnerable to strategic candidacy. First, it appears that they are vulnerable for any quota α. In particular, when $\alpha = \frac{1}{2}$, for large electorates the vulnerability is 6.25% for amendment voting procedure, and

50% and 18.75% for successive elimination voting procedure with maximin and with maximax, respectively. The amendment voting procedure is vulnerable to strategic candidacy only in the presence of Condorcet cycles.

For any number of voters, successive elimination voting procedure appears to be much more vulnerable to strategic candidacy than amendment voting rule. Besides, notice that the frequency under maximin (50%) is much more significant than under maximax (18.75%).

It is also worth noting that the vulnerability with respect to the number of voters is a decreasing function for opting out and an increasing one for opting in.

One can imagine many directions at which the results in this paper can be generalized and expanded; we only cite a few of them here: the evaluation of the rules studied in this paper under the hypothesis of sophisticated behavior as defined in Farquharson (1969) and subsequently developed in more recent research, the use of other probabilistic models (impartial culture, maximal culture), the possibility of different quotas in pairwise contests, etc. It will doubtless also be of interest to examine the vulnerability of positional rules (plurality, anti-plurality, Borda) to strategic candidacy.

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Appendix

Table 1. Frequencies of strategic candidacy for amendment and successive elimination rules for $\alpha = \frac{1}{2}$

Opting out				Opting in			
n	AP	SE min	SE max	n	AP	SE min	SE max
3	0.25	0.5	0.5	3	0	0	0
4	0.138889	0.333333	0.222222	4	0	0.222222	0
5	0.117647	0.372549	0.254902	5	0.019608	0.176471	0.039216
6	0.103896	0.34632	0.186147	6	0.021645	0.255411	0.034632
7	0.092105	0.366228	0.210526	7	0.026316	0.232456	0.057018
8	0.089133	0.361416	0.173382	8	0.029304	0.282051	0.052503
9	0.081633	0.372449	0.193878	9	0.030612	0.266764	0.069971
10	0.081267	0.374656	0.168044	10	0.033517	0.303489	0.065197
11	0.076087	0.380737	0.18599	11	0.033816	0.291667	0.080314
12	0.076512	0.385849	0.165775	12	0.036405	0.321061	0.075278
15	0.070513	0.396368	0.179487	15	0.038462	0.326923	0.096154
18	0.069659	0.410284	0.165164	18	0.04193	0.358797	0.097052
21	0.066986	0.414428	0.177033	21	0.043062	0.36106	0.112624
24	0.066867	0.426187	0.166917	24	0.045377	0.383308	0.111644
27	0.065385	0.427404	0.176923	27	0.046154	0.383654	0.124038
30	0.065447	0.437311	0.168872	30	0.047808	0.400536	0.122177
33	0.064516	0.43704	0.177419	33	0.048387	0.39983	0.132428
36	0.064625	0.445522	0.170638	36	0.049628	0.413316	0.130144
39	0.06399	0.444446	0.17806	39	0.050079	0.412018	0.138857
42	0.064105	0.45183	0.172158	42	0.051045	0.423178	0.136382
45	0.063647	0.450306	0.178703	45	0.051408	0.421542	0.143941
48	0.063756	0.456828	0.173457	48	0.5218	0.431021	0.141399
51	0.063411	0.455053	0.1793	51	0.052478	0.429196	0.148063
54	0.06351	0.460886	0.174569	54	0.05311	0.437409	0.14552
57	0.063241	0.458976	0.179842	57	0.05336	0.435484	0.151473
60	0.06333	0.464246	0.175526	60	0.053886	0.442712	0.148966
63	0.063115	0.462271	0.180328	63	0.054098	0.440743	0.154339
66	0.063194	0.467074	0.176357	66	0.054544	0.447186	0.15189
∞	0.0625	0.5	0.1875	∞	0.0625	0.5	0.1875