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A global analysis of liquidity effects, interest rate rules, and deflationary traps

Noritaka Kudoh
Hokkaido University

Abstract

The prevailing models of liquidity traps suggest that a deflationary trap is a stable steady state in a multiple equilibria model. These models implicitly assume that the central bank accelerates the process of disinflation by following a Taylor rule even though there is a long run positive relationship between the nominal interest rate and inflation rate. This paper presents a reduced-form model that integrates liquidity effects into the analysis of interest rate rules to generalize the previous results about uniqueness, determinacy, and dynamic property of the economy.

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1. Interest Rate Rules and Liquidity Traps

Consider a simple dynamic economy with flexible prices in which (i) the level of output is exogenous, (ii) the representative household's behavior is expressed by the standard Euler equation, and (iii) the central bank follows a Taylor-type interest rate rule. In a steady state, the interest rate rule is expressed by $I = A\Pi^{\alpha}$, where I is the gross nominal interest rate, Π is the inflation factor, $A > 0$ is a scale parameter, and $\alpha > 0$. According to Leeper (1991), monetary policy is said to be "active" if $\alpha > 1$ and "passive" if $\alpha < 1$. Since the level of output is exogenous, monetary policy reacts only to inflation.[1]

The standard Euler equation implies that in a steady state, the gross real interest rate R equals the inverse of the household's discount factor, $1/\beta$.[2] Then the Fisher equation is $I = R\Pi = \beta^{-1}\Pi$. Substitute this into $I = A\Pi^{\alpha}$ to obtain $\Pi = (A\beta)^{1/(1-\alpha)}$. Thus, the inflation rate is easily expressed in terms of parameters. According to this simple model, the central bank should be able to control inflation perfectly by choosing appropriately the policy parameters A and α.

It has become conventional to describe a deflationary trap in this environment by adding the zero lower bound (ZLB) on the nominal interest rate to the analysis. When the rate of inflation starts falling, sooner or later the nominal interest rate will hit the ZLB. In this case, the Fisher equation implies that $\Pi = R^{-1} = \beta$, which is the household's discount factor and is therefore less than one. In this simple economy, deflation occurs whenever the nominal interest rate hits the ZLB.

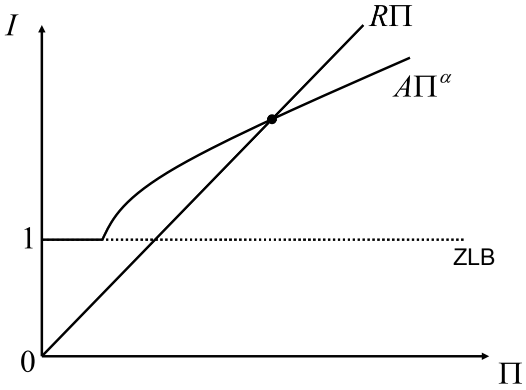

Figure 1: Monetary policy is passive

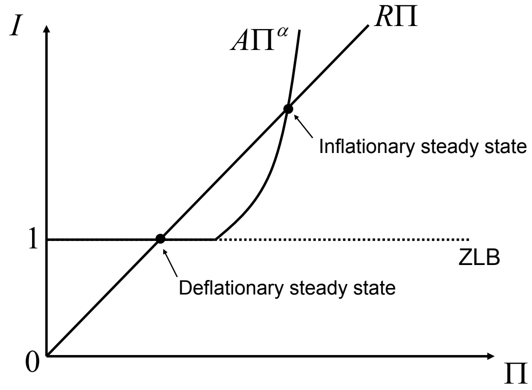

Figure 2: Monetary policy is active

Figures 1 and 2 depict how interest rate rules combined with the ZLB can generate a deflationary steady state. Figure 1 depicts the case in which monetary policy is passive, or $\alpha \in (0, 1)$. As is clear from the figure, the presence of the ZLB has no implication for the

[1] Clarida *et al.* (1998) found that the interest rate does not react strongly to the output gap in major countries. For example, the estimated output gap coefficient is 0.08 for the Bank of Japan, while the coefficient on the inflation gap is 2.04.

[2] This amounts to say that the real interest rate equals the natural rate of interest.

number of steady-state equilibria. However, it causes multiple equilibria if monetary policy is active, or $\alpha > 1$. As in Figure 2, active policy combined with the ZLB implies two steady states. Whether or not the economy actually hits the ZLB, and hence the deflationary trap, can be verified by considering the model's dynamic properties. The model's dynamics and determinacy are strongly influenced by (i) whether the interest rate rule is forward-looking or backward-looking, and (ii) whether there is an equilibrium rejection device such as the transversality condition (TVC).[3]

Proposition 1 *If monetary policy is passive ($\alpha \in (0, 1)$), then (i) there is a unique steady state, (ii) the steady state is stable under the backward-looking rule, and (iii) the steady state is unstable under the forward-looking rule.*

Part (i) is evident from Figure 1. Let us prove part (ii). The case of the backward-looking interest rate rule is described by the following system of difference equations, $I_t = \beta^{-1}\Pi_t$ and $I_t = A\Pi_{t-1}^{\alpha}$. From these equations it is easy to obtain $\Pi_t = \beta A\Pi_{t-1}^{\alpha}$, from which it is easy to verify that the steady state is stable since $\alpha \in (0, 1)$. Once the stability properties of the model are established, it is easy to show the following.

Proposition 2 *Suppose monetary policy is passive and there is an equilibrium rejection device such as the TVC, then (i) the steady state is indeterminate under the backward-looking rule, and (ii) the steady state is determinate under the forward-looking rule.*

The key is that the TVC rejects any divergent path as an equilibrium.[4] Thus, a unique unstable steady state implies that the steady state is the *only* possible equilibrium for any t. In contrast, a stable steady state implies that there is an infinity of convergent paths to the steady state—the steady state is indeterminate.

Proposition 3 *If monetary policy is active ($\alpha > 1$), then (i) there are two steady states, one of which is deflationary, (ii) the inflationary steady state is stable under the forward-looking rule, and (iii) the inflationary steady state is unstable under the backward-looking rule.*

A stable deflationary steady state, or a deflationary trap, arises when monetary policy is active ($\alpha > 1$) and backward-looking. Suppose that there is a decrease in inflation near the inflationary steady state. The central bank reduces the nominal interest rate according

[3]It might be hard to imagine the role of TVC in this reduced-form model since the household optimization is implicit. Since TVC is usually model-specific, it may be replaced with any other condition that rules out unbounded sequences of candidate equilibria.

[4]This statement is not universally accepted. For example, Cochrane (2006) argues, "the expectation of an explosion is perfectly reasonable—If the Fed were committed to raising interest rates more than 1–1 with inflation, *if* we lived in a world of constant real rates, so this translates into a commitment to raise future inflation more than 1–1 with past inflation, then my expectation is that we'll see hyperinflation." According to Cochrane (2006) and Woodford (2003), a non-Ricardian fiscal regime is needed as an equilibrium rejection device. Since the predictions of the model depend profoundly on whether there is an equilibrium rejection device, I choose to present both cases in this paper.

to the interest rate rule, $I_t = A\Pi_{t-1}^{\alpha}$. However, in the frictionless pure-exchange economy, a reduction in the nominal interest rate reduces the inflation rate further because of the Fisher equation with a predetermined real interest rate, $\Pi_t = \beta I_t$. Thus, in this simple economy, the interest rate rule accelerates the process of disinflation. Thus, the nominal interest rate will eventually hit the ZLB, at which $\Pi = \beta < 1$ holds.

Ruling out equilibrium paths in an environment with two steady states is a subject of debate (Cochrane, 2006). There seems to be no single way to interpret mathematical results. To limit the debate, I focus on the region between the inflationary steady state and the deflationary steady state. In this region, there is no divergent path, so the question to be addressed is whether the deflationary steady state is stable or not. If the deflationary steady state is stable, then there is a deflationary trap. With the TVC, stability implies that there is an infinity of paths leading to the trap. It is important to note here that Taylor's (2001) own interpretation of Figure 2 is that the deflationary steady state is unstable, so the issue is *not* whether it is a trap, but whether the economy gets into the *deflationary spiral*—the region on the left of the (unstable) deflationary steady state. Thus, the interpretation of Figure 2 differs profoundly, depending upon whether or not the analysis uses a device that rules out divergent paths.

2. Liquidity Effects and Interest Rate Rules

Although the above argument has become the building block of the analyses of liquidity traps,[5] it misses an important aspect. When disinflation accelerates and the central bank wishes to avoid getting into a deflationary trap, why does not the central bank *raise* the nominal interest rate, instead of reducing it? If the long-run Fisher relation is exploitable, then the central bank should be able to control inflation simply by pegging the nominal interest rate. In other words, in the basic model, it is the central bank that creates a deflationary trap (Benhabib *et al.*, 2001, 2002). In this sense, it is plausible to think that a Taylor rule, or monetary policy in general, presumes the existence of the *liquidity effect* or monetary nonneutrality.[6]

To generalize the analysis, suppose that the standard Fisher equation is replaced with

$$I = F(\Pi),$$

which is a generalization of the Fisher equation because it includes the Fisher equation as a special case when $F(\Pi) = R\Pi$. In what follows I do not exclude the case where there is a *persistent liquidity effect*: $F'(\cdot) < 0$. This states that the nominal interest rate and inflation rate are negatively related, and I do not argue whether there *is* such an effect at all (Melvin, 1983; Reichenstein, 1987; Gordon and Leeper, 1994; Christiano, 1995; Pagan and Robertson,

[5] Figure 2 has been popularized by Taylor (2001), Benhabib *et al.* (2001, 2002) and textbooks such as Walsh (2003) and Woodford (2003).

[6] Christiano (1995) defines the liquidity effect as "an exogenous, persistent, upward shock in the growth rate of the monetary base, engineered by the central bank and not associated with any current or prospective adjustment in distortionary taxes, drives the nominal rate of interest down for a significant period of time." See also Mishkin (2007).

1995; Christiano *et al.*, 1996). A related result appears in Nakajima (2006), in which the negative relationship is derived from a dynamic general equilibrium model with sticky prices and segmented markets.[7] Here, the exact mechanism that generates the liquidity effect is not described. Instead, this paper presents implications of the presence of the liquidity effect for the conduct of monetary policy.

A steady state of the model is given by the solution to $I = A\Pi^\alpha$ and $I = F(\Pi)$. If $F(\Pi)$ is decreasing and $\alpha > 0$, there is a *unique* steady state even with the ZLB. In other words, if there is a persistent liquidity effect, then the analysis above suggests that there is no liquidity trap because the steady state is unique.

In what follows I present some dynamic properties of the model. First, consider a backward-looking Taylor rule. The law of motion for this economy is driven by the difference equations $I_t = A\Pi_{t-1}^\alpha$ and $I_t = F(\Pi_t)$, from which it is easy to verify that $A\Pi_{t-1}^\alpha = F(\Pi_t)$. Then,

$$\frac{d\Pi_t}{d\Pi_{t-1}} = \frac{\alpha A \Pi^{\alpha-1}}{F'(\Pi)} = \frac{\alpha}{\varepsilon(\Pi)},$$

where $\varepsilon(\Pi) \equiv F'(\Pi)\Pi/F$ is the elasticity of $F(\Pi)$ with respect to Π. Note that with a persistent liquidity effect, $\varepsilon < 0$. Note also that, with the standard Fisher equation, $\varepsilon(\Pi) = 1$. It is then routine (Azariadis (1993), Chapter 2) to establish that the sequence $\{\Pi_t\}$ exhibits damped oscillations and converges to the steady state if $-1 < \alpha/\varepsilon < 0$, and it exhibits explosive oscillations if $\alpha/\varepsilon < -1$. Consider a forward-looking Taylor rule. The law of motion is given by $I_t = A\Pi_{t+1}^\alpha$ and $I_t = F(\Pi_t)$, from which it is easy to verify that $A\Pi_{t+1}^\alpha = F(\Pi_t)$. Then, $d\Pi_{t+1}/d\Pi_t = \varepsilon(\Pi)/\alpha$. Thus, the sequence $\{\Pi_t\}$ exhibits damped oscillations and converges to the steady state if $-1 < \varepsilon/\alpha < 0$, and it exhibits explosive oscillations if $\varepsilon/\alpha < -1$. To summarize:

Proposition 4 *Suppose there is no equilibrium rejection device. (i) Under a backward-looking rule, the steady state is stable if $\alpha < -\varepsilon$. It exhibits explosive oscillations if $-\varepsilon < \alpha$. (ii) Under a forward-looking rule, the steady state is stable if $-\varepsilon < \alpha$. It exhibits explosive oscillations if $\alpha < -\varepsilon$.*

If we live in a world without an equilibrium rejection device, then, for *stability*, the central bank should set α small under a backward-looking rule and it should set α large under a forward-looking rule. An interesting feature of the model is that the dynamic properties are characterized by comparing the two elasticity measures, α and ε. This generalizes the analysis in the literature, in which $\varepsilon = 1$ holds.

Proposition 5 *Suppose there is an equilibrium rejection device such as the TVC. (i) Under a backward-looking rule, the steady state is determinate if $-\varepsilon < \alpha$. (ii) Under a forward-looking rule, the steady state is determinate if $\alpha < -\varepsilon$.*

[7] It is important to note that in the existing general equilibrium models with liquidity effects, the effects do not last. By contrast, this paper allows for the scenario in which a liquidity effect is persistent. According to Mishkin (2007), such a case arises when the liquidity effect dominates the offsetting effects such as the price level effect and the expected inflation effect. Using identified VARs, Gordon and Leeper (1994) found that the liquidity effect lasts 8 months. Christiano *et al.* (1996) used other VARs to find a strong liquidity effect.

As usual, the implication will be reversed if there is an equilibrium rejection device. For *determinacy*, the central bank should set α large under a backward-looking rule and it should set α small under a forward-looking rule.

3. Conclusion

An important point made in this paper is that although it is convenient to describe a deflationary trap by combining Taylor rules and the ZLB of the nominal interest rate, such a model implicitly assumes that the central bank accelerates the process of disinflation even though there is a positive relationship between the nominal interest rate and inflation rate. In this sense, it is plausible to think that policy makers presume the presence of the liquidity effect. Based on this observation, this paper presented a reduced-form model that generalizes the previous models to include liquidity effects.

The exercises performed in this paper suggest that if there is a persistent liquidity effect, then there is no deflationary trap. One of the keys to understanding a deflationary trap is that there might be a gap between the liquidity effect perceived by the central bank and the actual liquidity effect. This suggests a direction for future research. It is worthwhile to explore a model in which there is uncertainty regarding (i) the strength of the liquidity effect and (ii) how long the effect will last. Such an analysis requires a fully specified dynamic general equilibrium model.

A limitation of this study is to try to understand a deflationary trap within an endowment economy. An important line of future research involves a study of a Taylor rule with the ZLB in a production economy that generates a strong liquidity effect. Another important line of inquiry is to consider whether policy switching (Davig and Leeper, 2007) will eliminate a deflationary trap. Although it is ultimately necessary to build full DSGE models to tackle these issues, writing down a simple reduced-form model is a useful starting point.

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