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Capacity Choice in a Mixed Duopoly with Managerial Delegation

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Abstract

This paper studies capacity choice in a mixed duopoly with differentiated goods under quantity competition and price competition, taking into account the separation between ownership and management. In this paper, we show that in equilibrium, under quantity competition, both the public firm and the private firm choose over capacity, while under price competition, both choose under capacity. Moreover, in both the competition types, we found that the results do not depend on the degree of product differentiation. Furthermore, under both the competition types, we conduct detailed analysis of each firm`s delegation parameter of managerial contract, and we compare the equilibrium market outcomes obtained in our model with those in the entrepreneurial case, which is considered in several existing literature, and those in the private duopolistic case, which corresponds to the one after privatization of the public firm.

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1 Introduction

This paper examines capacity choice in a mixed duopoly with differentiated goods under quantity competition and price competition, taking into account the separation between ownership and management. Departing from existing literature on capacity choice in mixed duopoly where both firms are entrepreneurial, in this paper, we focus on the effect of the managerial contract on the firms' capacity scale to explicitly consider the situation where the owners enter into delegation contracts with their respective managers.

Many researchers studied capacity choice in various mixed oligopolistic environments.[1] Adopting the cost function introduced in Vives (1986) in which case both excess capacity and under capacity yield inefficiency, Nishimori and Ogawa (2004) obtained the result that a welfare-maximizing public firm chooses under capacity in a homogeneous-good mixed duopoly.[2] Ogawa (2006) and Bárcena-Ruiz and Garzón (2007) considered this problem in a mixed duopoly with differentiated goods under quantity competition and price competition, respectively. However, there has been no attempt to study capacity choice in a mixed duopoly taking into consideration the separation between ownership and management as in Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985) (the so-called $FJSV$ contracts). This paper studies this capacity choice and investigates the influence of managerial delegation on capacity in a mixed duopoly.[3]

One of the main aims of this paper is to examine capacity choice in a mixed duopoly with differentiated goods under quantity competition and price competition when the $FJSV$ delegation contract within each firm is explicitly taken into account. For this purpose, we consider the following game-theoretical setting: In the first stage, the owners simultaneously determine the weight of their quantity relative to their profit in the $FJSV$ delegation contract. In the second stage, the managers simultaneously decide their firm's capacity levels. In the third stage, both the managers simultaneously determine their quantities.[4] In this paper, we show that in equilibrium, under quantity competition, both the public firm and the private firm choose excess capacity, whereas under price competition, both of the firms choose under capacity. Sur-

[1]Many works have studied the capacity choice in private oligopoly. Dixit (1980), Brander and Spencer (1983), and Horiba and Tsutsui (2000) showed that the investment for output expansion tends to be excessive. Further, Stewart (1991), Zhang (1993), and Haruna (1996) considered labour-managed industries in the context of the capacity choice.

[2]Lu and Poddar (2005) extended Nishimori and Ogawa's (2004) model to sequential-move competitions in terms of quantity decision regarding each firm's quantity and capacity choice. Lu and Poddar (2006) extended the same model to the case where the firms makes a capacity choice under demand uncertainty. Moreover, Lu and Poddar (2009) analyzed endogenous production timing with respect to the determinants of the firms' capacity and quantity levels.

[3]Many researchers have adopted the $FJSV$ contract to study mixed oligopoly. Barros (1995) focused on asymmetric information in managerial delegation contracts in mixed duopoly. White (2001) reconsidered the situation in Barros's (1995) model under complete information in order to focus on the strategic benefit of the $FJSV$ contracts. Furthermore, Heywood and Ye (2009) introduced the delegation contract that weighs both profit and welfare as the one for the public firm. For other works on the issue, see Nakamura and Inoue (2007), Nakamura and Inoue (2009), and Bárcena-Ruiz (2009).

[4]Note that the managers determine their capacity in the second stage and quantity in the third stage, seeking to maximize their objective functions, *i.e.*, the delegation contracts provided by their owners.

prisingly, under both competition types, the results are independent of the degree of product differentiation. This is unlike the results obtained in the existing literature — that the ratio of capacity to the public firm's output depends on the degree of product differentiation.

The other and central aim of this paper is to investigate how the equilibrium delegation contracts change with the degree of substitutability of the relevant goods. This investigation reveals the importance of the properties of goods when it comes to determining the managerial contracts between owners and managers. Indeed, irrespective of the competition type, all the owners — except for the private owner under price competition — reduce the weight of their firms' quantity in the $FJSV$ delegation contracts as substitutability increases. It is noteworthy that under price competition, the private owner offers a positive weight when the good is very substitutable or complementary; and otherwise, a negative weight. In addition, in both quantity competition and price competition, we compared the equilibrium market outcomes with the $FJSV$ delegation contract in this paper and those without the $FJSV$ delegation contract in Ogawa (2006) and in Bárcena-Ruiz and Garzón (2007), respectively. In the quantity-setting mixed duopoly with capacity choice, although the introduction of the $FJSV$ delegation contract leads to the efficiency of the production allocation with respect to the equilibrium social welfare in almost all areas of the degree of product differentiation, we obtain a new finding that this introduction may deteriorate the efficiency of the production allocation with respect to the equilibrium social welfare, only when the relation of the goods produced by both the firms is highly substitutable. On the other hand, in the price-setting mixed duopoly with capacity choice, the presence of the $FJSV$ delegation contract always decreases the equilibrium social welfare. Furthermore, in order to analyze the privatization effect of the public firm, in both quantity and price competition, we conduct a comparison between the equilibrium market outcomes in this model and those in the private oligopoly with the $FJSV$ delegation contract under which the firms' managers choose their capacity and output levels. Consequently, in the mixed duopoly with both the $FJSV$ delegation contract and the capacity choice, we show that the privatization of public firm deteriorates the equilibrium social welfare in terms of both quantity and price competition.

2 Model

As in Singh and Vives (1984), we consider an economy comprising one monopolistic sector and a competitive *numeraire* sector. In the monopolistic sector, there is one public firm and one private firm indexed by $i = 0, 1$, respectively, and each firm produces a differentiated good. There exists a continuum of consumers of the same type with a utility function that is separable and linear in the *numeraire* good. The representative consumer maximizes $U(q_0, q_1) - p_0 q_0 - p_1 q_1$, where q_i is the quantity of the good i and p_i is its price ($i = 0, 1$). We suppose that the utility function $U(q_0, q_1)$ is quadratic, strictly concave, and symmetric in q_0 and q_1:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}\Big(q_0^2 + 2bq_0q_1 + q_1^2\Big), \qquad a > 0,\ b \in (-1, 1),$$

where parameter b denotes the degree of product differentiation. If $b \in (0, 1)$, the products are substitutable; on the other hand, if $b \in (-1, 0)$, the products are complementary. This utility function yields a linear demand structure. Inverse demand is given by

$$p_i = a - q_i - bq_j, \quad i, j = 0, 1, \; i \neq j,$$

and we can derive the following direct demand function:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1 - b^2}, \quad i, j = 0, 1, \; i \neq j.$$

Both firms have identical technology represented by the cost function $C(q_i, x_i)$, where q_i and x_i are the production quantity and capacity of firm i, respectively. Following Vives (1986), Ogawa (2006), and Bárcena-Ruiz and Garzón (2007), we assume that the cost function is given by

$$C_i(\Delta_i, q_i) = mq_i + \Delta_i^2,$$

where $\Delta_i = q_i - x_i$.[5] This cost function implies that if production quantity equals capacity, *i.e.*, $q_i = x_i$, then the long-run average cost is minimized. The profit of firm i is given by

$$\pi_i = p_i q_i - mq_i - \Delta_i^2, \quad i = 0, 1.$$

Social welfare (W) is measured as the sum of consumer surplus (CS) and producer surplus (PS). That is,

$$W = CS + PS,$$

where $PS = \pi_0 + \pi_1$ and CS under quantity competition is given by

$$CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = \frac{1}{2}(q_0^2 + q_1^2 + 2bq_0 q_1),$$

and under price competition, by

$$CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = \frac{2a^2(1-b) + p_0^2 - 2bp_0 p_1 + p_1^2 - 2a(1-b)(p_0 + p_1)}{2(1-b^2)}.$$

Futhermore, our paper focuses on the managerial aspect of the firms. In order to analyze this, we assume that the owners decide to delegate the control authority to managers à la Fershtman and Judd (1987), Sklivas (1987), and Vickers (1985). In the model, we consider that the owner within firm i delegates to the manager the decision authority with regard to quantity q_i and capacity x_i to the manager. Following Lambertini (2000) and Nakamura and Inoue (2007, 2009), the owner of firm i offers an incentive contract $V_i(\pi_i, q_i)$ to the manager:

$$V_i = \pi_i + \theta_i q_i, \quad \theta_i \in \mathbb{R}, \; i = 0, 1, \tag{1}$$

where the parameter θ_i measures the relevance of the sales. Note that this parameter reduces the manager's marginal cost, except for Δ_i, in the cost function of the firm i. This can be interpreted as owners providing non-physical subsidies to their managers. The manager of firm i maximizes his or her payoff, *i.e.*, V_i, by choosing output q_i and capacity x_i ($i = 0, 1$).

[5] Similar to Ogawa (2006) and Bárcena-Ruiz and Garzón (2007), we assume $a > m$ such that the outputs of both the public and private firms are strictly positive in the two types of competition.

In this paper, we propose the following three-stage game. In the first stage, the owner of each firm i sets the parameter θ_i in the incentive contract for the manager of firm i, independently and simultaneously. In the second stage, the managers of both the firms independently and simultaneously make a decision with regard to capacity. At the end of the game, each firm's manager engages in quantity or price competition. In the next section, by backward induction, we obtain the subgame perfect equilibrium under quantity and price competition.

3 Quantity Competition

We solve the game by backward induction from the last stage to obtain a subgame perfect Nash equilibrium. In the third stage, the managers of the public and private firms simultaneously select their outputs. In the second stage, knowing that the decision on the capacity level has effects on firms' output decision in the third stage, they simultaneously choose the capacities of their firms, which results in the following equilibrium outcomes:[6]

$$
\begin{aligned}
q_i^*(\theta_i, \theta_j) &= \frac{(16-b^2)\left[(32-16b-4b^2+b^3)(a-m)+32\theta_i-4b^2\theta_i-16b\theta_j+b^3\theta_j\right]}{1024-512b^2+48b^4-b^6}, \\
x_i^*(\theta_i, \theta_j) &= \frac{16\left[(32-16b-4b^2+b^3)(a-m)+32\theta_i-4b^2\theta_i-16b\theta_j+b^3\theta_j\right]}{1024-512b^2+48b^4-b^6}, \quad \text{and} \\
\Delta_i^*(\theta_i, \theta_j) &= q_i^*(\theta_i, \theta_j) - x_i^*(\theta_i, \theta_j) = -\left(\frac{b^2}{16}\right) x_i^* \leq 0, \quad i, j = 0, 1, \ i \neq j. \qquad (2)
\end{aligned}
$$

As expected intuitively, an increase in the delegation parameter expands the firm's output and capacity because such an increment reduces the marginal cost of the firm. On the other hand, such an increase lowers the other firm's output and capacity through strategic substitution when the relevant good is substitutable ($b > 0$), and increases the same through strategic complementation when the relevant good is complementary ($b < 0$). Moreover, surprisingly, excess capacity follows irrespective of managerial delegation parameters θ_0 and θ_1.[7]

Proposition 1. *Under quantity competition in a mixed duopoly with managerial delegation, both the firms choose excess capacity irrespective of the substitution parameter b and managerial delegation parameters θ_0 and θ_1.*

We should note that Δ_i^* decreases as θ_i increases ($i = 0, 1$). Further, Δ_i^* decreases as θ_j ($i \neq j$) increases when the good is complementary ($b < 0$), and increases when the good is substitutable ($b > 0$).

We now conduct an analysis of the first stage. In this stage, the owner of firm i decides the delegation parameter θ_i ($i = 0, 1$). The public firm's owner chooses parameter θ_0 such that social welfare is maximized, while the private firm's owner chooses parameter θ_1 such that his

[6]We use the superscript "$*$" to denote the equilibrium outputs, capacities, and their differences of both the firms in the second stage, which are solved by backward induction.

[7]This result in the proposition 1 does not depend on our specific model. Indeed, in setting demand functions $p_i = p_i(q_0, q_1)$ and cost functions $C_i = C_i(\Delta_i, q_i)$, the property of excess capacity can be derived as long as the first order conditions in the second stage are satisfied.

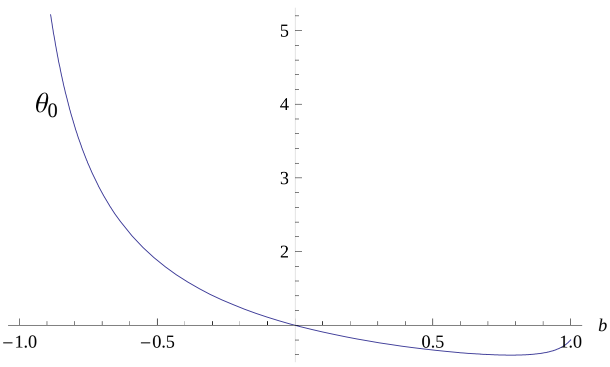

Figure 1: Equilibrium parameter of firm 0

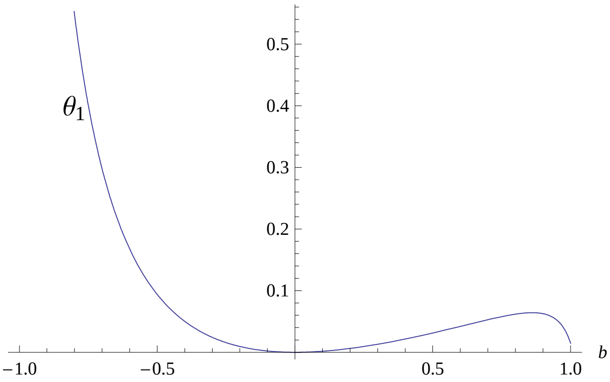

Figure 2: Equilibrium parameter of firm 1

or her profit is maximized.[8] Solving these problems, we obtain

$$\begin{cases} \theta_0^C = \dfrac{\begin{pmatrix} 134217728-134217728b-83886080b^2+109051904b^3+7864320b^4 \\ -31457280b^5+2424832b^6+4489216b^7-659456b^8-354304b^9 \\ +68608b^{10}+15872b^{11}-3648b^{12}-384b^{13}+96b^{14}+4b^{15}-b^{16} \end{pmatrix}(a-m)}{\begin{pmatrix} 134217728-201326592b^2+94896128b^4-20643840b^6 \\ +2445312b^8-166912b^{10}+6464b^{12}-128b^{14}+b^{16} \end{pmatrix}} > 0, \\ \text{and} \quad \theta_1^C = \dfrac{4b^2(8-b^2)(256-256b-32b^2+32b^3+3b^4-b^5)(3072-640b^2+48b^4-b^6)(a-m)}{\begin{pmatrix} 134217728-201326592b^2+94896128b^4-20643840b^6 \\ +2445312b^8-166912b^{10}+6464b^{12}-128b^{14}+b^{16} \end{pmatrix}} > 0. \end{cases} \quad (3)$$

Figures 1 and 2 illustrate how θ_0 and θ_1 change as the substitution parameter b increases when $a - m = 1$. Both θ_0 and θ_1 are consistently positive. In addition, the parameter of the public firm is higher than that of the private firm for any $b \in (-1, 1)$. Furthermore, the two parameters tend to decrease as b becomes large. The intuition behind these results is as follows. The welfare-maximizing government takes into account not only the profit of the public firm but also the consumers' benefits. Thus, for any $b \in (-1, 1)$, it has an incentive to produce more and to set θ_0 higher than θ_1, since q_0^* is an increasing function of θ_0. Moreover, in the case where the good is complementary, $\partial q_1^*/\partial\theta_0 > 0$ and $\partial\Delta_1^*/\partial\theta_0 < 0$, which implies that setting θ_0 higher than θ_1 improves the consumers' benefits from the good provided by the private firm along with the private firm's cost inefficiency. These welfare-improving effects become smaller as b becomes larger, and change into welfare-deteriorating effects when $b > 0$. Therefore, θ_0 is likely to decrease as b increases. Likewise, θ_1 also tends to decrease, because θ_1 is below θ_0 for any b.

Proposition 2. *Both owners propose positive delegation parameters to their managers. Moreover, these parameters tend to decrease as b becomes large.*

Furthermore, by comparing the equilibrium market outcomes with the $FJSV$ delegation contract in this model with those of the non-delegation quantity-setting model in Ogawa (2006), we obtain the following result:[9]

[8] Superscript "C" is used to denote the subgame perfect Nash equilibrium under quantity competition. Furthermore, we represent the equilibrium market outcomes under quantity competition, except for the delegation parameters of both the firms, in the Appendix.

[9] In the rest of this paper, we use the superscript "O" to represent the equilibrium market outcomes obtained

Proposition 3. *A comparison between the equilibrium market outcomes in this model and those in Ogawa (2006) yields the rankings of each firm's output, difference between the output and capacity, and social welfare, as follows:*

(1) $\begin{cases} q_0^O \geq q_0^C, & \forall b \in [0,1), \\ q_0^C > q_0^O, & \forall b \in (-1,0), \end{cases} \quad q_1^C \geq q_1^O, \quad \forall b \in (-1,1),$

(2) $\begin{cases} 0 \geq \Delta_0^C \geq \Delta_0^O, & \forall b \in (-0.68034, 0], \\ \Delta_0^O > \Delta_0^C, & otherwise, \end{cases} \quad \begin{cases} 0 \geq \Delta_1^C \geq \Delta_1^O, & \forall b \in [-0.773907, 0.764971], \\ 0 \geq \Delta_1^O > \Delta_1^C, & otherwise, \end{cases}$

(3) $W^C \geq W^O, \quad \forall b \in (-1, 0.949606], \quad W^O > W^C, \quad \forall b \in (0.949606, 1).$

From the above proposition, in the context of a quantity-setting mixed duopoly with capacity choice, the introduction of the $FJSV$ delegation contract increases the efficiency of production allocation with respect to the equilibrium social welfare in a sufficiently wide range of degrees of product differentiation. White (2001) compared the equilibrium social welfare between the quantity-setting mixed duopoly with the $FJSV$ delegation contract and the one without the $FJSV$ delegation contract, for which the two types of competitions are not taken into account in each firm's capacity choice, and he obtained the result that the equilibrium social welfare is higher in the delegation case than in the non-delegation case.[10] In contrast with the above result in White (2001), we obtain the one where the equilibrium social welfare is higher in the non-delegation case than in the delegation case, in the context of capacity choice, when b is sufficiently near 1, *i.e.*, the case where both the firms produce almost similar goods with each other. The intuition of this result is given as follows: When b is sufficiently large, the delegation parameters of both the firms become low.[11] Thus, the level of each firm's output becomes small, and then, the consumer surplus is relatively low. On the other hand, the value of Δ_i is large due to the low value of θ_i, as indicated in Proposition 1 ($i = 0, 1$). The above two effects yield a result that is the inverse of that in White (2001), that the equilibrium social welfare is higher in the non-delegation case than in the delegation case, in the context of the capacity-choice problem, only when b is sufficiently high.

Finally, in the context of the capacity choice, we analyze the privatization effect through a comparison of the equilibrium market outcomes between the mixed duopolistic case considered in this model and the private duopolistic case, with the $FJSV$ dlegation contract.[12] We obtain the following result:

Proposition 4. *A comparison between the equilibrium market outcomes in this model and those in the quantity-setting private duopoly with the $FJSV$ delegation contract, under which each*

in the quantity-setting, mixed-duopolistic model of Ogawa (2006) without managerial delegation. Moreover, the concrete values of the equilibrium market outcomes in Ogawa (2006) are presented in the Appendix.

[10]Note that White (2001) considered only the situation that both the firms produce homogeneous goods.

[11]These facts are realized from Figures 1 and 2.

[12]This private duopolistic case corresponds to the market structure after the privatization of the public firm in this model. Moreover, in the rest of this paper, we use the superscript pC to denote the equilibrium market outcomes in a private duopoly with the $FJSV$ delegation contract under which both the firms' managers choose the levels of their capacities and outputs, and the equilibrium market outcomes in the case pC are presented in the Appendix.

firm's manager chooses not only the output level but also the capacity scale, yields the rankings of each firm's output, difference between the output and capacity, and social welfare, as below:

$$(1)\quad q_0^C > q_0^{pC}, \quad \forall b \in (-1,1), \quad \begin{cases} q_1^{pC} \geq q_1^C, & \forall b \in [0,1), \\ q_1^C > q_1^{pC}, & \forall b \in (-1,0), \end{cases}$$

$$(2)\quad 0 \geq \Delta_0^{pC} \geq \Delta_0^C, \quad \forall b \in (-1,1), \quad \begin{cases} 0 \geq \Delta_1^C \geq \Delta_1^{pC}, & \forall b \in [0,1), \\ 0 > \Delta_1^{pC} > \Delta_1^C, & \forall b \in (-1,0), \end{cases}$$

$$(3)\quad W^C > W^{pC}, \quad \forall b \in (-1,1).$$

The result obtained in White (2001) through the comparison of equilibrium outcomes before and after the privatization of the public firm — that the public output falls and the private output rises — is robust against the introduction of capacity choice, as long as the degree of product differentiation is more than 0, *i.e.*, the relation of the outputs produced by both the firms is substitutable. Moreover, the reverse ranking of the private output in the case of the complementary goods relative to the case of substitutable goods causes a change in the strategic relation between the public firm and the private firm in the market (from strategic substitutability to strategic complementarity). Similar to the case of privatization in a mixed duopoly with the $FJSV$ delegation contract and without the capacity choice, which was considered in White (2001), the privatization of the public firm decreases the equilibrium social welfare, even if each firm's capacity choice is modelled as well.

4 Price Competition

Next, we investigate price competition. Similar to the case of quantity competition considered in the previous section, we solve the game by backward induction from the last stage in order to obtain a subgame perfect Nash equilibrium under the price competition as well. As under quantity competition, in the third stage, the two managers simultaneously select their prices, and in the second stage, knowing that the decision on the capacity level has effects on the firms' price setting in the third stage, they simultaneously choose the capacity levels of their firms, which leads to the following equilibrium outcomes:[13]

$$p_i^{**}(\theta_i,\theta_j) = \frac{\left[\begin{pmatrix} 512-256b-1056b^2 \\ +448b^3+932b^4 \\ -315b^5-443b^6 \\ +107b^7+119b^8-17b^9 \\ -17b^{10}+b^{11}+b^{12} \end{pmatrix} a + \begin{pmatrix} 512+256b-992b^2 \\ -448b^3+716b^4 \\ +315b^5-238b^6 \\ -107b^7+36b^8 \\ +17b^9-2b^{10}-b^{11} \end{pmatrix} m - \begin{pmatrix} 512-992b^2 \\ +716b^4-238b^6 \\ +36b^8-2b^{10} \end{pmatrix} \theta_i - \begin{pmatrix} 256b-448b^3 \\ +315b^5-107b^7 \\ +17b^9-b^{11} \end{pmatrix} \theta_j \right]}{1024-2048b^2+1648b^4-681b^6+155b^8-19b^{10}+b^{12}},$$

$$x_i^{**}(\theta_i,\theta_j) = \frac{2(8-6b^2+b^4)\left[\begin{pmatrix} 32-16b-28b^2 \\ +9b^3+9b^4-b^5-b^6 \end{pmatrix}(a-m) + (32-28b^2+9b^4-b^6)\theta_i - b(16-9b^2+b^4)\theta_j\right]}{1024-2048b^2+1648b^4-681b^6+155b^8-19b^{10}+b^{12}},$$

$$\text{and}\quad \Delta_i^{**}(\theta_i,\theta_j) = q_i^{**}(\theta_i,\theta_j) - x_i^{**}(\theta_i,\theta_j) = \left[\frac{b^2(3-b^2)}{2(8-6b^2+b^4)}\right] x_i^{**}(\theta_i,\theta_j) > 0,$$

[13]Similar to quantity competition, under price competition, we distinguish the subgame perfect Nash equilibrium market outcomes with the equilibrium ones in the second stage of the game, based on the backward induction. Then, we use the superscript "$**$" to denote the equilibrium prices, capacities, and their differences of both the firms at the second stage, which are solved by backward induction.

$$i, j = 0, 1, \ i \neq j. \tag{4}$$

p_i^{**} decreases with θ_i, because an increase in θ_i lowers the marginal cost, $(i = 0, 1)$. As under quantity competition, owing to strategic interaction, as θ_i increases, p_j^{**} $(i \neq j)$ decreases when $b \in (0, 1)$ and increases when $b \in (-1, 0)$. On the other hand, x_i^{**} increases with θ_i. In the second stage, the manager seeks to improve the market share and cost efficiency since the capacity choice does not affect the price of the firm in the first order. Thus, a decrease in the marginal cost through an increase in θ_i induces the manager to set the capacity at a higher level. As a result, x_j^{**} either increases or decreases depending upon whether the strategic interaction is substitutable or complementary. As under quantity competition, surprisingly, we obtain the result that capacity does not depend on the degree of product differentiation. Further, under capacity follows, irrespective of managerial delegation parameters, θ_0 and θ_1.

Proposition 5. *Under price competition in a mixed duopoly with the managerial delegation, both the firms choose under capacity irrespective of the substitution parameter b and managerial delegation parameters θ_0 and θ_1.*

Note that Δ_i^{**} increases as θ_i increases. Further, Δ_i^{**} also increases as θ_j increases when the good is complementary $(b < 0)$, and decreases when the good is substitutable $(b > 0)$.

On the basis of the above analyses, we now focus on the first stage. As under quantity competition, we obtain the following subgame perfect equilibrium outcomes:[14]

$$\begin{cases} \theta_0^B = \dfrac{\begin{pmatrix} 134217728(1-b)-620756992b^2+6123683384b^3+13416552992b^4-12876511328b^5-18045337760b^6 \\ +16566517776b^7+16941834224b^8-14594560000b^9-11779020800b^{10}+933201408b^{11} \\ +6264682888b^{12}-447746048b^{13}-2591300016b^{14}+164293196b^{15}+838374311b^{16} \\ -465368847b^{17}-211587111b^{18}+101907751b^{19}+4122439b^{20}-1714311b^{21}-608167b^{22} \\ +218031b^{23}+65757b^{24}-20341b^{25}-4925b^{26}+1317b^{27}+229b^{28}-53b^{29}-5b^{30}+b^{31} \end{pmatrix}(a-m)}{\begin{pmatrix} 134217728-671088640b^2+1541931008b^4-2156331008b^6+2050789376b^8 \\ -1404310528b^{10}+715071296b^{12}-275669056b^{14}+81108361b^{16} \\ -18204607b^{18}+3087813b^{20}-387635b^{22}+34667b^{24}-2061b^{26}+71b^{28}-b^{30} \end{pmatrix}} > 0, \\ \text{and} \quad \theta_1^B = \dfrac{b^2\begin{pmatrix} -(1-b)8388608+450887768b^2(1-b)-1075773344b^4+1069875520b^5+1532231168b^6 \\ -1503232000b^7-1469532116b^8+1407744000b^9+1010759936b^{10}-934117744b^{11} \\ -516705400b^{12}+454458892b^{13}+200169885b^{14}-165072999b^{15}-5921101b^{16} \\ +4506231b^{17}+1333433b^{18}-921243b^{19}-225333b^{20}+138999b^{21} \\ +27731b^{22}-15025b^{23}-2351b^{24}+1101b^{25}+123b^{26}-49b^{27}-3b^{28}+b^{29} \end{pmatrix}(a-m)}{\begin{pmatrix} 134217728-671088640b^2+1541931008b^4-2156331008b^6+2050789376b^8 \\ -1404310528b^{10}+715071296b^{12}-275669056b^{14}+81108361b^{16} \\ -18204607b^{18}+3087813b^{20}-387635b^{22}+34667b^{24}-2061b^{26}+71b^{28}-b^{30} \end{pmatrix}}. \end{cases} \tag{5}$$

Figure 3 illustrates the relationship between the equilibrium delegation parameter of each firm and substitution parameter b when $a - m = 1$. The figure demonstrates that θ_0 is always positive and is higher than θ_1 owing to the welfare maximization behavior of the government; this is as under quantity competition. Further, the reason for the downward slope of θ_0 in Figure 3 can be explained in a manner similar to that in the case of quantity competition. Interestingly, under price competition, θ_1 is peculiarly curved in a tiny range of the vertical axis; this is shown in Figure 4.[15] Two effects determine such curving: one is the effect of price competition on

[14]The superscript "B" is used to denote the subgame perfect Nash equilibrium under price competition. Furthermore, we represent the equilibrium market outcomes under price competition, except for the delegation parameters of both the firms, in the Appendix.

[15]Similar to the other three Figures, in figure 4, the change in θ_1 is described in accordance with that of b in the case where $a - m = 1$.

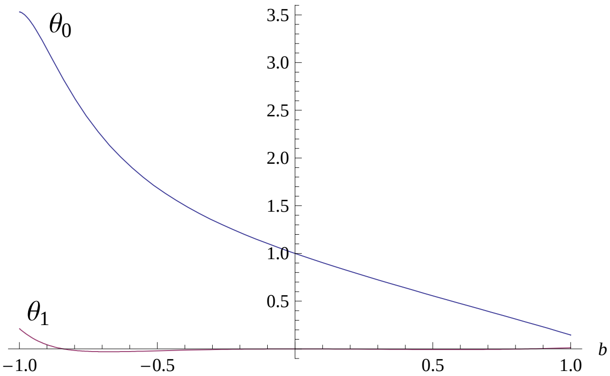

Figure 3: Equilibrium parameters of firms 0 and 1

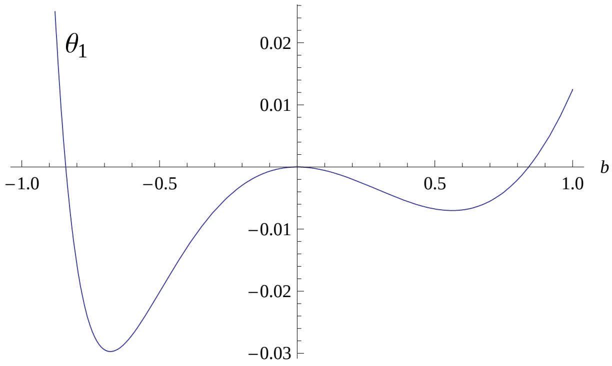

Figure 4: Equilibrium parameter of firm 1

profit and the other is the effect of an improvement of cost inefficiency Δ_1 on consumer benefits. For exposition, we concentrate only on substitutable goods ($b > 0$). Recalling that $\Delta_1 = 0$ if $b = 0$, we find that the latter effect is small when b is relatively low. Thus, for such b, the private owner attempts to set a lower θ_1 to raise the price because $\partial p_1^{**}/\partial\theta_1 < 0$. However, as b becomes large, price competition intensifies, and as a result, the former effect weakens. In this case, the latter effect matters when it comes to deciding θ_1. Therefore, the private owner selects a higher θ_1 so as to improve cost inefficiency ($\partial\Delta_1^{**}/\partial\theta_1 < 0$).

The results are summarized in the following proposition.

Proposition 6. *The government offers a positive delegation parameter regardless of* b*, whereas the private owner offers a positive delegation parameter when the good is very substitutable or complementary, and a negative parameter otherwise.*

We point out that Propositions 1 and 5 have one important implication. As stated above, delegation parameters θ_i have no influence regardless of whether there is excess or under capacity. This means that the results regarding the relationships between quantities and the capacity levels in Propositions 1 and 5 stand even when the public firm is privatized. In other words, the tendencies of excess capacity under quantity competition and under capacity under price competition are not related to the ownership of the firms.

Furthermore, by comparing the equilibrium market outcomes with the $FJSV$ delegation contract in this model with those without managerial delegation in Bárcena-Ruiz and Garzón (2007), we obtain the following result:[16]

Proposition 7. *A comparison between the equilibrium market outcomes in this model and those in Bárcena-Ruiz and Garzón (2007) yields the rankings of each firm's output, difference between*

[16] In the rest of this paper, we use superscript "BG" to represent the equilibrium market outcomes obtained in the price-setting mixed duopolistic model of Bárcena-Ruiz and Garzón (2007) without managerial delegation. Moreover, the concrete values of the equilibrium market outcomes obtained in Bárcena-Ruiz and Garzón (2007) are presented in the Appendix.

the output and capacity, and social welfare, as follows:

$$(1)\quad \begin{cases} q_0^{BG} \geq q_0^B, & \forall b \in [0,1)\,, \\ q_0^B > q_0^{BG}, & \forall b \in (-1,0)\,, \end{cases} \qquad \begin{cases} q_1^B \geq q_1^{BG}, & \forall b \in (-1,-0.762042] \cup [0.629955,1)\,, \\ q_1^{BG} \geq q_1^B, & otherwise, \end{cases}$$

$$(2)\quad \begin{cases} \Delta_0^{BG} \geq \Delta_0^B \geq 0, & \forall b \in [-0.541333,0]\,, \\ \Delta_0^B > \Delta_0^{BG}, & otherwise, \end{cases} \qquad \Delta_1^B \geq \Delta_1^{BG} \geq 0, \quad \forall b \in (-1,1)\,,$$

$$(3)\quad W^{BG} \geq W^B, \quad \forall b \in (-1,1)\,.$$

From the above proposition, in the context of a price-setting mixed duopoly with capacity choice, we realize that the introduction of the $FJSV$ delegation contract deteriorates the efficiency of the production allocation with respect to the equilibrium social welfare. In contrast to the result under quantity competition that the equilibrium social welfare is higher in the delegation case than in the non-delegation case in a wide range of degrees of product differentiation, under price competition, we find that the equilibrium social welfare is higher in the non-delegation case than in the delegation case, irrespective of the degree of product differentiation. Therefore, in the context of a mixed duopoly with capacity choice, which is composed of managerial firms, the change in each firm's strategic variable has a strikingly different influence on the equilibrium social welfare.

Finally, similar to the case of quantity competition, under price competition, we also study the privatization effect through the comparison of the equilibrium market outcomes between the mixed duopolistic case considered in this model and the private duopolistic case.[17]

Proposition 8. *A comparison between the equilibrium market outcomes in this model and those in the price-setting private duopoly with the $FJSV$ delegation contract that each firm's manager chooses not only her/his output level but also her/his capacity scale yields the following rankings of each firm's output, difference between her/his output and capacity, and social welfare:*

$$(1)\quad q_0^B > q_0^{pB}, \quad \forall b \in (-1,1)\,, \qquad \begin{cases} q_1^{pB} \geq q_1^B, & \forall b \in [0,1)\,, \\ q_1^B > q_1^{pB}, & \forall b \in (-1,0)\,, \end{cases}$$

$$(2)\quad \Delta_0^B \geq \Delta_0^{pB} \geq 0, \quad \forall b \in (-1,1)\,, \qquad \begin{cases} \Delta_1^{pB} \geq \Delta_1^B \geq 0, & \forall b \in [0,1)\,, \\ \Delta_1^B > \Delta_1^{pB} > 0, & \forall b \in (-1,0)\,, \end{cases}$$

$$(3)\quad W^B > W^{pB}, \quad \forall b \in (-1,1)\,.$$

The rankings of each firm's equilibrium output, the absolute value on the difference between the output and capacity scale, and social welfare obtained in the above proposition in the case of price competition are the same as those in Proposition 4, which applies to quantity competition. Similar to the case of quantity competition, the privatization of the public firm also deteriorates the equilibrium social welfare in price competition.

[17]Similar to the case of the quantity-setting competition, in the case of the price-setting competition, the private duopoly also corresponds to the market structure after the privatization of the public firm in this model. Moreover, we use the superscript pB to denote the equilibrium market outcomes in a private duopoly with the $FJSV$ delegation contract under which both firm's managers choose their capacity and output levels.

5 Conclusion

This paper examined capacity choice in a mixed duopolistic industry with differentiated goods under quantity competition and price competition, taking into account the separation between ownership and management. More precisely, we considered the influence of $FJSV$ managerial delegation on capacity scales under quantity and price competition in a mixed duopoly with differentiated goods. In a mixed duopoly with differentiated goods, as indicated in Ogawa (2006) and Bárcena-Ruiz and Garzón (2007), the capacity choice of the public firm heavily depends on the degree of product differentiation under quantity and price competition. However, the results obtained in this paper are strikingly different from those in the above literature. We found that both the firms always choose excess capacity under quantity competition and under capacity under price competition for any degree of substitutability of goods. Thus, by taking the modern internal organization of firms into account, we confirmed that irrespective of the degree of product differentiation, the firms' capacity to quantity ratios are constant under each quantity- and price-setting mixed duopoplies. Furthermore, we showed that as the degree of product differentiation increases, the owners, except for the private owner in price competition, lower their delegation parameters under quantity and price competition. In particular, under price competition, the private owner offers a positive parameter when the good is highly substitutable or complementary, and a negative parameter otherwise.

In addition, in order to clarify the role of the $FJSV$ delegation contract in the context of the capacity-choice problem in a mixed duopoly, we compared the equilibrium market outcomes with the $FJSV$ delegation contract in this model, with those without the $FJSV$ delegation contract in Ogawa (2006) and in Bárcena-Ruiz and Garzón (2007) under quantity and price competition, respectively. Under quantity competition in particular, we showed that the equilibrium social welfare is higher in Ogawa's (2006) non-delegation model than in this model with managerial delegation, only when the relation of the goods of both the firms is highly substitutable. This result is strikingly different from the one obtained in the mixed duopolistic model with the $FJSV$ delegation contract in White (2001) — that the capacity choice is not taken into account. On the other hand, under price competition, the introduction of the $FJSV$ delegation contract deteriorates the equilibrium social welfare in the context of a mixed duopoly with capacity choice. Furthermore, in order to analyze the privatization effect of the public firm under both quantity and price competition, we conducted a comparison between the equilibrium market outcomes in this model and those in the private oligopoly with the $FJSV$ delegation contract under which the firms' managers choose their capacity and output levels; we found that the privatization of the public firm decreases the equilibrium social welfare in the case of both quantity and price competition.

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Appendix

Quantity competition

Third stage: Given the delegation parameters and the production capacities, the manager of firm i maximizes (1) with respect to q_i $(i = 0, 1)$. Solving the maximization problem, we obtain the following result:

$$q_i(\theta_k, x_k; k = 0, 1) = \frac{a(4-b) - (4-b)m + 8x_i - 2bx_j + 4\theta_i - b\theta_j}{16 - b^2}, \quad i = 0, 1;\ i \neq j. \tag{6}$$

Second stage: In this stage, the managers of both the firms know how the decisions of the capacity scales influence each firm's output levels in the third stage, and hence, in the second stage, the maximization issue of the manager of firm i is given as follows:

$$\begin{cases} \max_{x_i} V_i(x_i, x_j) = (a - q_i - bq_j - m + \theta_i) q_i - (q_i - x_i)^2, \\ s.t. \quad eq.(6), \quad i = 0, 1;\ i \neq j. \end{cases} \tag{7}$$

Solving the problem of eq. (7), we obtain the results that are represented in eq. (2).

First stage: In this stage, the owners of both the firms take into account the functions of their managers' outputs and capacity scales. The maximization issue for the owner of the public firm, *i.e.*, the government is given as follows:

$$\begin{cases} \max_{\theta_0} W(\theta_0, \theta_1) = \frac{1}{2}\left(q_0^2 + 2bq_0q_1 + q_1^2\right) + \sum_{i=0}^{1} \left[(a-q_i-bq_j-m)q_i-(q_i-x_i)^2\right], \\ s.t. \quad eq.(2), \quad i = 0, 1;\ i \neq j, \end{cases} \tag{8}$$

yielding

$$\theta_0(\theta_1) = \frac{\begin{bmatrix} a\left(32-16b-4b^2+b^3\right)^2\left(256-64b^2+b^4\right)-\left(32-16b-4b^2+b^3\right)^2\left(256-64b^2+b^4\right)m \\ -b^3\left(49152-14336b^2+1600b^4-72b^6+b^8\right)\theta_1 \end{bmatrix}}{262144 - 294912b^2 + 80896b^4 - 8448b^6 + 368b^8 - 5b^{10}}. \tag{9}$$

On the other hand, the maximization issue for the owner of the private firm is given as follows:

$$\begin{cases} \max_{\theta_1} \Pi_1(\theta_0, \theta_1) = (a - bq_0 - q_1 - m)\, q_1 - (q_1 - x_1)^2, \\ s.t. \quad eq.\,(2)\,. \end{cases} \tag{10}$$

yielding

$$\theta_1(\theta_0) = \frac{b^2\left(3072 - 640b^2 + 48b^4 - b^6\right)\left[a\left(32-16b-4b^2+b^3\right)-\left(32-16b-4b^2+b^3\right)m-b\left(16-b^2\right)\theta_0\right]}{8\left(65536 - 57344b^2 - 13824b^4 + 1408b^6 - 64b^8 + b^{10}\right)}. \tag{11}$$

By simultaneously solving the first order conditions of the owners of both firms, which are represented in eqs. (9) and (11), we obtain the equilibrium delegation parameters of both the firms as in eq. (3). Furthermore, we obtain the following equilibrium market outcomes except for the delegation parameters of both the firms:

$$\begin{aligned}
q_0^C &= \frac{\left(16 - b^2\right)^2 \begin{pmatrix} 524288-393216b-327680b^2+204800b^3+69632b^4 \\ -34304b^5-6656b^6+2528b^7+288b^8-84b^9-4b^{10}+b^{11} \end{pmatrix} (a - m)}{\bar{C}}, \\
q_1^C &= \frac{16\left(16 - b^2\right)^2 \left(8 - b^2\right)^2 \left(256 - 256b - 32b^2 + 32b^3 + 3b^4 - b^5\right)(a - m)}{\bar{C}}, \\
x_0^C &= \frac{16 \begin{pmatrix} 8388608-6291456b-5767168b^2+3670016b^3+1441792b^4-753664b^5 \\ -176128b^6+74752b^7+11264b^8-3872b^9-352b^{10}+100b^{11}+4b^{12}-b^{13} \end{pmatrix} (a - m)}{\bar{C}}, \\
x_1^C &= \frac{256\left(8 - b^2\right)^2 \left(4096 - 4096b - 768b^2 + 768b^3 + 80b^4 - 48b^5 - 3b^6 + b^7\right)(a - m)}{\bar{C}}, \\
p_0^C &= \frac{abC_1 + \left(16 - b^2\right)^2 C_2 m}{\bar{C}}, \quad p_1^C = \frac{4aC_3 + \left(16 - b^2\right)^2 C_4 m}{\bar{C}}, \\
\Pi_0^C &= \frac{b\left(16 - b^2\right)^2 C_5 (a - m)^2}{\left(\bar{C}\right)^2}, \quad \Pi_1^C = \frac{64\left(8 - b^2\right)^3 C_6 (a - m)^2}{\left(\bar{C}\right)^2}, \\
CS^C &= \frac{\left(16 - b^2\right)^4 C_7 (a - m)^2}{2\left(\bar{C}\right)^2}, \quad W^C = \frac{\left(16 - b^2\right)^2 C_8 (a - m)^2}{2\left(\bar{C}\right)^2}, \\
\Delta_0^C &= -\frac{(4 - b)\, b^2 (4 + b)\, C_9 (a - m)}{\bar{C}} \leq 0, \quad \forall b \in (-1, 1)\,, \\
\Delta_1^C &= -\frac{16(4 - b)\, b^2 (4 + b)\left(8 - b^2\right)^2 \left(256 - 256b - 32b^2 + 32b^3 + 3b^4 - b^5\right)(a - m)}{\bar{C}} \leq 0, \\
&\quad \forall b \in (-1, 1)\,,
\end{aligned}$$

where

$\bar{C} = 134217728-201326592b^2+94896128b^4-20643840b^6+2445312b^8-166912b^{10}+6464b^{12}-128b^{14}+b^{16}$,

$C_1 = 33554432-335544432b-31457280b^2+325058856b^3+8388608b^4-9568256b^5-1032192b^6$
$+1368064b^7+68096b^8-108032b^9-2656b^{10}+4768b^{11}+68b^{12}-108b^{13}-b^{14}+b^{15}$,

$C_2 = 524288-131072b-589824b^2+106496b^3+167936b^4-18944b^5-19968b^6+1248b^7+1056b^8-36b^9-20b^{10}+b^{11}$,

$C_3 = 16777216-16777216b-16777216b^2+16777216b^3+5636096b^4-5505024b^5-999424b^6+884736b^7$
$+106752b^8-78592b^9-6848b^{10}+3904b^{11}+236b^{12}-100b^{13}-3b^{14}+b^{15}$,

$$C_4 = 262144+262144b-491520b^2-229376b^3+220160b^4+56320b^5-35584b^6-5888b^7+2576b^8+272b^9-84b^{10}-4b^{11}+b^{12},$$

$$C_5 = 17592186044416-307863255777728b-142936511161088b^2+470041220874 24b^3-2199023255552b^4-287075614064 64b^5$$

$$+5394478923776b^6+9607841841152b^7-2295391584256b^8-2021788745728b^9+516385931264b^{10}+287150440448b^{11}$$

$$-73347891200b^{12}-28543549440b^{13}+7041843200b^{14}+2013528064b^{15}-469778432b^{16}-100345856b^{17}+21858304b^{18}$$

$$+3448320b^{19}-697088b^{20}-77536b^{21}+14560b^{22}+1024b^{23}-180b^{24}-6b^{25}+b^{26},$$

$$C_6 = \left(4096-4096b-768b^2+768b^3+80b^4-48b^5-3b^6+b^7\right)^2\left(8192-6144b^2+960b^4-56b^6+b^8\right),$$

$$C_7 = 343597383680-274877906944b-652835028992b^2+506806140928b^3+445065986048b^4-326954385408b^5$$

$$-155960999936b^6+105730015232b^7+33043775488b^8-19958595584b^9-4544266240b^{10}+2364801024b^{11}$$

$$+413466624b^{12}-181370880b^{13}-24382464b^{14}+9017344b^{15}+870912b^{16}-282112b^{17}-15920b^{18}+5088b^{19}+72b^{20}-40b^{21}+b^{22},$$

$$C_8 = 123145302310912-105553116266496b-252887674388480b^2+206708186021888b^3+207738978172928b^4-157917357539328b^5$$

$$-94506460381184b^6+65455301591040b^7+26984474214400b^8-16772384161792b^9-5179361460224b^{10}+2852193828864b^{11}$$

$$+694833643520b^{12}-335290564608b^{13}-66400026624b^{14}+27830255616b^{15}+4534534144b^{16}-1640693760b^{17}$$

$$-218737664b^{18}+68126720b^{19}+7240704b^{20}-1943552b^{21}-155760b^{22}+36256b^{23}+1960b^{24}-400b^{25}-11b^{26}+2b^{27},$$

$$C_9 = 524288-393216b-327680b^2+204800b^3+69632b^4-34304b^5-6656b^6+2528b^7+288b^8-84b^9-4b^{10}+b^{11}.$$

Price competition

Third stage: Given the delegation parameters and the production capacities, the manager of firm i maximizes (1) with respect to p_i $(i = 0, 1)$. Solving the maximization problem, we obtain the following result:

$$p_i\left(\theta_k, x_k; k = 0, 1\right) = \frac{1}{16}\left(16 - 9b^2 + b^4\right)\left[\begin{matrix} a\left(12-3b-7b^2+b^3+b^4\right)+\left(4+3b-2b^2-b^3\right)m \\ -8x_i+4b^2x_i-6bx_j+2b^3x_j-4\theta_i+2b^2\theta_i-3b\theta_j+b^3\theta_j \end{matrix}\right], \quad i = 0, 1;\ i \neq j. \tag{12}$$

Second stage: In this stage, the managers of both the firms recognize how the decisions of the capacity scales influences each firm's price levels in the third stage, and then, in the second stage, the maximization issue of the manager of firm i is given as follows:

$$\begin{cases} \max_{x_i} V_i\left(x_i, x_j\right) = \left(p_i - m + \theta_i\right)\left[\frac{a(1-b)-p_i+bp_j}{1-b^2}\right] - \left[\frac{a(1-b)-p_i+bp_j}{(1-b^2)} - x_i\right]^2, \\ s.t. \quad eq. (12), \quad i = 0, 1;\ i \neq j. \end{cases} \tag{13}$$

Solving the issue in eq. (13), we obtain the results, which are represented in eq. (4).

First stage: In this stage, the owners of both the firms take into account the functions of their managers' prices and capacity scales. The maximization issue for the owner of the public firm, *i.e.*, the government, is given as follows:

$$\begin{cases} \max_{\theta_0} W\left(\theta_0, \theta_1\right) = \left[\frac{a(1-b)+bp_0-p_1}{1-b^2}\right]\left(p_1 - m\right) + \left[\frac{a(1-b)-p_0+bp_1}{1-b^2}\right]\left(p_0 - m\right) \\ + \frac{2a^2(1-b)+p_0^2-2bp_0p_1+p_1^2-2a(1-b)(p_0+p_1)}{2(1-b^2)} - \left[\frac{a(1-b)-p_0+bp_1}{1-b^2} - x_0\right]^2 - \left[\frac{a(1-b)+bp_0-p_1}{1-b^2} - x_1\right]^2, \\ s.t. \quad eq. (4), \quad i = 0, 1;\ i \neq j, \end{cases} \tag{14}$$

yielding

$$\theta_0(\theta_1) = \frac{\begin{bmatrix} \left(32-16b-28b^2+9b^3+9b^4-b^5-b^6\right)^2\left(256-448b^2+297b^4-95b^6+15b^8-b^{10}\right)(a-m) \\ +b^3\left(16384-55296b^2+83136b^4-70728b^6+37145b^8-12501b^{10}+2714b^{12}-370b^{14}+29b^{16}-b^{18}\right)\theta_1 \end{bmatrix}}{\begin{pmatrix} 262144-950272b^2+1522688b^4-1415936b^6+846384b^8 \\ -340461b^{10}+93617b^{12}-17450b^{14}+2122b^{16}-153b^{18}+5b^{20} \end{pmatrix}}. \tag{15}$$

On the other hand, the maximization issue for the owner of the private firm is given as follows:

$$\begin{cases} \max_{\theta_1} \Pi_1(\theta_0,\theta_1) = \frac{a(1-b)+bp_0-p_1}{1-b^2}(p_1-m) - \left[\frac{a(1-b)+bp_0-p_1}{1-b^2} - x_1\right]^2, \\ s.t. \quad eq.\,(4), \end{cases} \tag{16}$$

yielding

$$\theta_1(\theta_0) = \frac{\begin{Bmatrix} -b^2\left(1024-3456b^2+4400b^4-2827b^6+1010b^8-204b^{10}+22b^{12}-b^{14}\right) \\ \times\left[\left(32-16b-28b^2+9b^3+9b^4-b^5-b^6\right)(a-m)-b\left(16-9b^2+b^4\right)\theta_0\right] \end{Bmatrix}}{2\begin{pmatrix} 262144-884736b^2+1325056b^4-1159680b^6+657280b^8-252316b^{10} \\ +66511b^{12}-11908b^{14}+1390b^{16}-96b^{18}+3b^{20} \end{pmatrix}}. \tag{17}$$

By simultaneously solving the first order conditions of the owners of both the firms, which are represented in eqs. (15) and (17), we obtain the equilibrium delegation parameters of both the firms in eq. (5). Furthermore, we obtain the following equilibrium market outcomes, except for the delegation parameters, for both the firms:

$$q_0^B = \frac{\left(16-9b^2+b^4\right)^2\begin{pmatrix} 524288-393216b-1638400b^2+1155072b^3+2314240b^4 \\ -1506816b^5-1939968b^6+1146464b^7+1066912b^8-562652b^9 \\ -401900b^{10}+186174b^{11}+105017b^{12}-42149b^{13}-18816b^{14} \\ +6468b^{15}+2218b^{16}-646b^{17}-156b^{18}+38b^{19}+5b^{20}-b^{21} \end{pmatrix}(a-m)}{\bar{B}},$$

$$q_1^B = \frac{\left(16-9b^2+b^4\right)^2\left(32-28b^2+9b^4-b^6\right)^2\begin{pmatrix} 256-256b-288b^2+288b^3+131b^4 \\ -113b^5-30b^6+18b^7+3b^8-b^9 \end{pmatrix}(a-m)}{\bar{B}},$$

$$x_0^B = \frac{2\left(8-6b^2+b^4\right)B_1(a-m)}{\bar{B}}, \quad x_1^B = \frac{2\left(4-b^2\right)^3\left(2-b^2\right)\left(8-5b^2+b^4\right)^2 B_2(a-m)}{\bar{B}},$$

$$p_0^B = \frac{2abB_3+\left(16-9b^2+b^4\right)^2 mB_4}{\bar{B}}, \quad p_1^B = \frac{4aB_5+\left(16-9b^2+b^4\right)^2 mB_6}{\bar{B}},$$

$$\Pi_0^B = \frac{b\left(4+b-b^2\right)^2\left(4-b-b^2\right)^2(2-b)(a-m)^2 B_7B_8}{\left(\bar{B}\right)^2},$$

$$\Pi_1^B = \frac{\left(32-28b^2+9b^4-b^6\right)^3\begin{pmatrix} 8192-20480b^2+21184b^4-11688b^6 \\ +3715b^8-685b^{10}+69b^{12}-3b^{14} \end{pmatrix}(a-m)^2\left(B_9\right)^2}{\left(\bar{B}\right)^2},$$

$$CS^B = \frac{\left(16-9b^2+b^4\right)^4(a-m)^2 B_{10}}{\left(\bar{B}\right)^2}, \quad W^B = \frac{\left(16-9b^2+b^4\right)^2(a-m)^2 B_{11}}{\left(\bar{B}\right)^2},$$

$$\Delta_0^B = \frac{b^2\left(3-b^2\right)\left(4+b-b^2\right)\left(4-b-b^2\right)B_{12}(a-m)}{\bar{B}} \geq 0, \quad \forall b \in (-1,1),$$

$$\Delta_1^B = \frac{b^2\left(32-28b^2+9b^4-b^6\right)^2 B_{13}(a-m)}{\bar{B}} \geq 0, \quad \forall b \in (-1,1),$$

where

$\bar{B} = 134217728-671088640b^2+1541931008b^4-2156331008b^6+2050789376b^8-1404310528b^{10}+715071296b^{12}$

$$-275669056b^{14}+81108361b^{16}-18204607b^{18}+3087813b^{20}-387635b^{22}+34667b^{24}-2061b^{26}+71b^{28}-b^{30},$$

$$B_1 = 8388608-6291456b-30932992b^2+22020096b^3+52297728b^4-34897920b^5-53506048b^6+33059840b^7+36844544b^8$$
$$-20827424b^9-17972576b^{10}+9189116b^{11}+6364284b^{12}-2912602b^{13}-1648109b^{14}+669003b^{15}$$
$$+309849b^{16}-110697b^{17}-41274b^{18}+12890b^{19}+3702b^{20}-1004b^{21}-201b^{22}+47b^{23}+5b^{24}-b^{25},$$

$$B_2 = 4096-4096b-6912b^2+6912b^3+4944b^4-4656b^5-1947b^6+1593b^7+449b^8-291b^9-57b^{10}+27b^{11}+3b^{12}-b^{13},$$

$$B_3 = 16777216-16777216b-70254592b^2+74973184b^3+130285568b^4-150536192b^5-141090816b^6+180629504b^7$$
$$+98438400b^8-145025792b^9-45538928b^{10}+82550800b^{11}+13435990b^{12}-34393138b^{13}-1954679b^{14}+10660536b^{15}$$
$$-217647b^{16}-2468980b^{17}+191679b^{18}+423950b^{19}-50621b^{20}-52714b^{21}+7807b^{22}+4516b^{23}-745b^{24}-240b^{25}+41b^{26}+6b^{27}-b^{28},$$

$$B_4 = 524288-131072b-1900544b^2+401408b^3+3067904b^4-508416b^5-2919936b^6+343904b^7+1824928b^8-129004b^9-787516b^{10}$$
$$+21526b^{11}+239537b^{12}+2358b^{13}-51321b^{14}-1992b^{15}+7574b^{16}+436b^{17}-730b^{18}-46b^{19}+41b^{20}+2b^{21}-b^{22},$$

$$B_5 = (2-b)(2+b)\big(8-5b^2+b^4\big)\big(256-256b-288b^2+288b^3+131b^4-113b^5-30b^6+18b^7+3b^8-b^9\big)$$
$$\big(2048-5120b^2+5368b^4-3033b^6+999b^8-194b^{10}+21b^{12}-b^{14}\big),$$

$$B_6 = 262144+262144b-1146880b^2-884736b^3+2153472b^4+1334272b^5-2309376b^6-1181952b^7+1580112b^8+681296b^9$$
$$-727300b^{10}-267380b^{11}+230681b^{12}+72512b^{13}-50609b^{14}-13460b^{15}+7550b^{16}+1644b^{17}-730b^{18}-120b^{19}+41b^{20}+4b^{21}-b^{22},$$

$$B_7 = 524288-393216b-1638400b^2+1155072b^3+2314240b^4-1506816b^5-1939968b^6+1146464b^7+1066912b^8-562652b^9$$
$$-401900b^{10}+186174b^{11}+105017b^{12}-42149b^{13}-18816b^{14}+6468b^{15}+2218b^{16}-646b^{17}-156b^{18}+38b^{19}+5b^{20}-b^{21},$$

$$B_8 = 16777216-8388608b-744488896b^2+353389440b^3+149749760b^4-66715648b^5-180826112b^6$$
$$+746250224b^7+146193408b^8-554373312b^9-83514656b^{10}+29015344b^{11}+34668396b^{12}$$
$$-11079670b^{13}-10593485b^{14}+3152061b^{15}+2387902b^{16}-674356b^{17}-394139b^{18}$$
$$+107943b^{19}+46736b^{20}-12582b^{21}-3827b^{22}+1003b^{23}+198b^{24}-48b^{25}-5b^{26}+b^{27},$$

$$B_9 = \big(4+b-b^2\big)\big(4-b-b^2\big)\big(256-256b-288b^2+288b^3+131b^4-113b^5-30b^6+18b^7+3b^8-b^9\big),$$

$$B_{10} = 171798691840-137438953472b-1185410973696b^2+923417968640b^3+3873255194624b^4-2913330003968b^5$$
$$-7984478420992b^6+5747521880064b^7+11664724852736b^8-7961729564672b^9-12853507915776b^{10}+8240308551680b^{11}$$
$$+11095320592384b^{12}-6618093862912b^{13}-7688926569984b^{14}+4227545324544b^{15}+4347276336640b^{16}-2183649257216b^{17}$$
$$-2026423743224b^{18}+922102887024b^{19}+783584926352b^{20}-320516127892b^{21}-252032727146b^{22}+92028135562b^{23}$$
$$+67401169613b^{24}-21839687179b^{25}-14935123247b^{26}+4274261425b^{27}+2723360620b^{28}-686299156b^{29}-404195408b^{30}$$
$$+89631936b^{31}+48028830b^{32}-9398250b^{33}-4458010b^{34}+775894b^{35}+311228b^{36}-48928b^{37}-15358b^{38}+2238b^{39}$$
$$+477b^{40}-67b^{41}-7b^{42}+b^{43},$$

$$B_{11} = 61572651155456-527765581133248b-4958797441269766b^2+4200134418104326b^3+19070685586391046b^4-15929861902172166b^5$$
$$-46597388684492806b^6+38306469715640326b^7+81199724253675526b^8-65557030575800326b^9-107391489272709126b^{10}$$
$$+84969895200030726b^{11}+112013683693977606b^{12}-86669079126999046b^{13}-94525435543552006b^{14}+71367768014848006b^{15}$$
$$+65692396270878726b^{16}-48293616704225286b^{17}-38075504544455686b^{18}+27195863514982406b^{19}+18570400246187526b^{20}$$
$$-12859459841704966b^{21}-7668435732576246b^{22}+5137021756646566b^{23}+2691377971834406b^{24}-1740329700406886b^{25}$$
$$-804317895984706b^{26}+500915928176206b^{27}+204666505193376b^{28}-122477449774096b^{29}-44256153473876b^{30}+25385946682396b^{31}$$
$$+8098907237036b^{32}-4441425815676b^{33}-1246038339956b^{34}+651452211336b^{35}+159602074186b^{36}-79310493226b^{37}-16782366306b^{38}$$

$$+790115270b^{39}+141988286b^{40}-63124910b^{41}-9386524b^{42}+3928206b^{43}+463693b^{44}-182213b^{45}-15911b^{46}$$
$$+5867b^{47}+331b^{48}-115b^{49}-3b^{50}+b^{51},$$
$$B_{12} = 524288-393216b-1638400b^2+1155072b^3+2314240b^4-1506816b^5-1939968b^6+1146464b^7+1066912b^8-562652b^9$$
$$-401900b^{10}+186174b^{11}+105017b^{12}-42149b^{13}-18816b^{14}+6468b^{15}+2218b^{16}-646b^{17}-156b^{18}+38b^{19}+5b^{20}-b^{21},$$
$$B_{13} = 12288-12288b-24832b^2+24832b^3+21744b^4-20880b^5-10785b^6+9435b^7+3294b^8-2466b^9-620b^{10}+372b^{11}$$
$$+66b^{12}-30b^{13}-3b^{14}+b^{15}.$$

Equilibrium results in Ogawa (2006)

We present the equilibrium market outcomes, which are derived in Ogawa (2006):

$$q_0^O = \frac{\left(24-14b-4b^2+b^3\right)(a-m)}{24-18b^2+b^4}, \quad q_1^O = \frac{(1-b)\left(12-b^2\right)(a-m)}{24-18b^2+b^4},$$
$$x_0^O = \frac{\left(24-15b-3b^2+b^3\right)(a-m)}{24-18b^2+b^4}, \quad x_1^O = \frac{12(1-b)(a-m)}{24-18b^2+b^4},$$
$$p_0^O = \frac{2a(1-b)b+\left(24-2b-16b^2+b^4\right)m}{24-18b^2+b^4)},$$
$$p_1^O = \frac{3a\left(4-4b-b^2+b^3\right)+\left(12+12b-15b^2-3b^3+b^4\right)m}{24-18b^2+b^4},$$
$$\Pi_0^O = \frac{b\left(48-77b+22b^2+9b^3-2b^4\right)(a-m)^2}{\left(24-18b^2+b^4\right)^2}, \quad \Pi_1^O = \frac{2(1-b)^2\left(72-24b^2+b^4\right)(a-m)^2}{\left(24-18b^2+b^4\right)^2},$$
$$CS^O = \frac{\left(720-384b-788b^2+400b^3+161b^4-54b^5-8b^6+2b^7\right)(a-m)^2}{2\left(24-18b^2+b^4\right)^2},$$
$$W^O = \frac{\left(1008-864b-750b^2+636b^3+87b^4-66b^5-4b^6+2b^7\right)(a-m)^2}{2\left(24-18b^2+b^4\right)^2},$$
$$\Delta_0^O = \frac{(1-b)b(a-m)}{24-18b^2+b^4}\begin{cases}\geq 0, & \forall b \in [0,1), \\ < 0, & \forall b \in (-1,0),\end{cases}$$
$$\Delta_1^O = -\frac{(1-b)b^2(a-m)}{24-18b^2+b^4} \leq 0, \quad \forall b \in (-1,1).$$

Equilibrium results in Bárcena-Ruiz and Garzón (2007)

We present the equilibrium market outcomes derived in Bárcena-Ruiz and Garzón (2007):

$$q_0^{BG} = \frac{\left(24-18b-20b^2+14b^3+9b^4-5b^5-b^6+b^7\right)(a-m)}{24-38b^2+23b^4-6b^6+b^8},$$
$$q_1^{BG} = \frac{(1-b)\left(12-5b^2+b^4\right)(a-m)}{24-38b^2+23b^4-6b^6+b^8},$$
$$x_0^{BG} = \frac{\left(24-15b-23b^2+10b^3+13b^4-4b^5-2b^6+b^7\right)(a-m)}{24-38b^2+23b^4-6b^6+b^8},$$
$$x_1^{BG} = \frac{6\left(2-2b-b^2+b^3\right)(a-m)}{24-38b^2+23b^4-6b^6+b^8},$$

$$
\begin{aligned}
p_0^{BG} &= \frac{a(1-b)^2 b\left(6+6b-3b^2-3b^3+b^4+b^5\right)+\left(24-6b-32b^2+9b^3+14b^4-4b^5-2b^6+b^7\right)m}{24-38b^2+23b^4-6b^6+b^8}, \\
p_1^{BG} &= \frac{a\left(12-12b-15b^2+15b^3+8b^4-8b^5-b^6+b^7\right)+\left(12+12b-23b^2-15b^3+15b^4+8b^5-5b^6-b^7+b^8\right)m}{24-38b^2+23b^4-6b^6+b^8}, \\
\Pi_0^{BG} &= \frac{(1-b)^2 b\left(144+27b-300b^2-30b^3+276b^4+26b^5-143b^6-10b^7+47b^8+3b^9-9b^{10}+b^{12}\right)(a-m)^2}{\left(24-38b^2+23b^4-6b^6+b^8\right)^2}, \\
\Pi_1^{BG} &= \frac{(1-b)^2\left(144-240b^2+182b^4-69b^6+12b^8-b^{10}\right)(a-m)^2}{\left(24-38b^2+23b^4-6b^6+b^8\right)^2}, \\
CS^{BG} &= \frac{\begin{pmatrix} 720-576b-1620b^2+1344b^3+1493b^4-1274b^5-753b^6 \\ +698b^7+192b^8-234b^9-12b^{10}+46b^{11}-5b^{12}-4b^{13}+b^{14} \end{pmatrix}(a-m)^2}{2\left(24-38b^2+23b^4-6b^6+b^8\right)^2}, \\
W^{BG} &= \frac{\begin{pmatrix} 1008-864b-2334b^2+1884b^3+2571b^4-1930b^5-1639b^6+1136b^7 \\ +682b^8-434b^9-192b^{10}+114b^{11}+35b^{12}-20b^{13}-3b^{14}+2b^{15} \end{pmatrix}(a-m)^2}{2\left(24-38b^2+23b^4-6b^6+b^8\right)^2}, \\
\Delta_0^{BG} &= -\frac{(1-b)^2 b\left(3+3b-b^2-b^3\right)(a-m)}{24-38b^2+23b^4-6b^6+b^8}\begin{cases} \leq 0, & \forall b \in [0,1), \\ > 0, & \forall b \in (-1,0), \end{cases} \\
\Delta_1^{BG} &= \frac{b^2\left(1-b+b^2-b^3\right)(a-m)}{24-38b^2+23b^4-6b^6+b^8} \geq 0, \quad \forall b \in (-1,1).
\end{aligned}
$$

Private duopoly

In this section, we present, under both quantity and price competition, the equilibrium market outcomes in the private duopolistic setting with the $FJSV$ delegation contract when both the firms are private.

Quantity competition

$$
\begin{aligned}
\theta_i^{pC} &= \frac{b^2\left(3072-640b^2+48b^4-b^6\right)(a-m)}{16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8}, \\
q_i^{pC} &= \frac{4\left(16-b^2\right)^2\left(8-b^2\right)(a-m)}{16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8}, \\
x_i^{pC} &= \frac{64\left(128-24b^2+b^4\right)(a-m)}{16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8}, \\
p_i^{pC} &= \frac{a\left(8192-6144b^2+992b^4-60b^6+b^8\right)+4\left(16-b^2\right)^2\left(8+8b-b^2-b^3\right)m}{\left(16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8\right)^2}, \\
\Pi_i^{pC} &= \frac{4\left(16-b^2\right)^2\left(65536-57344b^2+13824b^4-1408b^6+64b^8-b^{10}\right)(a-m)^2}{\left(16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8\right)^2}, \\
CS^{pC} &= \frac{16(1+b)\left(16-b^2\right)^4\left(8-b^2\right)^2(a-m)^2}{\left(16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8\right)^2}, \\
W^{pC} &= \frac{8\left(16-b^2\right)^2\left(98304+32768b-69632b^2-12288b^3+15488b^4+1664b^5-1504b^6-96b^7+66b^8+2b^9-b^{10}\right)(a-m)^2}{\left(16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8\right)^2}, \\
\Delta_i^{pC} &= -\frac{4b^2\left(128-24b^2+b^4\right)(a-m)}{16384+8192b-8192b^2-2048b^3+1152b^4+160b^5-64b^6-4b^7+b^8} \leq 0, \quad \forall b \in (-1,1),
\end{aligned}
$$

$i = 0, 1.$

Price competition

$$\theta_i^{pB} = -\frac{b^2\left(1024 - 3456b^2 + 4400b^4 - 2827b^6 + 1010b^8 - 204b^{10} + 22b^{12} - b^{14}\right)(a-m)}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}},$$

$$q_i^{pB} = \frac{\left(16 - 9b^2 + b^4\right)^2\left(32 - 28b^2 + 9b^4 - b^6\right)(a-m)}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}},$$

$$x_i^{pB} = \frac{2\left(4-b^2\right)^2\left(2-b^2\right)\left(128 - 152b^2 + 69b^4 - 14b^6 + b^8\right)(a-m)}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}},$$

$$p_i^{pB} = \frac{\begin{bmatrix} 4a\left(2048-5120b^2+5368b^4-3033b^6+999b^8-194b^{10}+21b^{12}-b^{14}\right) \\ +\left(16-9b^2+b^4\right)^2\left(32+32b-28b^2-28b^3+9b^4+9b^5-b^6-b^7\right)m \end{bmatrix}}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}},$$

$$\Pi_i^{pB} = \frac{\left(16 - 9b^2 + b^4\right)^2\begin{pmatrix} 262144-884736b^2+1325056b^4-1159680b^6+657280b^8 \\ -252316b^{10}+66511b^{12}-11908b^{14}+1390b^{16}-96b^{18}+3b^{20} \end{pmatrix}(a-m)^2}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}^2},$$

$$CS^{pB} = \frac{(1+b)\left(16 - 9b^2 + b^4\right)^4\left(32 - 28b^2 + 9b^4 - b^6\right)^2(a-m)^2}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}^2},$$

$$W^{pB} = \frac{\left(16 - 9b^2 + b^4\right)^2\begin{pmatrix} 786432+262144b-2523136b^2-753664b^3+3630080b^4+979968b^5 \\ -3077376b^6-758016b^7+1700176b^8+385616b^9-639152b^{10} \\ -134520b^{11}+165527b^{12}+32505b^{13}-29172b^{14}-5356b^{15} \\ +3354b^{16}+574b^{17}-228b^{18}-36b^{19}+7b^{20}+b^{21} \end{pmatrix}(a-m)^2}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}^2},$$

$$\Delta_i^{pB} = \frac{b^2\left(1536 - 2720b^2 + 2020b^4 - 803b^6 + 179b^8 - 21b^{10} + b^{12}\right)(a-m)}{\begin{pmatrix} 16384+8192b-36864b^2-16384b^3+35456b^4+13984b^5-18720b^6-6588b^7 \\ +5837b^8+1841b^9-1079b^{10}-303b^{11}+111b^{12}+27b^{13}-5b^{14}-b^{15} \end{pmatrix}} \geq 0, \quad \forall b \in (-1, 1),$$

$i = 0, 1.$