Economics Bulletin

Volume 29, Issue 3

First Difference or Forward Orthogonal Deviation- Which Transformation Should be Used in Dynamic Panel Data Models?: A Simulation Study

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Abstract

This paper compares the performances of the generalized method of moments (GMM) estimator of dynamic panel data model wherein unobserved individual effects are removed by the forward orthogonal deviation or the first difference. The simulation results show that the GMM estimator of the model transformed by the forward orthogonal deviation tends to work better than that transformed by the first difference.

Citation: Kazuhiko Hayakawa, (2009) ''First Difference or Forward Orthogonal Deviation- Which Transformation Should be Used in Dynamic Panel Data Models?: A Simulation Study'', *Economics Bulletin*, Vol. 29 no.3 pp. 2008-2017.
Submitted: Jun 15 2009. **Published:** August 18, 2009.

1 Introduction

Since the seminal work of Arellano and Bond (1991), there have been many papers on the GMM estimation of dynamic panel data models. One of the typical studies in this literature is Arellano and Bover (1995) who show that the GMM estimator is invariant to the choice of transformation that removes individual effects if the transformation matrix is upper triangular and if *all* the available instruments are used.

However, in empirical studies, it is common practice not to use all instruments since it is well known that using too many instruments deteriorates the finite sample behavior, especially the bias, of the GMM estimator. In this case, the choice of transformation is considered to have an influence on the finite sample behavior of the GMM estimator. Therefore, in terms of empirical studies, the choice of transformation to be used is of great concern. However, to the best of author's knowledge, to date, no studies have investigated how different the performances of the GMM estimators are when different transformation methods are used. Thus, this paper compares the performances of the GMM estimators by Monte Carlo experiments when different transformation methods are used. Specifically, we consider the first difference (DIF) and the forward orthogonal deviation (FOD) as the transformation methods.

The rest of this paper is organized as follows. Section 2 provides the model and the GMM estimators. Section 3 provides Monte Carlo results, and Section 4 concludes the paper.

2 Setup

We consider the following dynamic panel data model:

$$\begin{aligned} y_{it} &= \alpha y_{i,t-1} + \beta x_{it} + \eta_i + v_{it} \qquad (i = 1, ..., N;\ t = 1, ..., T) \\ &= \boldsymbol{\delta}' \mathbf{w}_{it} + \eta_i + v_{it} \end{aligned}$$

where $\mathbf{w}_{it} = (y_{i,t-1}\ x_{it})'$, $\boldsymbol{\delta} = (\alpha\ \beta)'$, and $\boldsymbol{\delta}$ is the parameter of interest with $|\alpha| < 1$. η_i is the unobservable heterogeneity with $E(\eta_i) = 0$ and $var(\eta_i) = \sigma_\eta^2$, and v_{it} is an error term with $E(v_{it}) = 0$ and $var(v_{it}) = \sigma_v^2$. For the purpose of simplicity, we consider a scalar x_{it} and assume that x_{it} is a weakly exogenous variable.

We make standard assumptions in the sense of Ahn and Schmidt (1995), i.e., $E(v_{it}\eta_i) = 0$, $E(v_{it}v_{js}) = 0$, and $E(v_{it}y_{i0}) = 0$ for all i, j, t, and s with $t \neq s$.

Given a valid instrumental variable matrix $\mathbf{Z}_i$, the optimal one-step GMM estimator can be written as

$$\begin{aligned}\widehat{\delta} &= \left[\left(\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{K}_T'\mathbf{Z}_i\right)\left(\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{K}_T\mathbf{K}_T'\mathbf{Z}_i\right)^{-1}\left(\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{K}_T\mathbf{W}_i\right)\right]^{-1} \\ &\quad\times\left[\left(\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{K}_T'\mathbf{Z}_i\right)\left(\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{K}_T\mathbf{K}_T'\mathbf{Z}_i\right)^{-1}\left(\sum_{i=1}^{N}\mathbf{Z}_i'\mathbf{K}_T\mathbf{y}_i\right)\right]\end{aligned}$$

where $\mathbf{y}_i = (y_{i1}, ..., y_{iT})'$, $\mathbf{W}_i = (\mathbf{w}_{i1}, ..., \mathbf{w}_{iT})'$, and $\mathbf{v}_i = (v_{i1}, ..., v_{iT})$. $\mathbf{K}_T$ is an upper triangular matrix such that $\mathbf{K}_T\boldsymbol{\iota}_T = 0$ with $\boldsymbol{\iota}_T$ being a $T \times 1$ vector of ones. The typical examples of $\mathbf{K}_T$ are $\mathbf{D}_T$ of the first difference and $\mathbf{F}_T$ of the forward orthogonal deviation, which are defined by

$$\mathbf{D}_T = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix},$$

$$\mathbf{F}_T = diag\left[\sqrt{\frac{T-1}{T}}, ..., \sqrt{\frac{1}{2}}\right]\begin{bmatrix} 1 & -\frac{1}{T-1} & -\frac{1}{T-1} & \cdots & -\frac{1}{T-1} & -\frac{1}{T-1} & -\frac{1}{T-1} \\ 0 & 1 & -\frac{1}{T-2} & \cdots & -\frac{1}{T-2} & -\frac{1}{T-2} & -\frac{1}{T-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}.$$

Now, we define the IV matrices.[1] We consider two types of instruments, instruments in levels commonly used in practice, and instruments in backward orthogonal deviation recently suggested by Hayakawa (2009). Hayakawa (2009) shows that in AR(p) panel data models, instruments in backward orthogonal deviation is asymptotically equivalent to the infeasible optimal instruments when both N and T are large. Therefore, instruments in backward orthogonal deviation may work well in this context, too. Specifically, let us

[1]For the purpose of simplicity, we do not consider the additional moment conditions that arise from the homoskedasticity assumption (Ahn and Schmidt, 1995) and stationary initial conditions (Blundell and Bond, 1998).

define the backward orthogonal deviation of $\mathbf{w}_{it}$ as follows:

$$\mathbf{w}_{it}^{**} = \left[\mathbf{w}_{it} - \frac{\mathbf{w}_{i,t-1} + \cdots + \mathbf{w}_{i1}}{t-1}\right] \qquad t = 2, ..., T-1 \tag{1}$$

where $\mathbf{w}_{it}^{**} = (y_{i,t-1}^{**} \ x_{it}^{**})'$.

With regard to the number of instruments, we consider three types following Bun and Kiviet (2006). Let us define the following IV matrices:

$$\begin{aligned}
\mathbf{Z}_i^{LEV2} &= \text{diag}(\mathbf{z}_{i1}^{LEV2'}, ..., \mathbf{z}_{i,T-1}^{LEV2'}), \qquad \mathbf{Z}_i^{LEV1} = \text{diag}(\mathbf{z}_{i1}^{LEV1'}, ..., \mathbf{z}_{i,T-1}^{LEV1'}) \\
\mathbf{Z}_i^{BOD2} &= \text{diag}(\mathbf{z}_{i2}^{BOD2'}, ..., \mathbf{z}_{i,T-1}^{BOD2'}), \qquad \mathbf{Z}_i^{BOD1} = \text{diag}(\mathbf{z}_{i2}^{BOD1'}, ..., \mathbf{z}_{i,T-1}^{BOD1'})
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{z}_{it}^{LEV2} &= (y_{i0}, ..., y_{i,t-1}, x_{i1}, ..., x_{it})', \qquad \mathbf{z}_{it}^{LEV1} = (y_{i,t-1}, x_{it})' \\
\mathbf{z}_{it}^{BOD2} &= (y_{i1}^{**}, ..., y_{i,t-1}^{**}, x_{i2}^{**}, ..., x_{it}^{**})', \qquad \mathbf{z}_{it}^{BOD1} = (y_{i,t-1}^{**}, x_{it}^{**})'.
\end{aligned}$$

Note that the number of instruments of $\mathbf{Z}_i^{LEV2}$ and $\mathbf{Z}_i^{BOD2}$ are of order $O(T^2)$ and that of $\mathbf{Z}_i^{LEV1}$ and $\mathbf{Z}_i^{BOD1}$ are of order $O(T)$. Finally, we define $\mathbf{Z}_i^{LEV0}$ and $\mathbf{Z}_i^{BOD0}$ whose number of instruments are $O(1)$ as follows:

$$\mathbf{Z}_i^{LEV0} = \begin{pmatrix} y_{i0} & x_{i1} & 0 & 0 \\ y_{i1} & x_{i2} & y_{i0} & x_{i1} \\ \vdots & \vdots & \vdots & \vdots \\ y_{i,T-2} & x_{i,T-1} & y_{i,T-3} & x_{i,T-2} \end{pmatrix}, \ \mathbf{Z}_i^{BOD0} = \begin{pmatrix} y_{i1}^{**} & x_{i2}^{**} & 0 & 0 \\ y_{i2}^{**} & x_{i3}^{**} & y_{i1}^{**} & x_{i2}^{**} \\ \vdots & \vdots & \vdots & \vdots \\ y_{i,T-2}^{**} & x_{i,T-1}^{**} & y_{i,T-3}^{**} & x_{i,T-2}^{**} \end{pmatrix}.$$

We denote, say, the GMM estimator using IV matrix $\mathbf{Z}_i^{LEV2}$ as "GMM-LEV2," etc.

3 Monte Carlo experiments

We use the same simulation designs as Bun and Kiviet (2006). The two data generating processes (DGPs) are given by

$$y_{it} = \alpha y_{i,t-1} + \beta x_{it} + \eta_i + v_{it}$$

where

Scheme 1: $\quad x_{it} = \bar{x}_{it} + \phi_1 v_{i,t-1} + \pi\eta_i, \qquad \bar{x}_{it} = \rho\bar{x}_{i,t-1} + \xi_{it}$

Scheme 2: $\quad x_{it} = \rho x_{i,t-1} + \phi_2 y_{i,t-1} + \pi_2\eta_i + \xi_{it}.$

v_{it}, ξ_{it}, η_i are generated as $v_{it} \sim iidN(0,1)$, $\xi_{it} \sim iidN(0,\sigma_\xi^2)$ and $\eta_i \sim iidN(0,\sigma_\eta^2)$ with

$$\begin{aligned}
\sigma_\eta^2 &= \mu^2 \frac{(1-\alpha)(1+2\alpha\beta\phi_1+\beta^2\phi_1^2)}{(1+\alpha)(1+\beta\pi_1)^2} \\
\sigma_\xi^2 &= \frac{1}{\beta^2}\left[\zeta - \frac{(\alpha+\beta\phi_1)^2}{(1-\alpha^2)}\right]\frac{(1-\alpha^2)(1-\rho^2)(1-\alpha\rho)}{(1+\alpha\rho)}
\end{aligned}$$

for scheme 1, and

$$\begin{aligned}
\sigma_\eta^2 &= \mu^2\left(1+\rho^2-2\rho\frac{\alpha+\beta\phi_2+\rho}{1+\alpha\rho}\right)\left[\frac{1-\rho+\beta\pi_2}{(1-\alpha)(1-\rho)-\beta\phi_2}\right]^{-2} \\
&\quad \times\left[1-(\alpha\rho)^2-\frac{(1-\alpha\rho)}{(1+\alpha\rho)}(\alpha+\beta\phi_2+\rho)^2\right]^{-1} \\
\sigma_\xi^2 &= \frac{1}{\beta^2}(\zeta+1)\left[1-\alpha^2\rho^2-\frac{1-\alpha\rho}{1+\alpha\rho}(\alpha+\beta\phi_2+\rho)^2\right]-\frac{1}{\beta^2}\left[1+\rho^2-2\rho\frac{\alpha+\beta\phi_2+\rho}{1+\alpha\rho}\right] \\
\phi_2 &= \frac{\phi_1(1-\alpha)(1-\rho)}{1+\beta\phi_1} \\
\pi_2 &= \pi_1(1-\rho-\phi_2)-\frac{\phi_2}{1-\alpha}
\end{aligned}$$

for scheme 2. We consider $\alpha = \{0.25, 0.75\}$, $\beta = 1-\alpha$, $\rho = \{0.5, 0.95\}$, $\phi_1 = \{-1, 0, 1\}$, $\pi_1 = \{-1, 0, 1\}$, $\mu = \{0, 1, 5\}$, $\zeta = \{3, 9\}$. Thus, we have 216 designs in total. However, for scheme 2, 6 desings have negative variances for σ_ξ^2 and σ_η^2. Hence, we deleted these cases in the simulation. For T and N, we set $T = 6, N = 200$ and $T = 15, N = 200$. The number of replications is 1000.

Since reporting all the results requires large space, we report the summary of the simulation results[2]. The summary of simulation results are given in Table 1–3. In these tables, we provide the biases(BIAS), standard deviations (STD. DEV.), and root mean squared errors (RMSE).

Table 1 shows the number of times that "FOD" beats "DIF" and "DIF" beats "FOD" over 216 designs. For instance, in terms of the bias of α with scheme 1 and $T = 6$, GMM-LEV1 from the FOD model has smaller bias in absolute value than the GMM-LEV1 from the DIF model in 130 designs, and GMM-LEV1 from the DIF model has smaller bias in absolute value than the GMM-LEV1 from the FOD model in 86 designs (see the "total" part). We decompose the total result into two cases, i.e., the cases $\alpha = 0.25$, $\beta = 0.75$ and $\alpha = 0.75$, $\beta = 0.25$.

From Table 1, the followings are observed:

[2]Complete simulation results are available from the author upon request.

1. In terms of bias of α, with some exceptions in scheme 2 with $T = 6$, the GMM estomators from FOD have smaller bias than that from DIF model.
2. As T gets larger, the GMM estimator of α from FOD model tends to perform better than that from DIF model. However, for β, this tendency is not always true.
3. In terms of standard deviation, the GMM estimator from FOD model outperforms that from the DIF model in all cases.
4. In terms of RMSE, the GMM estimator from FOD model outperforms that from the DIF model in all cases.
5. There is not a significant result between two cases of $\alpha = 0.25,\ \beta = 0.75$ and $\alpha = 0.75,\ \beta = 0.25$
6. If instruments in backward orthogonal deviation is used, the GMM estimator from FOD model works better than that from DIF model.

In Table 2 and 3, we provide an average of the bias, standard deviation and RMSE over 216 designs for scheme 1 and 210 designs for scheme 2. Some remarks are in order as follows:

1. The GMM estimator from the FOD outperforms that from DIF in many cases. In some cases, the difference is significant.
2. In terms of RMSE, GMM-L2 performs best in many cases.
3. The GMM estimators using instruments in backward orthogonal deviation do not outperform that using instruments in levels. This results may be explained from the fact that the GMM estimator using instruments in backward orthogonal deviation uses $T - 1$ periods while that using instruments in levels uses T periods. Also, the nice property of the GMM estimator using instruments in backward orthogonal deviation is obtained from large N and T asymptotics. Hence, if we consider large T, say $T = 50$, the result may change.

These results suggest that the GMM estimator from FOD model tend to outperform that from the DIF model. With regard to the choice of instruments, when T is as large as $T = 15$, using all instruments in levels is the best choice in terms of RMSE. However, it should be noted that if T is large, this result may change.

4 Conclusion

In this paper, we compared the performances of the GMM estimators of the DIF and FOD models using six types of IV matrices, by Monte Carlo experiments. The simulation results showed that overall the GMM estimator of the FOD model performs better than that of the DIF model in many cases. In terms of RMSE, we found that the GMM estimator using all instruments in levels tends to perform well.

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Table 1: Number of times FOD(DIF) beats DIF(FOD)

Scheme 1: $T = 6$

IV		BIAS α		STD. DEV. α		RMSE α		BIAS β		STD. DEV. β		RMSE β	
		FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF
LEV1	$\alpha = 0.25,\ \beta = 0.75$	60	48	70	38	70	38	64	44	75	33	76	32
LEV1	$\alpha = 0.75,\ \beta = 0.25$	70	38	91	17	92	16	52	56	90	18	90	18
LEV1	total	130	86	161	55	162	54	116	100	165	51	166	50
LEV0	$\alpha = 0.25,\ \beta = 0.75$	59	49	97	11	97	11	69	39	98	10	98	10
LEV0	$\alpha = 0.75,\ \beta = 0.25$	70	38	105	3	105	3	60	48	100	8	100	8
LEV0	total	129	87	202	14	202	14	129	87	198	18	198	18
BOD1	$\alpha = 0.25,\ \beta = 0.75$	99	9	90	18	99	9	72	36	90	18	90	18
BOD1	$\alpha = 0.75,\ \beta = 0.25$	108	0	99	9	108	0	99	9	81	27	81	27
BOD1	total	207	9	189	27	207	9	171	45	171	45	171	45
BOD0	$\alpha = 0.25,\ \beta = 0.75$	72	36	99	9	99	9	54	54	81	27	81	27
BOD0	$\alpha = 0.75,\ \beta = 0.25$	90	18	108	0	108	0	63	45	108	0	108	0
BOD0	total	162	54	207	9	207	9	117	99	189	27	189	27

Scheme 1: $T = 15$

IV		BIAS α		STD. DEV. α		RMSE α		BIAS β		STD. DEV. β		RMSE β	
		FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF
LEV1	$\alpha = 0.25,\ \beta = 0.75$	74	34	70	38	70	38	75	33	74	34	74	34
LEV1	$\alpha = 0.75,\ \beta = 0.25$	85	23	86	22	86	22	61	47	89	19	89	19
LEV1	total	159	57	156	60	156	60	136	80	163	53	163	53
LEV0	$\alpha = 0.25,\ \beta = 0.75$	81	27	106	2	106	2	82	26	103	5	103	5
LEV0	$\alpha = 0.75,\ \beta = 0.25$	82	26	107	1	107	1	78	30	104	4	104	4
LEV0	total	163	53	213	3	213	3	160	53	207	9	207	9
BOD1	$\alpha = 0.25,\ \beta = 0.75$	108	0	99	9	108	0	90	18	90	18	90	18
BOD1	$\alpha = 0.75,\ \beta = 0.25$	108	0	99	9	99	9	99	9	72	36	72	36
BOD1	total	216	0	198	18	207	9	189	27	162	54	162	54
BOD0	$\alpha = 0.25,\ \beta = 0.75$	99	9	108	0	108	0	36	72	108	0	108	0
BOD0	$\alpha = 0.75,\ \beta = 0.25$	108	0	108	0	108	0	54	54	108	0	108	0
BOD0	total	207	9	216	0	216	0	90	126	216	0	216	0

Scheme 2: $T = 6$

IV		BIAS α		STD. DEV. α		RMSE α		BIAS β		STD. DEV. β		RMSE β	
		FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF
LEV1	$\alpha = 0.25,\ \beta = 0.75$	9	93	51	51	51	51	61	41	97	5	97	5
LEV1	$\alpha = 0.75,\ \beta = 0.25$	33	75	81	27	81	27	54	54	108	0	108	0
LEV1	total	42	168	132	78	132	78	115	95	205	5	205	5
LEV0	$\alpha = 0.25,\ \beta = 0.75$	22	80	102	0	102	0	36	66	102	0	102	0
LEV0	$\alpha = 0.75,\ \beta = 0.25$	66	42	108	0	108	0	63	45	108	0	108	0
LEV0	total	88	122	210	0	210	0	99	111	210	0	210	0
BOD1	$\alpha = 0.25,\ \beta = 0.75$	84	18	102	0	102	0	15	87	102	0	102	0
BOD1	$\alpha = 0.75,\ \beta = 0.25$	108	0	108	0	108	0	105	3	108	0	108	0
BOD1	total	192	18	210	0	210	0	120	90	210	0	210	0
BOD0	$\alpha = 0.25,\ \beta = 0.75$	48	54	102	0	102	0	31	71	75	27	75	27
BOD0	$\alpha = 0.75,\ \beta = 0.25$	105	3	108	0	108	0	54	54	108	0	108	0
BOD0	total	153	57	210	0	210	0	85	125	183	27	183	27

Scheme 2: $T = 15$

IV		BIAS α		STD. DEV. α		RMSE α		BIAS β		STD. DEV. β		RMSE β	
		FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF	FOD	DIF
LEV1	$\alpha = 0.25,\ \beta = 0.75$	61	42	59	43	59	43	61	41	97	5	97	5
LEV1	$\alpha = 0.75,\ \beta = 0.25$	71	37	72	36	72	36	48	60	108	0	108	0
LEV1	total	132	78	131	78	131	78	108	101	204	5	204	5
LEV0	$\alpha = 0.25,\ \beta = 0.75$	59	43	102	0	102	0	91	11	102	0	102	0
LEV0	$\alpha = 0.75,\ \beta = 0.25$	99	9	108	0	108	0	96	12	108	0	108	0
LEV0	total	158	52	210	0	210	0	187	23	210	0	210	0
BOD1	$\alpha = 0.25,\ \beta = 0.75$	102	0	102	0	102	0	55	47	102	0	102	0
BOD1	$\alpha = 0.75,\ \beta = 0.25$	108	0	108	0	108	0	107	1	108	0	108	0
BOD1	total	210	0	210	0	210	0	162	48	210	0	210	0
BOD0	$\alpha = 0.25,\ \beta = 0.75$	102	0	102	0	102	0	14	88	102	0	102	0
BOD0	$\alpha = 0.75,\ \beta = 0.25$	108	0	108	0	108	0	76	32	108	0	108	0
BOD0	total	210	0	210	0	210	0	90	120	210	0	210	0

Table 2: Average over 216 designs

Scheme 1: $T = 6$

IV		BIAS α FOD	BIAS α DIF	STD. DEV. α FOD	STD. DEV. α DIF	RMSE α FOD	RMSE α DIF	BIAS β FOD	BIAS β DIF	STD. DEV. β FOD	STD. DEV. β DIF	RMSE β FOD	RMSE β DIF
LEV2	$\alpha = 0.25,\ \beta = 0.75$	0.010	0.010	0.039	0.039	0.041	0.041	0.005	0.005	0.075	0.075	0.076	0.076
LEV2	$\alpha = 0.75,\ \beta = 0.25$	0.052	0.052	0.081	0.081	0.098	0.098	-0.014	-0.014	0.077	0.077	0.080	0.080
LEV2	total	0.031	0.031	0.060	0.060	0.069	0.069	-0.005	-0.005	0.076	0.076	0.078	0.078
LEV1	$\alpha = 0.25,\ \beta = 0.75$	0.016	0.023	0.076	0.079	0.078	0.084	0.015	0.024	0.122	0.134	0.125	0.139
LEV1	$\alpha = 0.75,\ \beta = 0.25$	0.060	0.087	0.145	0.186	0.161	0.210	-0.021	-0.038	0.121	0.142	0.126	0.152
LEV1	total	0.038	0.055	0.111	0.133	0.120	0.147	-0.003	-0.007	0.121	0.138	0.125	0.145
LEV0	$\alpha = 0.25,\ \beta = 0.75$	0.003	0.003	0.059	0.061	0.059	0.061	0.003	0.004	0.111	0.118	0.111	0.118
LEV0	$\alpha = 0.75,\ \beta = 0.25$	0.011	0.015	0.131	0.144	0.131	0.145	-0.003	-0.005	0.114	0.124	0.114	0.124
LEV0	total	0.007	0.009	0.095	0.103	0.095	0.103	0.000	0.000	0.112	0.121	0.113	0.121
BOD2	$\alpha = 0.25,\ \beta = 0.75$	0.015	0.015	0.059	0.059	0.061	0.061	0.011	0.011	0.168	0.168	0.171	0.171
BOD2	$\alpha = 0.75,\ \beta = 0.25$	0.106	0.106	0.158	0.158	0.193	0.193	-0.032	-0.032	0.169	0.169	0.175	0.175
BOD2	total	0.061	0.061	0.108	0.108	0.127	0.127	-0.011	-0.011	0.168	0.168	0.173	0.173
BOD1	$\alpha = 0.25,\ \beta = 0.75$	0.019	0.027	0.090	0.095	0.093	0.101	0.025	0.043	0.276	0.313	0.281	0.324
BOD1	$\alpha = 0.75,\ \beta = 0.25$	0.063	0.085	0.211	0.243	0.221	0.260	-0.025	-0.031	0.250	0.291	0.253	0.294
BOD1	total	0.041	0.056	0.150	0.169	0.157	0.180	0.000	0.006	0.263	0.302	0.267	0.309
BOD0	$\alpha = 0.25,\ \beta = 0.75$	0.005	0.006	0.073	0.075	0.073	0.075	0.003	0.005	0.359	0.354	0.359	0.354
BOD0	$\alpha = 0.75,\ \beta = 0.25$	0.023	0.028	0.223	0.241	0.224	0.243	-0.014	-0.012	0.369	0.395	0.369	0.395
BOD0	total	0.014	0.017	0.148	0.158	0.149	0.159	-0.005	-0.004	0.364	0.375	0.364	0.375

Scheme 1: $T = 15$

IV		BIAS α FOD	BIAS α DIF	STD. DEV. α FOD	STD. DEV. α DIF	RMSE α FOD	RMSE α DIF	BIAS β FOD	BIAS β DIF	STD. DEV. β FOD	STD. DEV. β DIF	RMSE β FOD	RMSE β DIF
LEV2	$\alpha = 0.25,\ \beta = 0.75$	0.006	0.006	0.015	0.015	0.017	0.017	0.003	0.003	0.021	0.021	0.022	0.022
LEV2	$\alpha = 0.75,\ \beta = 0.25$	0.024	0.024	0.022	0.022	0.032	0.032	-0.002	-0.002	0.021	0.021	0.022	0.022
LEV2	total	0.015	0.015	0.019	0.019	0.025	0.025	0.000	0.000	0.021	0.021	0.022	0.022
LEV1	$\alpha = 0.25,\ \beta = 0.75$	0.005	0.006	0.029	0.028	0.030	0.029	0.005	0.007	0.037	0.040	0.037	0.041
LEV1	$\alpha = 0.75,\ \beta = 0.25$	0.014	0.023	0.038	0.054	0.041	0.059	-0.001	-0.007	0.028	0.040	0.029	0.041
LEV1	total	0.010	0.015	0.034	0.041	0.035	0.044	0.002	0.000	0.032	0.040	0.033	0.041
LEV0	$\alpha = 0.25,\ \beta = 0.75$	0.001	0.001	0.020	0.025	0.020	0.025	0.000	0.002	0.028	0.040	0.028	0.040
LEV0	$\alpha = 0.75,\ \beta = 0.25$	0.001	0.001	0.034	0.047	0.034	0.047	0.000	0.001	0.027	0.038	0.027	0.038
LEV0	total	0.001	0.001	0.027	0.036	0.027	0.036	0.000	0.001	0.028	0.039	0.028	0.039
BOD2	$\alpha = 0.25,\ \beta = 0.75$	0.008	0.008	0.018	0.018	0.020	0.020	0.002	0.002	0.032	0.032	0.033	0.033
BOD2	$\alpha = 0.75,\ \beta = 0.25$	0.037	0.037	0.030	0.030	0.048	0.048	-0.008	-0.008	0.032	0.032	0.034	0.034
BOD2	total	0.022	0.022	0.024	0.024	0.034	0.034	-0.003	-0.003	0.032	0.032	0.034	0.034
BOD1	$\alpha = 0.25,\ \beta = 0.75$	0.003	0.022	0.028	0.038	0.028	0.046	0.001	0.033	0.059	0.097	0.059	0.111
BOD1	$\alpha = 0.75,\ \beta = 0.25$	0.011	0.035	0.042	0.063	0.044	0.073	-0.008	-0.014	0.055	0.079	0.056	0.081
BOD1	total	0.007	0.028	0.035	0.050	0.036	0.059	-0.003	0.010	0.057	0.088	0.058	0.096
BOD0	$\alpha = 0.25,\ \beta = 0.75$	0.000	0.001	0.021	0.025	0.021	0.025	0.002	0.001	0.079	0.100	0.079	0.100
BOD0	$\alpha = 0.75,\ \beta = 0.25$	0.001	0.002	0.037	0.055	0.037	0.055	0.001	0.002	0.075	0.151	0.075	0.151
BOD0	total	0.000	0.001	0.029	0.040	0.029	0.040	0.002	0.002	0.077	0.126	0.077	0.126

Table 3: Average over 210 designs

Scheme 2: $T = 6$

IV		BIAS α FOD	BIAS α DIF	STD. DEV. α FOD	STD. DEV. α DIF	RMSE α FOD	RMSE α DIF	BIAS β FOD	BIAS β DIF	STD. DEV. β FOD	STD. DEV. β DIF	RMSE β FOD	RMSE β DIF
LEV2	$\alpha = 0.25,\ \beta = 0.75$	0.009	0.009	0.037	0.037	0.038	0.038	-0.001	-0.001	0.116	0.116	0.116	0.116
LEV2	$\alpha = 0.75,\ \beta = 0.25$	0.052	0.052	0.075	0.075	0.093	0.093	-0.021	-0.021	0.138	0.138	0.140	0.140
LEV2	total	0.031	0.031	0.057	0.057	0.066	0.066	-0.011	-0.011	0.127	0.127	0.129	0.129
LEV1	$\alpha = 0.25,\ \beta = 0.75$	0.015	0.008	0.066	0.055	0.069	0.056	-0.001	-0.004	0.139	0.150	0.139	0.150
LEV1	$\alpha = 0.75,\ \beta = 0.25$	0.052	0.046	0.114	0.115	0.129	0.126	-0.027	-0.031	0.166	0.179	0.170	0.184
LEV1	total	0.034	0.028	0.091	0.086	0.100	0.092	-0.015	-0.018	0.153	0.165	0.155	0.168
LEV0	$\alpha = 0.25,\ \beta = 0.75$	0.001	0.001	0.049	0.052	0.049	0.052	0.004	0.003	0.140	0.148	0.140	0.148
LEV0	$\alpha = 0.75,\ \beta = 0.25$	0.007	0.011	0.110	0.121	0.110	0.121	0.000	-0.003	0.175	0.192	0.175	0.192
LEV0	total	0.004	0.006	0.080	0.087	0.081	0.088	0.002	0.000	0.158	0.171	0.158	0.171
BOD2	$\alpha = 0.25,\ \beta = 0.75$	0.009	0.009	0.053	0.053	0.054	0.054	-0.006	-0.006	0.319	0.319	0.319	0.319
BOD2	$\alpha = 0.75,\ \beta = 0.25$	0.100	0.100	0.146	0.146	0.178	0.178	-0.038	-0.038	0.358	0.358	0.362	0.362
BOD2	total	0.055	0.055	0.101	0.101	0.118	0.118	-0.023	-0.023	0.339	0.339	0.341	0.341
BOD1	$\alpha = 0.25,\ \beta = 0.75$	0.002	0.003	0.056	0.064	0.056	0.064	-0.014	-0.005	0.458	0.545	0.459	0.545
BOD1	$\alpha = 0.75,\ \beta = 0.25$	0.044	0.066	0.170	0.206	0.176	0.217	-0.026	-0.034	0.536	0.641	0.536	0.643
BOD1	total	0.023	0.036	0.115	0.137	0.118	0.142	-0.020	-0.020	0.498	0.595	0.499	0.595
BOD0	$\alpha = 0.25,\ \beta = 0.75$	0.002	0.002	0.066	0.069	0.066	0.069	-0.008	-0.003	0.818	0.800	0.818	0.800
BOD0	$\alpha = 0.75,\ \beta = 0.25$	0.026	0.033	0.223	0.239	0.225	0.242	-0.030	-0.024	0.907	0.965	0.907	0.965
BOD0	total	0.015	0.018	0.147	0.156	0.148	0.158	-0.019	-0.014	0.864	0.885	0.864	0.885

Scheme 2: $T = 15$

IV		BIAS α FOD	BIAS α DIF	STD. DEV. α FOD	STD. DEV. α DIF	RMSE α FOD	RMSE α DIF	BIAS β FOD	BIAS β DIF	STD. DEV. β FOD	STD. DEV. β DIF	RMSE β FOD	RMSE β DIF
LEV2	$\alpha = 0.25,\ \beta = 0.75$	0.007	0.007	0.015	0.015	0.017	0.017	0.002	0.002	0.028	0.028	0.028	0.028
LEV2	$\alpha = 0.75,\ \beta = 0.25$	0.023	0.023	0.020	0.020	0.031	0.031	-0.001	-0.001	0.030	0.030	0.030	0.030
LEV2	total	0.015	0.015	0.018	0.018	0.024	0.024	0.000	0.000	0.029	0.029	0.029	0.029
LEV1	$\alpha = 0.25,\ \beta = 0.75$	0.005	0.003	0.028	0.022	0.029	0.022	0.002	0.001	0.035	0.042	0.035	0.042
LEV1	$\alpha = 0.75,\ \beta = 0.25$	0.014	0.012	0.035	0.037	0.038	0.039	-0.002	-0.003	0.034	0.048	0.034	0.048
LEV1	total	0.010	0.008	0.032	0.030	0.034	0.031	0.000	-0.001	0.034	0.045	0.034	0.045
LEV0	$\alpha = 0.25,\ \beta = 0.75$	0.001	0.000	0.020	0.022	0.020	0.022	0.000	0.002	0.033	0.045	0.033	0.045
LEV0	$\alpha = 0.75,\ \beta = 0.25$	0.001	0.001	0.030	0.039	0.030	0.039	0.001	0.002	0.036	0.053	0.036	0.053
LEV0	total	0.001	0.001	0.025	0.031	0.025	0.031	0.000	0.002	0.034	0.049	0.034	0.049
BOD2	$\alpha = 0.25,\ \beta = 0.75$	0.008	0.008	0.017	0.017	0.019	0.019	-0.004	-0.004	0.049	0.049	0.049	0.049
BOD2	$\alpha = 0.75,\ \beta = 0.25$	0.035	0.035	0.028	0.028	0.045	0.045	-0.011	-0.011	0.057	0.057	0.058	0.058
BOD2	total	0.022	0.022	0.023	0.023	0.033	0.033	-0.008	-0.008	0.053	0.053	0.054	0.054
BOD1	$\alpha = 0.25,\ \beta = 0.75$	0.001	0.004	0.018	0.024	0.018	0.024	-0.006	-0.003	0.082	0.149	0.082	0.149
BOD1	$\alpha = 0.75,\ \beta = 0.25$	0.006	0.025	0.031	0.052	0.032	0.057	-0.009	-0.021	0.098	0.171	0.098	0.173
BOD1	total	0.004	0.015	0.025	0.038	0.025	0.041	-0.007	-0.012	0.090	0.161	0.090	0.161
BOD0	$\alpha = 0.25,\ \beta = 0.75$	0.000	0.001	0.020	0.022	0.020	0.022	0.003	0.003	0.170	0.225	0.170	0.225
BOD0	$\alpha = 0.75,\ \beta = 0.25$	0.000	0.002	0.036	0.059	0.036	0.059	0.004	0.006	0.178	0.384	0.178	0.384
BOD0	total	0.000	0.001	0.028	0.041	0.028	0.041	0.003	0.005	0.174	0.307	0.174	0.307