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Exchange rate volatility and noise traders: Currency Transaction Tax as an eviction device

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Abstract

The aim of the paper is to identify the impact of the currency transaction tax on the foreign exchange structure and thus its impact on exchange rate volatility. In a noise trading framework a la Jeanne and Rose (2002), we explain that the exchange rate volatility depends on fundamentals volatility and extra volatility due to the behaviour of noise traders. The exchange rate volatility is lower after introducing a Currency Transaction Tax as it increases the entry cost of noise traders and influences the range of possible equilibria. While there are multiple equilibria of exchange rate volatility without Currency Tax, there are only two aggregate exchange rate volatility corner equilibria after introducing a CTT. One of them is a low exchange rate volatility equilibrium. Moreover, we prove analytically the existence of an optimal tax rate for which the exchange rate volatility depends solely on fundamentals variance. In this case, few noise traders enter the market and there is consequently a low excess volatility.

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1 Introduction

Government intervention through financial transaction taxes is an old idea. Keynes already suggested it in the General Theory in 1936. But the idea is mainly associated with the name of Tobin who put it in concrete terms in 1972 during his Janeways Lectures. More recently Stiglitz (1989) and Summers/Summers (1989) also seize about the issue. Tobin (1972, 1974, 1978, 1996) suggested the introduction of a small transaction Tax (1% originally) on each conversion of domestic currency into any foreign currency, the primary aim of the Currency Transaction Tax (CTT) is indeed to increase monetary autonomy and decrease exchange rate volatility. The tax would, in theory, break-up the link between domestic and foreign interest rate and discourage destabilizing speculation and irrationnaly market traders activity, leading to higher market efficiency and thus exchange rate stability.

Except the major contribution of Haq, Kaul and Krunberg (1996) and the seminal paper of Frankel (1996), this topic was more often than not ideologically considered so that there are only few robust analysis. In overall however, those contributions have not reached a consensus on the likely effects of such a tax on exchange rate volatility. Moreover, while the approach of Frankel (1996) has been very fruitful, it still possesses a number of drawbacks: especially the Frankel's model is a macro monetary model of exchange rate determination with no micro specification of the trader's portfolio optimization. The model of Frankel holds only for the covered interest parity condition and thus the expectations of the traders are no taken into account[1].

In this paper, we consider the effects of a CTT on the exchange rate volatility by using and extending the macro-microstructure framework of Jeanne and Rose (2002). This work is thus in line with Xu (2005) who built a DSGE (Dynamic Stochastic General Equilibrium) Model based on the Jeanne and Rose (2002) framework in which a financial tax is introduced[2]. The benchmark model of Jeanne and Rose is very interesting because it mixes elements from the macroeconomic theory of exchange rate determination and from the noise trading approach of asset price volatility. Therefore, the aggregate exchange rate volatility can be expressed as a function of fundamental (macro) volatility and extra volatility (due to the noise trading).

We show that CTT increases the entry cost of noise traders, CTT makes decrease the benefit of entering the foreign exchange market for the noise traders and therefore leads to lower extra volatility in exchange rate. Furthermore, CTT introduction influences the range of possible equilibria. After introducing a CTT, there are only two aggregate exchange rate volatility corner equilibria. One equilibrium with high aggregate exchange rate volatility if the fundamental volatility is high and one equilibrium with low aggregate exchange rate volatility if fundamental volatility is low to moderate. Contrary to the Jeanne and Rose (2002) framework, after introducing a CTT, the multiple equilibria problem for intermediate levels of fundamental volatility disappears.

The rest of the paper is structured as follows. After presenting the model foundations in section 2, we next present the main results in section 3 and section 4 points out the concluding remarks.

[1] Bird and Rajan (2001) have extended the model of Frankel (1996) adding some expectations only in the interest parity condition, not in the trading behaviour.

[2] Since the writting of this article, an other study in line with the Jeanne and Rose (2002) model and considering the CTT effects on exchange rate volatility has been developed by Shi and Xu (2009, forthcoming at Canadian Journal of Economics).

2 Model foundations

Following Jeanne and Rose (2002), our theoretical baseline melts two separate branches of exchange rate theory: the macroeconomic theory of exchange rate determination (2.1) and the noise trading approach to financial markets (2.2). Our contribution consists in adding a CTT to study its impact on exchange rates determination.

2.1 Macroeconomic side: a monetary model

On the macroeconomic side, we used the conventional monetary model (see Mussa (1976, 1979) and Frenkel (1976) and see Neely and Sarno (2002) for a survey).

We consider two countries "'domestic"' and "'foreign"' and assume that the domestic country is a small open economy (periphery) and the foreign country is a developed country which is the core of the international financial system. Following Jeanne and Rose (2002, see equations (1) to (4)), prices are assumed to be perfectly flexible and purchasing power parity is only satified in average: thus the log of the foreign exchange rate (e) is the ratio of prices levels and a normal shock (ϵ_t), so that:

$$e_t = (m_t - m_{t*}) + \alpha(i_t - i*_t) + \epsilon_t \tag{1}$$

where α is a parameter, m is the natural logarithm of the money stock and i is the natural logarithm of the interest rate. The foreign country is denoted with an asterisk and all foreign variables are assumed to be in a constant steady state in order to put the emphasize on the small domestic country. Foreign price level is constant, then all foreign variables are expressed in real terms.

Two different kinds of bonds are assumed: the domestic bonds are issued by the domestic country in local currency (Peso as in Jeanne and Rose (2002)) and foreign bonds are denominated in foreign currency (in dollars).

2.2 Microeconomic side: trading behavior

On the microeconomic side, our model incorporates microstructural mechanisms (see Lyons (2001) about microstructure theory). Consider that traders are modelled as overlapping generations of investors. They live for two periods in a mean-variance *à la Markowitz* (1952) framework and allocate their endowment between a domestic risky bond and a foreign risky less bond. At each period t, the new born traders j receive an endowment W denominated in dollars and form a continuum. At t, each trader knows the nominal interest rates and decides wether or not to enter the Peso bond (and consequently the foreign exchange) market. We assume that the entry decision is made at time $t-1$, i.e. before the shocks are revealed.

According to Jeanne and Rose (2002), there are two kinds of traders : N_I informed traders ($j = 1, ..., N_I$) and N_N noise traders $N_N(j = N_I + 1, ..., N)$. They have the same endowments, they are both risk adverse, but they are heterogeneous according to their expectations of exchange rate and excess return: informed traders are knowledgeable about the economic fundamentals, can process information costlessly and make rational expectations about the future; non informed traders or so called noise traders have imperfect knowledge of fundamentals determinants of exchange rate. Thus, following Jeanne and Rose (2002), noise traders are restricted to unbiased signals:

- informed traders $j = I$ have rational expectations about both expected and variance of the excess return

$$E_t^I(\delta_{t+1}) = E_t(\delta_{t+1}) \tag{2}$$

$$Var_t^I(\delta_{t+1}) = Var_t(\delta_{t+1}) \tag{3}$$

- non informed traders $j = N_N$ perceive only the the second moment of the excess return correctly; the first moment is assumed to be affected by a non systematic (unlike De Long et al., 1990) noise like in Jeanne and Rose (2002)

$$E_t^N(\delta_{t+1}) = E_t(\delta_{t+1}) + \nu_t = \overline{\delta} + \nu_t \tag{4}$$

$$Var_t^N(\delta_{t+1}) = Var_t(\delta_{t+1}) \tag{5}$$

where $\overline{\delta}$ is the unconditional mean of the excess return (or average risk premium), ν_t is a stochastic $i.i.n$ shock consecutive to news in t. Because this noise is not correlated with macroeconomic monetary fondamentals (like m in the model), it is a kind of excess exchange rate volatility.

Following Jeanne and Rose (2002), we link the random error to economic uncertainty by linking noise to unconditionnal variance of the exchange rate:

$$Var(\nu_t) = \lambda Var(e_t), \tag{6}$$

with $\lambda \in R^+$. In other words, the more the uncertainty is, the more the noise trader error is. The λ parameter is like an acceleration factor.

A trader who has entered in the foreign exchange market invests B_t^{j*} in Peso bonds to maximise the expected utility of his portfolio. We make the assumption that, at each period, all informed traders enter in the foreign exchange market because the entry is costless. In contrary to informed traders, the noise traders decide whether or not to enter in this market by comparing the entry cost denoted k (due to their processing information cost) and expected benefit of diversifying their portfolio. In this way, the Peso bonds market entry decision is endogenised. If ϕ is a binary decision parameter, the decision of wether or not entering in the domestic bond market can be written as:

$$E_{t-1}^j\left(U_t^j/\phi_t^j = 1\right) \geq E_{t-1}^j\left(U_t^j/\phi_t^j = 0\right) \tag{7}$$

where U_t^j denotes the trader utility at time t and E_{t-1}^j the investor expectation conditional on the information available at time $t-1$.

The optimal demand for Peso bonds (B_t^{j*} or $B_t^{j*\tau}$) is a solution of the following maximization problem[3]:

$$\max_{B_t^{j*}} E_t^j\left(W_{t+1}^j\right) - \frac{a}{2}Var_t^j\left(W_{t+1}^j\right), \tag{8}$$

$$\text{s.t } W_{t+1}^j = (1+i^*)W + \phi(B_t^{j*}\delta_{t+1} - k) \text{ (8')}$$

where W_{t+1}^j is a random variable that denotes the end-of-life portfolio wealth, $E_t^j\left(W_{t+1}^j\right)$ is the conditional (to j and t) expected value of the portfolio wealth and a is the coefficient of absolute risk aversion, with $a > 0$.

Finally, we assume that traders need a risk premium to hold domestic Peso bonds. Note that δ_{t+1} is the excess return (or the risk premium) on domestic bonds and is defined by:

[3]We make the assumption that the risk premium is normally distributed, see Jeanne and Rose (2002) or Bauer and Herz (2006).

$$\delta_{t+1} = i_t - (e_{t+1} - e_t) - i^*$$

2.3 Currency Transaction Tax introduction

Assume that a Currency Tax is introduced for all traders. In this case, the constraint (8') in the previous system should be substituted by a new one (8"):

$$W_{t+1}^j = (1+i^*)W + \phi((B_t^{j*\tau}\delta_{t+1} - \tau X) - k) \text{ (8")}$$

Note that X is the taxed part of the excess return: $X = \gamma\delta$ where γ is a coefficient of fiscal avoidance close to 1. In addition, the CTT rate τ is given as: $0 < \tau < 1$. Besides, considering currency taxes literature, the tax rate would be at a very low level (0.1% for instance) so as to avoid drowning the transaction costs on the Forex which are currently at very low level.

3 Results

Equilibrium

An equilibrium in the model of Jeanne and Rose (2002) consists of a stochastic process for the exchange rate $\{e_t\}$, the excess return $\{\delta\}$ and individual and optimal trader's decisions $\left\{\phi_t^j\right\}$ and $\left\{B_t^{j*}\right\}$, such that at each period ϕ_t^j satisfies the entry condition (7), B_t^{j*} is the solution of the portfolio maximization problem and the market clearing condition for domestic bonds holds ($\overline{B} = \sum \phi_t^j B_t^{j*}$, where $\overline{B}$ is the constant supply of Peso bonds). Following Jeanne and Rose (2002), we solve the model by a "'guess and verifiy"' technique, first assuming the shocks as i.i.d. distributed, then checking that those properties are satisfied.

The analysis is executed in two steps. In the first step, the demand for foreign bonds is computed at each period, before and after the introduction of a CTT. Considering first the number of noise traders as exogenous (i.e. there is no entry condition), we can deduce the optimal demand for bonds denominated in Peso ($B_t^{j*\tau}$), the average risk premium ($\overline{\delta}$) and the variance of exchange rate ($Var\left(e_t\right)$). On this basis, the number of noise trader is endogenized in the second step.

Step 1: Exogenous number of noise traders

First, we can get the optimal demand for Peso bond by an individual trader by maximizing the mean-variance objective function (8). Next, using market clearing condition in the Peso bond market, we can derive the expression of the average risk premium and consequently solve for the equilibrium exchange rate and finally its variance (i.e. the volatility). By resolving for the model anew after introducing a CTT, the following results are derived:

1) In line with Jeanne and Rose (2002), we show that the exchange rate volatility[4] is increasing in the fundamental volatility (money demand and prices denoted $Var(m+\epsilon)$) and in the number of noise traders (n):

$$Var\left(e_t\right) = \frac{Var(m+\epsilon)}{(1+\alpha^2) - \lambda\left(\frac{n}{N_I}\right)\alpha^2}.$$

[4]See the equation (20) in the appendix.

In others words, the aggregate exchange rate volatility ($Var(e_t)$) consists in a fundamental volatility (in the numerator) and in an extra volatility due to the noise traders n. Given the fundamental volatility, the minimal exchange rate variance is thus reached when there is zero noise traders entering the market and conversely the maximal exchange rate volatility is reached when the number of noise traders (n) is maximum. An exogenous increase in the number of noise traders increases unambiguously the variance of the exchange rate and tends to raise the risk premium. However[5], increasing the number of noise traders leads also to decrease the risk and the average risk premium since $\overline{\delta}$ is decreasing in n:

$$\overline{\delta} = \frac{aVar(e)\overline{B}}{(N_I+n)}.$$

Thus, the impact of noise traders in equilibrium is non monotonic. The noise traders have indeed two counter-acting roles in the model: they both create and share risk at the same time.

2) Our first main result is that the exchange rate volatiliy is lower after introducing a CTT when the number of noise traders is exogenous:

$$Var^{\tau}(e_t) = \frac{Var(m+\epsilon)}{\left\{(1+\alpha^2)-\alpha^2\left[\lambda\left(\frac{n}{N_I}\right)^2-\tau^2\left(\frac{N_I+n}{N_I}\right)^2+\frac{\tau^4(N_I+n)^2-N_I^2}{N_I^2}\right]\right\}}.$$

Proposition 1. *The CTT decreases the exchange rate volatility level when the number of noise traders is given.* See **Proof 1** in appendix.

So, this result is in line with of Tobin conjectures (1978, 1996) and in accordance with the main experimental (or simulation based) studies like Ehrenstein et al. (2005), Westerhoff and Dieci (2006), Kaiser et al. (2006) and Bianconi et al. (2007) but it opposes to Hanke et al. (2007).

Step 2: Endogenous number of noise traders

Now, we consider that the entry decision of the noise trader is endogenized. As in step 1, an equilibrium in the model is first computed without CTT and then a CTT is introduced.

Some remarks to address:

1) In line with Jeanne and Rose (2002) and Chen (2007), we note that the informed traders always enter the domestic bond market since they bear no entry costs, while noise traders enter the market only if the gross benefit of entry exceeds their entry cost. We establish that, when they enter the domestic bond market, the benefit of entry for noise traders is an increasing function with the expected unconditionnal mean of the excess return (i.e. the risk premium) and decreasing with the adverse risk coefficient and the exchange rate variance (see (23) in proposition 2).

However, as we seen above, in equilibrium, both the risk premium and variance of the exchange rate are functions of the number of noise traders. So, we have a ciruclarity process: $n = f(\overline{\delta}, Var(e))$ with $\overline{\delta} = g(n)$ and $var(e) = h(n)$.

2) This circularity process generates multiple equilibria with a range of entry by noise traders as derived in the **Corollary 1**. Since the variance of exchange rate is a monotonic function of the number of noise traders, there are a range of excess volatility equilibria with respect to the level of fundamental variance:

- First, low fundamental variance

In this context, we show that an unique stable low exchange rate volatility equilibrium emerges in which no noise traders enter the Peso bond market.

[5]See equation (16) in the appendix.

- Second, high fundamental variance

In this context, we derive a stable high exchange rate volatility equilibrium and a high number of noise traders which are attracted by a high risk premium they generated themselves.

- Third, intermediate levels of fundamental variance

There exist multiple equilibria (with low to high entry by noise traders and low to high aggregate exchange rate volatility). This class of equilibrium is at the heart of the study of Jeanne and Rose (2002), because in such a context their model can generate different levels of aggregate exchange rate volatility for the same level of macroeconomic volatility. The excess variance is indeed a function of the number of noise traders.

3) After introducing a CTT, there is some evidence that the entry benefit for noise traders always decreases (see **Proposition 2**) and thus the number of noise traders entering the market follows the same pattern.
4) Based on Proposition 2, we get the main result of our analysis. When a CTT is introduced, we get two class of stable corner equilibria[6]: i) a unique stable low equilibrium of aggregate exchange rate volatility with a low level of fundamental variance and, ii) a unique high aggregate exchange rate volatility equilibrium with a high level of fundamental variance. But, and this is the more interesting result of our analysis, considering the magnitude of the decline of the entry benefit caused by the tax rate, there is only a unique stable low equilibrium for intermediate levels of fundamental variance. This result is opposed to the Jeanne and Rose (2002) results where there exist a multiple equilibria situation for intermediate levels of fundamental variance. The following **Proposition 3** can thus be derived.

Proposition 3. *When a low level CTT is introduced, there exist a unique stable equilibrium for intermediate levels of fundamental variance. In this case, few noise traders enter the market and there is consequently a low excess volatility.* See **Proof 3** in appendix.

This analytical result can be illustrated by some simulation exercise (see figure 1). The set of simulation in line with Jeanne and Rose (2002) study except for the introduction of a 0.1% CTT rate. It emerges that the new noise trader j enters the market only if the number of noise traders already in the Peso bond market is low (lower than 0.2% in our simulation).

insert figure 1

Investigating the gross benefit curve (GNB) with high and very high levels of tax rate (1% to 10%), we get the result that no noise trader is willing to enter in the foreign exchange market, because the benefit curve is always under the cost curve (see figure 1 in the left setting). In an intuitive manner, the more higher the tax rate is, the more lower the number of noise traders entering the market is.
Overall, considering the **Proposition 3** and our simulation exercise, there is some evidence that there are only two unique stable equilibria when a CTT is introduced: one with a low level of aggregate exchange rate volatility when the fundamental volatility is low to high; one with high aggregate exchange rate volatility for high levels of fundamental variance. The CTT introduction influences the range of possible exchange rate equilibria and eliminates the problem of multiple equilibria for intermediate levels of fundamental volatility.

[6]Stable "‘corner equilibrium"’ means that there is zero (or respectively a maximum number of) noise traders entering the market

The figure 2 summarizes the effects of a CTT on the aggregate exchange rate volatility equilibria. It points out the multiple equilibria generated by the model without CTT for intermediate levels of fundamental variance (in the middle of the left setting). Nevertheless, after introducing a CTT (in the right setting), there is only two unique corner equilibria: for low to moderate level of fundamental variance, the exchange rate volatility stands always at low level. On the contrary, for high level of fundamental variance, the aggregate exchange rate variance remains at high level.

insert figure 2

5) Our simulations show that the GNB function move downward when the tax rate is increased. We indeed show in **Propostion 4** that the GNB function is decreasing in the tax rate.

We can finally analytically prove the existence of an optimal low level tax rate for which the marginal benefit is ever lower than the entry cost as underlined by the **Proposition 4**.

Proposition 4. *There is an optimal CTT rate τ^* for which the marginal benefit level, GNB^τ, is always smaller than the entry cost k. In this case, no noise trader enters the foreign exchange market. Therefore, the aggregate volatility is solely a function of the fundamental volatility.* See **Proof 4** in appendix.

The optimal tax rate is thus an increasing function of the entry cost (k), the supply of bonds ($\overline{B}$), the amont taxed (X) and the number of noise traders (n):

$$\tau^* = \frac{(-4kN_I^4 - 8kN_I^3)(\frac{1}{2}+\alpha+\alpha^2) + \left[(n^2k)((2\lambda-4)\alpha^{2-4\alpha-2)+(aVar(m+\epsilon)\overline{B}^2}\right]N_I^2 + 2k\alpha^2\lambda(2N_In^3+n^4)}{4\overline{B}(N_I+n-1)((\frac{1}{2}+\alpha+\alpha^2)N_I^2 - \frac{\lambda\alpha^2n^2}{2})X}.$$

In other words, the higher the entry cost, the amount invested and the number of noise traders aldready in the market are, the higher the optimal tax rate should be to unseat the noise traders. In this case, there is no more noise traders in the Peso bond market and since the *excess* (contrary to *fundamental*) volatility is a function of the number of noise traders, there is only an unique low stable exchange rate volatility equilibrum for low to intermediate fundamental variance.

4 Concluding remarks

A Currency Transaction Tax changes the structure of the foreign exchange market by crowding out the noise traders, whatever their types. Noise traders can be speculators or not, short termists or not. This property leads to diminish excess volatility and *in fine* dampens the aggregate volatility. All in all, the CTT influences the range of possible exchange rate equilibria. The introduction of a CTT in the Jeanne and Rose framework (2002) corrects for the multiple equilibria problem for moderate levels of fundamental volatility. Indeed, in such a context, with CTT there is a unique equilibrium with few noise traders.

Moreover, we derive an optimal level of tax rate for which zero noise traders enter the market. The excess volatility is thus at its minimum. The CTT indeed decreases the excess return and consequently the risk premium that noise traders create when they trade. In a sense, our study is in line with Summers and Summers (1989), Griffith Jones (1996), Frankel (1996) and Arestis and Sawyer (1997) among others, who consider Transaction Taxes as a mean to diminish the mimetic behaviors of traders in order to enhance the financial stability.

5 Appendix

We will show how to obtain the results outlined in Section 3.

Step 1 Results.

Proposition 1. *The CTT decreases the exchange rate volatility when the number of noise traders is given.*

Proof 1.

Assuming that the risk premium δ_t is normally distributed, we get the optimal demand for domestic Peso bonds by each individual trader by maximizing the problem (8).

The mean-variance function is quadratic and thus concave. Given the strict positivity of the bond demand, B_t^{j*} is a point which achieves its maximum.

Assuming $Var(\delta_{t+1}) = Var(e_{t+1})$, the optimal Peso bonds demand can be rewritten as:

$$B_t^{j*} = \frac{1}{a} \frac{E_t^j(\delta_{t+1})}{Var_t^j(e_{t+1})}. \tag{9}$$

We can get the following lemma in accordance with Jeanne and Rose (2002).

Lemma 1. *The optimal demand for bonds is increasing with the risk premium but decreasing with the exchange rate volatility and the coefficient of absolute risk aversion.*

We now introduce a CTT and maximize (8') under (8") constraint.

We can derive the first order condition:

$$E_t^j(\delta_{t+1}) - \tau X - aB_t^{j*} Var_t^j(\delta_{t+1}) + aB_{t+1}^{j*}\tau^2 Var_t^j(X) = 0. \tag{10}$$

Hence, we can obtain anew the optimal demand for Peso bonds[7]:

$$B_t^{j*\tau} = \frac{E_t^j(\delta_{t+1}) - \tau X}{aVar_t^j(\delta_{t+1}) - a\tau^2 Var_t^j(X)}. \tag{11}$$

Assuming that the excess return, the exchange rate variance and the taxed part of the risk premium are equal to the same constant, that is $Var(\delta_{t+1}) = Var(e_{t+1}) = Var(X)$, the optimal Peso bonds is equivalent to:

$$B_t^{j*\tau} = \frac{E_t^j(\delta_{t+1}) - \tau X}{aVar_t^j(e_{t+1})(1-\tau^2)}. \tag{12}$$

Lemma 1'. *In an intuitive manner, the demand for Peso bonds after introduced the CTT is lower than before introducing the CTT.*

Therefore, the effect of the CTT can be mesuring by evaluating the sign of the following expression:

$$\frac{1}{a} \frac{E_t^j(\delta_{t+1})}{Var_t^j(e_{t+1})} - \frac{E_t^j(\delta_{t+1}) - \tau X}{aVar_t^j(e_{t+1})(1-\tau^2)}. \tag{13}$$

The sign of (14) is positive if $\tau < \frac{X}{E_t^j(\delta_{t+1})}$, that is the demand for Peso bonds decreases after introducing a CTT. Hence, given the strict following condition $\tau < 1$, the sign of (14) is always positive.

[7]Note that all variables computed after introducing a CTT are expanded with τ.

Considering **lemma 1** and **lemma 1'**, we can solve for the average risk premium and the variance of exchange rate, before and after introducing a CTT as follows:

The Peso bonds market clearing condition is:

$$\overline{B} = N_I \frac{E_t^j(\delta_{t+1})}{aVar_t^j(\delta_{t+1})} + (1 - N_I) \frac{E_t^j(\delta_{t+1})}{aVar_t^j(\delta_{t+1})}. \tag{14}$$

Rearranging yields the market clearing condition:

$$\overline{B} = \frac{E_t(\delta_{t+1}) + (1 - N_I)\nu_t}{aVar_t(\delta_{t+1})}. \tag{15}$$

Applying the same scheme given (12) and assuming that $Var(\delta_{t+1}) = Var(e_{t+1})$, we can deduce the market clearing condition after introducing a CTT:

$$\overline{B}^{\tau} = \frac{E_t(\delta_{t+1}) + (1 - N_I)\nu_t - \tau X}{aVar_t(\delta_{t+1}) - a\tau^2 Var_t(X)}. \tag{16}$$

From (16), we get the average risk premium without CTT:

$$\overline{\delta} = \frac{aVar(e)\,\overline{B}}{(N_I + n)}. \tag{17}$$

Applying the same scheme with (17), we derive the risk premium with CTT:

$$\overline{\delta}^{\tau} = \frac{aVar(e)\left(1 - \tau^2\right)\overline{B}^{\tau} + \tau X}{(N_I + n)}. \tag{18}$$

Next, following the "'Guess and Verify"' technique and assuming that the domestic supply m_t, follows a iid normal process centered on $\overline{m}$, in accordance with Jeanne and Rose (2002), we get the volatility expressions[8].

Assuming that $Var_t^j(\delta_{t+1}) = Var_t^j(e_{t+1})$ and[9] $\sqrt{\lambda} < \frac{(1+\alpha)N_I}{\alpha n}$,by combining (9), (16), (17) and rearranging, we get the variance of exchange rate:

$$Var(e_t) = \frac{Var(m + \epsilon)}{(1 + \alpha^2) - \lambda\left(\frac{n}{N_I}\right)\alpha^2}. \tag{19}$$

In the same scheme and assuming $\lambda < \left(\frac{N_I}{n}\right)^2\left(2 + \frac{1}{\alpha^2}\right) - \tau^2\left(\frac{N_I+n}{n}\right)^2$ to ensure the exchange rate volatility is positive for all n, we next turn the variance of exchange rate with CTT:

$$Var^{\tau}(e_t) = \frac{Var(m + \epsilon)}{\left\{(1 + \alpha^2) - \alpha^2\left[\lambda\left(\frac{n}{N_I}\right)^2 - \tau^2\left(\frac{N_I+n}{N_I}\right)^2 + \frac{\tau^4(N_I+n)^2 - N_I^2}{N_I^2}\right]\right\}}. \tag{20}$$

Finally, that the difference between (20) and (21) is positive if:

$$(1 + \alpha)^2 - \alpha^2\left[\lambda\left(\frac{n}{N_I}\right)^2 - \tau^2\left(\frac{N_I + n}{N_I}\right)^2 + \frac{\tau^4(N_I + n)^2 - N_I^2}{N_I^2}\right] > (1 + \alpha)^2 - \alpha^2\lambda\left(\frac{n}{N_I}\right)^2. \tag{21}$$

[8]The integral proof will be provided upon request.

[9]The intuition behind this follows the fact that we assume only positive exchange rate variance.

It follows that:

$$\frac{\alpha\tau\left(N_I+n\right)\left(1-\tau\right)+_I}{N_I}>0. \tag{22}$$

Therefore, given the positive signs of all parameters (α, τ, N_I, n), the preceding condition is always satisfied. This implies that the exchange rate volatiliy is always minimal after introducing a CTT when the number of noise traders is exogenous.

Step 2 Results.
Proposition 2. *The entry benefit of noise traders always decreases after introducing a CTT.*
Proof 2.
Contrary to the informed traders, the noise traders decide wether or not to enter the market by comparing the entry cost and her gross benefit of entry, that is $GNB(\overline{\delta}, Var(e)) \geq k$. Combinig (7) and (8) and assuming that the entry costs does not affect the optimal demand for Peso bonds, we get:

$$GNB=\frac{E\left(\delta_{t+1}\right)^2}{2aVar\left(e\right)}=\frac{\overline{\delta}^2}{2aVar\left(e\right)}. \tag{23}$$

Introducing a CTT, we get:

$$GNB^{\tau}=\frac{\overline{\delta}^2-2\overline{\delta}\tau X+\tau X^2}{2aVar\left(e\right)\left(1-\tau^2\right)}. \tag{24}$$

The CTT can reduce the entry of noise traders if the difference between (24) and (25) is positive, that is:

$$\frac{2E\left(\delta_{t+1}\right)\tau X-\left(\tau X\right)^2-\tau E\left(\delta_{t+1}\right)^2}{2aVar\left(e_{t+1}\right)\left(1-\tau^2\right)}>0. \tag{25}$$

Since $a > 0$ and $Var\left(e_{t+1}\right) > 0$, the denominator of (26) is > 0 when $\tau < 1$. Seeing that the tax rate is always smaller than 1, the denominator is always positive.
From (24), it is thus easy to show that the model generate multiple equilibria as denoted by the Corollary 1.

Corollary 1[10]. *For intermediate levels of fundamental variance, there exist multiple equilibria with a range of entry by noise traders.*
Combining (18), (20) and (24) and by varying n, we can see that GNB is a non-monotonic function of n:

$$GNB=\underbrace{\frac{1}{2\left[\left(1+\alpha\right)^2-\frac{\lambda n^2\alpha^2}{N_I^2}\right]}}_{\uparrow when\ n\uparrow}\underbrace{\frac{a\overline{B}^2Var\left(m+\epsilon\right)}{\left(N_I+n\right)}}_{\downarrow when\ n\uparrow}. \tag{26}$$

There are consequently more than one interception of GNB and k. See also the figure 1 in the left setting.

Proposition 3. *When a low level CTT is introduced, there exist an unique equilibrium with a few number of noise traders entering the market for intermediate levels of fundamental variance and a low excess volatility.*

[10] See also Chen (2007) who used the Jeanne and Rose model (2002).

Proof 3.
Considering the Corollary 1, we compute the benefit entry function with CTT by substituting the risk premium (19) and the volatility (21) in the entry benefit (25), we get :

$$GNB^{\tau} = \frac{\left(K - 2K\tau X + \tau^2 X^2\right)\left\{(1+\alpha^2) - \alpha^2\left[\lambda\frac{n^2}{N_I^2} - \frac{\tau^2(N_I+n)}{n^2} + \frac{\tau^4(N_I+n)^2 - N_I^2}{N_I^2}\right]\right\}}{2aVar\,(m+\epsilon)\,(1-\tau^2)} \tag{27}$$

where $K = \left(\frac{aVar(m+\epsilon)\left(1-\tau^2\right)\overline{B}^{\tau}}{(1+\alpha^2)-\alpha^2\left[\lambda\frac{n^2}{N_I^2} - \frac{\tau^2(N_I+n)}{n^2} + \frac{\tau^4(N_I+n)^2-N_I^2}{N_I^2}\right]} + \tau X\right) / \left(N_I + n\right)$.
It is possible to approximate (28) by employing second-order Taylor series expansion. By this, we assume a very low tax rate τ. The new function is denoted GNB^{Tayl}:

$$GNB^{Tayl} = \frac{\overline{B}}{2(N+n)} + \frac{\frac{X}{N+n} - \frac{2aVar(m+\epsilon)\overline{B}X}{\left(1+\alpha^2-\alpha^2\left(\frac{\lambda n^2}{N^2}-1\right)\right)}}{2aVar(m+\epsilon)} \tag{28}$$

where $Z = \left(1+\alpha^2 - \alpha^2\left(\frac{\lambda n^2}{N^2} - 1\right)\right)$.

Hence, differentiating GNB^{Tayl} with respect to n and collecting to τ, we get:

$$\frac{\underbrace{\left(-X\lambda\alpha^2 n^2 - 2XN\lambda\alpha^2 n - 2XN^2\alpha^2 + 2XaVar(m+\epsilon)\overline{B}N^2 - XN^2\right)\tau}_{\approx 0}\underbrace{-aVar(m+\epsilon)N^2}_{<0}}{\underbrace{2aVar(m+\epsilon)N^2(N+n)^2}_{>0}}.(29)$$

Since τ is near zero and all parameters are positive, (30) is negative. The GNB function is thus purely decreasing in n. Therefore, a low CTT may end up with a unique benefit function equilibrium[11]

Proposition 4. *There is an optimal CTT rate τ^* for which the benfit marginal level, GNB^{τ}, is always smaller than the cost of entry k. In this case, no noise trader enters the foreign exchange market. Therefore, the aggregate volatility is solely a function of the fundamental volatility.*

Proof 4.
We can differentiate GNB^{Tayl} with respect to the tax rate τ:

$$\frac{\left(X\left(Z - 2aVar(m+\epsilon)\right)/(N+n)\right)}{2aVar(m+\epsilon)}. \tag{30}$$

Note that the denominator of (31) is always positive and the numerator is negative if $2aVar(m+\epsilon)BN^2 + \lambda\left(\alpha n\right)^2 > N^2\left(1+2\alpha^2\right)$.
Considering the parameters, these conditions are satisfied and the benefit function is positive and thus decreasing in low levels of tax rate.

[11] The same result can be derived by employing a first-order Taylor expansion: in this case, the derivative we get is $-\frac{\overline{B}}{(N+n)^2}$.

Finally, we can compute the optimal tax rate for which the entry benefit is always below the entry cost:

$$GNB - k < 0. \tag{31}$$

By taking (31) and substituting out τ, we can thus compute the optimal level tax rate for which the the benefit GNB is samller than the cost of entry k:

$$\tau^* = \frac{(-4kN_I^4 - 8kN_I^3)(\frac{1}{2} + \alpha + \alpha^2) + \left[(n^2 k)((2\lambda - 4)\alpha^{2-4\alpha-2)+(aVar(m+\epsilon)\overline{B}^2}\right] N_I^2 + 2k\alpha^2\lambda(2N_I n^3 + n^4)}{4\overline{B}(N_I + n - 1)((\frac{1}{2} + \alpha + \alpha^2)N_I^2 - \frac{\lambda\alpha^2 n^2}{2})X}. \tag{32}$$

The optimal tax rate is thus an increasing function with the entry cost (k), the supply of bonds ($\overline{B}$), the amount taxed part (X) and the number of noise traders (n).

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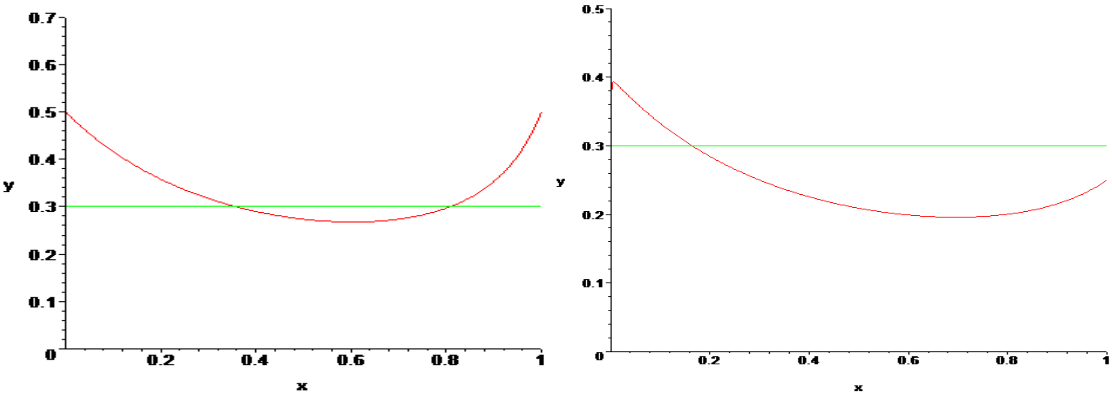
Figure 1: Benefit of entry without and with CTT

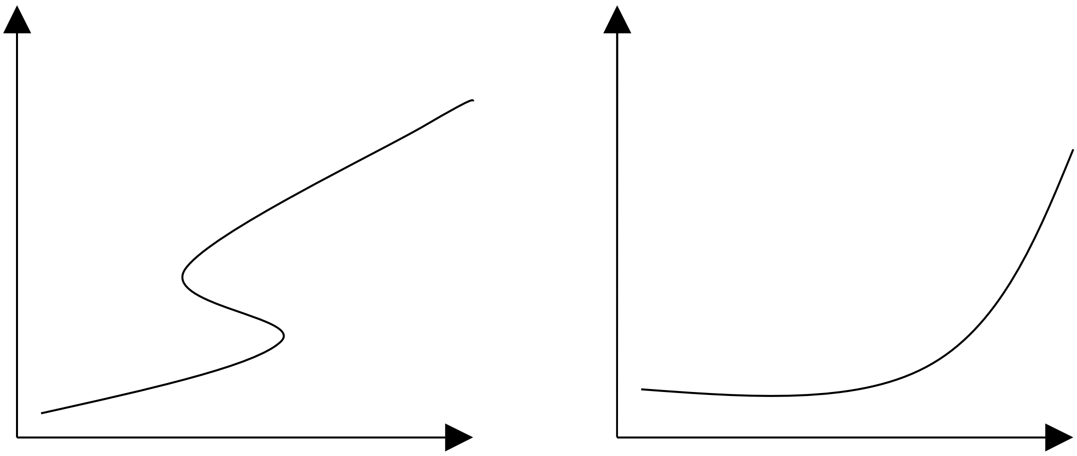
The introduction of a CTT of 0.1% makes decrease the benefit of the noise traders



Lecture: x is the number of noise traders (x-axis) and y is the benefit of entry (y-axis)

Figure 2: Exchange rate volatility equilibria before and after introducing a CTT

For intermediate levels of fundamental variance, multiple equilibria disappear after introducing a low CTT



Lecture: exchange rate variance (y-axis) is a function of fundamental variance (x-axis)