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Mixed oligopoly and spatial agglomeration in quasi-linear city

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Abstract

We apply a spatial model that includes both circular-city and linear-city models as special cases to the analysis of location-quantity model in mixed oligopoly. We find that the equilibrium pattern continuously moves from that of the circular-city to that of the linear-city and that the linear-city result is more likely in our setting as the equilibrium location.

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1. Introduction

Since the seminal work of Hotelling (1929), a rich and diverse literature on spatial competition has emerged. There are two standard models of the space, the linear-city and the circular-city models. Two models often yield completely different equilibrium outcomes. For example, consider a two stage location-quantity models formulated by Hamilton et al. (1989) and Anderson and Neven (1991). They show that in the linear-city model, two firms agglomerate at the center of the city. On the contrary, Pal (1998) shows that in the circular-city model, each firm locates at the opposite side of the rival (maximal distance).[1]

In this paper, we adopt the model that contains both models as a special case. Takahashi and de Palma (1993) originally created a similar setting with a mill pricing model. The model is of unit length circular-city with a caveat at point 0. When transporting across point 0, an additional cost of amount $\beta \in [0, 1]$ must be incurred. This cost can be interpreted as a barrier such as mountain, river, or a congested bridge. If $\beta = 0$, the model corresponds to the circular-city model. If β is sufficiently large (in our setting $\beta = 1$), no firm transports across point 0, corresponding the model to the linear-city case.

So as to highlight the usefulness of this model formulation, we investigate a location-quantity model in mixed oligopoly where a public firm competes against private firms.[2] Matsushima and Matsumura (2003) show that in the circular-city model all private firms agglomerate at the single point while in the linear-city model the public firm locates at the central point and half of private firms locate at point 1/10 and the other half of private firms locate at point 9/10.[3] We show that the equilibrium pattern continuously moves from that of the circular-city to that of the linear-city as β increases. We also find that the equilibrium location pattern is that of the circular-city only when $\beta = 0$, while it is that of the linear-city for a wide range of β, implying that the linear-city result is more robust in our setting.

2. The model

We formulate a mixed oligopoly model with $n + 1$ firms. Firm 0 is the state-owned public firm and other firms are private firms (firms 1, ..., n). Let n be even. Let $N := \{1, 2, \cdots , n\}$ be the set of all private firms and $N^+ := \{0, 1, 2, \cdots , n\}$ be the set of all firms. These firms engage in a two-stage location-quantity game.

[1] For discussions on location-quantity models in oligopoly, see Gupta (2004), Gupta et al. (2004, 2006), Matsumura et al. (2005), Matsushima (2001), Shimizu and Matsumura (2003), and Yu and Lai (2003). All models are delivered pricing models (shipping models). We can interpret that the shipping models as models of flexible manufacturing system (FMS). See Eaton and Schmitt (1994). For the discussion of FMS in mixed oligopoly, see Gil-Moltó and Poyago-Theotoky (2008).

[2] For the recent discussions of mixed oligopoly, see Cato (2008a,b), Ishida and Matsushima (2009), Ogawa and Kato (2006), Ogawa (2006), Tomaru (2006) and works cited by them.

[3] See Li (2006) and Matsushima and Matsumura (2006) for other discussions on the location-quantity model in mixed oligopoly. Location models are intensively used in the analysis of mixed oligopoly. See, Cremer et al. (1991), Heywood and Ye (2009a, 2009b), Inoue et al. (2009), Nilssen and Sørgard (2002), and Ogawa and Sanjo (2007). Delivered pricing (shipping) models are widely adopted for analyzing competition and public policies. See, Chen and Lai (2008), Gupta et al. (1994), Gupta and Venkatu (2002), Matsumura (2003), Matsushima (2001), and Nikae and Ikeda (2006).

We formulate a model whose setting has the linear-city and the circular-city models as special cases. The shape of the city is a circle of unit size, with a caveat at point 0 that we will explain below when discussing transport cost. The points on the circle are identified with numbers in $[0, 1]$, the most northern point being 0 and the values increasing in a clockwise direction. Thus the most northern point is considered both 0 and 1.

In the first stage, each firm i ($i \in N^+$) simultaneously chooses its location $x_i \in [0, 1]$. In the second stage, each firm observes its competitors' locations and chooses its output $q_i(x) \in [0, \infty)$ for each point (market) $x \in [0, 1]$. Let $p(x)$ be the price of the product at x and $q(x) := \sum_{i=0}^{n} q_i(x)$ be the total quantity supplied at x. We assume a linear demand at each market, given by $p(x) = a - q(x)$, where $a > 0$. Let $t(x, x_i)$ denote the unit transport cost incurred when transporting a good between x_i and x. Each firm transports its product along the perimeter, either clockwise or counter-clockwise and t is equal to the distance of transportation unless it involves crossing point 0. One caveat we mentioned above is that there is a barrier to transportation at point 0. Transporting across point 0 requires an additional cost of amount $\beta \in [0, 1]$.[4] Then $t(x, x_i)$ is:

$$t(x, x_i) = \begin{cases} |x - x_i| & \text{if } x_i \leq x \leq x_i + (1+\beta)/2 \text{ or } \ x_i - (1+\beta)/2 < x < x_i, \\ 1 - |x - x_i| + \beta & \text{if } x_i + (1+\beta)/2 < x \text{ or } x + (1+\beta)/2 \leq x_i. \end{cases}$$

Note that if a firm locates between $(1 - \beta)/2$ and $(1 + \beta)/2$, it never chooses to transport across the barrier, as transporting in the other direction will always be less costly.

Each private firm maximizes its profit, while the public firm maximizes social surplus (consumer surplus plus profit of all firms). The profits for firm $i \in N^+$ and social surplus at market x are given by:

$$\pi_i(x) = (a - q(x) - t(x, x_i))q_i(x), \quad w(x) = \int_0^{q(x)} (a - r)dr - \sum_{i \in N^+} t(x, x_i)q_i(x).$$

When choosing locations, the public firm maximizes the aggregate social surplus $W = \int_0^1 w(x)dx$ and each private firm maximizes the profit from all markets $\Pi_i = \int_0^1 \pi_i(x)dx$, respectively.

We assume that consumer arbitrage is prohibitively costly.[5] All firms have identical constant marginal costs of production, which is normalized to zero. Finally, we assume that $a \geq (n + 1)(1 + \beta)/2$ to ensure that the public firm serves all markets. These types of assumptions are standard in the literature.

3. Equilibrium

We solve for equilibrium of this game. The equilibrium concept is subgame perfection.

3.1 Quantity competition

[4]Note that $\beta = 0$ reverts to the circular-city setting, while $\beta = 1$ reverts to the linear-city setting, as no one will find it beneficial to transport across point 0.

[5]This assumption is not essential. Unless transport costs for consumers are strictly smaller than those of firms, consumer arbitrage plays no role in our model. For this discussion, see Hamilton et al. (1989).

Because marginal cost is constant, each local market can be analyzed independently. The first-order conditions of welfare/profit maximization for the public/private firms, respectively, at market x are:

$$\frac{\partial w(x)}{\partial q_0} = 0 \iff a - q(x) - t(x, x_0) = 0 \qquad \text{(the public firm)}, \tag{1}$$

$$\frac{\partial \pi_i(x)}{\partial q_i} = 0 \iff a - q(x) - q_i(x) - t(x, x_i) = 0, \quad i = 1, \cdots, n \quad \text{(private firms)}. \tag{2}$$

The second-order conditions are satisfied. We introduce three lemmas on the equilibrium outcomes in this stage. These are derived in circular and linear-city models, which are special cases of our analysis, in Matsushima and Matsumura (2003).

Lemma 1: *In equilibrium,* $p(x) = t(x, x_0)$.
Proof: From (1) and the demand function, we have the result. ■

Lemma 1 shows that the price at each local market is equal to the unit transportation cost of the public firm. If this is lower than a private firm's unit transportation cost, the private firm will not supply to the market. Therefore we have the following.

Lemma 2: *The output for private firm* i *(*$i \in N$*) is given by*

$$q_i(x) = \begin{cases} t(x, x_0) - t(x, x_i) & \text{if } t(x, x_0) \geq t(x, x_i), \\ 0 & \text{if } t(x, x_0) < t(x, x_i). \end{cases}$$

Proof: From (1), (2), the demand function, and the constraint $q_i(x) \geq 0$, we have it. ■

Lemma 2 implies that the output level at market x for a private firm i depends only on its location and the location of the public firm, and not on the location of other private firms. Calculating the local profit, we derive that $\pi_i(x) = q_i(x)^2$. Using Lemma 2 and the definition of the total profit, we have the following.

Lemma 3: *The total profit for a private firm* i*,* Π_i*, does not depend on the location of other private firms.*

3.2 Location Equilibrium

We examine the locational equilibrium of the game. Matsushima and Matsumura (2003) showed that in a circular-city setting, the private firms agglomerating at a point most distant from the location of the public firm is an equilibrium. They also showed that in a linear-city setting, the public firm locating at the center and half of the private firms locate at 1/10 and 9/10 respectively is an equilibrium. Here, we would like to examine the equilibrium location pattern that takes these two results as special cases. The following proposition shows the equilibrium location pattern.

Proposition: *Let* $T(\beta) := (1 + \beta - \sqrt{1 - 2\beta + 5\beta^2})/4$. *The location pattern where the public* *firm locates at 1/2, n/2 private firms locate at x^L, and the other n/2 private firms locate at x^R constitutes an equilibrium, where*

$$(x^L, x^R) = \begin{cases} (T(\beta), 1 - T(\beta)), & \textit{if } \; 0 \leq \beta < 2/5 \\ (1/10, 9/10) & \textit{if } \; 2/5 \leq \beta \leq 1. \end{cases}$$

Proof: See Appendix.

Note that $T(0) = 0, T(2/5) = 1/10$ and $T(\beta)$ is increasing for $\beta \in [0, 2/5]$. From Matsushima and Matsumura (2003) we already know that $x^L = 0$ when $\beta = 0$ (circular-city) and $x^L = 1/10$ when $\beta = 1$ (linear-city). When β is small, the result is similar to the circular-city result. The partial agglomeration points for the private firms move away from each other as β increases. When these points reach 1/10 and 9/10 respectively, however, the gradual separation stops and the private firms locate at those points despite the increase in β. Since $T(2/5) = 1/10$, x^L is continuous with respect to β. We believe that our model formulation is a natural one in generalizing of circular and linear spaces.

Proposition 1 also states that the equilibrium location is the linear-city type ($x^L = 1/10$) for wide range of β, ($\beta \in [2/5, 1]$), while it is circular-city type only when $\beta = 0$. This might imply the broad applicability of the linear-city model.[6]

4. Concluding Remarks

In this paper, we adopt the model that contains both linear and circular city models as special cases and investigate spatial Cournot competition in mixed oligopoly. In the circular-city model, there is a unique equilibrium where all private firms agglomerate at one point. We find that this holds true only for the circular-city model. We also find that the equilibrium location pattern corresponds to that of the linear-city for a wide range of the parameter value, implying that the linear-city result is more robust in our setting.

In contrast to the mill pricing (shopping) setting, linear-city model and circular-city model often yield quite different equilibrium location and thus yield different welfare implications in delivered pricing (shipping) setting. We should pay more attention to the robustness of welfare implications in spatial models in future research.

[6]Although the analysis is much more complicated, we can show that this holds true in private oligopoly, too. However, in private oligopoly, the continuity result of equilibrium location with respect to β does not hold.

Appendix

Proof of Proposition: First, we discuss the optimal locations of private firms given $x_0 = 1/2$. From Lemma 3, we have that private firm i's profits depend only on x_i, not x_j ($i \neq j$). Thus, the possible optimal locations of each private firm are common for all private firms. We look for the optimal location for firm 1. By symmetry, we assume $x_1 \in [0, 1/2]$ without loss of generality. (By symmetry, if $x^L \in [0, 1/2]$ is an optimal location of firm 1, $x^R = 1 - x^L$ is another optimal location of firm 1.)

$$\Pi_1 = \begin{cases} \int_0^{x_1} \{(x_0 - x) - (x_1 - x)\}^2 dx + \int_{x_1}^{(x_0+x_1)/2} \{(x_0 - x) - (x - x_1)\}^2 dx \\ \qquad \text{if } x_1 \in ((1-\beta)/2, 1/2] \\ \int_0^{x_1} \{(x_0 - x) - (x_1 - x)\}^2 dx + \int_{x_1}^{(x_0+x_1)/2} \{(x_0 - x) - (x - x_1)\}^2 dx \\ \qquad \text{if } x_1 \in [1/2 - \beta, (1-\beta)/2] \\ \int_0^{x_1} \{(x_0 - x) - (x_1 - x)\}^2 dx + \int_{x_1}^{(x_0+x_1)/2} \{(x_0 - x) - (x - x_1)\}^2 dx \\ \quad + \int_{(x_0+x_1+1+\beta)/2}^{1} \{(x - x_0) - (1 - x + \beta + x_1)\}^2 dx \quad \text{if } x_1 < 1/2 - \beta. \end{cases} \tag{3}$$

Substituting $x_0 = 1/2$ and differentiation it with respect to x_1 yields:

$$\frac{\partial \Pi_1}{\partial x_1} = \begin{cases} (1/2 - 5x_1)(1/2 - x_1)/2 & \text{if } x_1 \in [1/2 - \beta, 1/2] \\ 2x_1^2 - (1+\beta)x_1 + \beta(1-\beta)/2 = 2(x_1 - T(\beta))(x_1 + T(\beta)) & \text{if } x_1 < 1/2 - \beta, \end{cases} \tag{4}$$

where $T(\beta)$ is given in Proposition. Note that $T(\beta) \geq 0$ for all $\beta \in [0, 1]$. For $x_1 < 1/2 - \beta$, Π_1 is increasing (decreasing) in x_1 for $x_1 <$ ($>$) $T(\beta)$. For $x_1 \geq 1/2 - \beta$, Π_1 is increasing (decreasing) in x_1 for $x_1 <$ ($>$) $1/10$. Thus, either $x_1 = T(\beta)$, $x_1 = 1/2 - \beta$ or $x_1 = 1/10$ maximizes Π_1. Substituting these values into Π_1, we have:

$$x^L = \begin{cases} T(\beta) & \text{if} \quad 0 \leq \beta < 2/5 \\ 1/10 & \text{if} \quad 2/5 \leq \beta \leq 1. \end{cases} \tag{5}$$

Next, we discuss the optimal location of firm 0 given $n/2$ firms locate at x^L and $n/2$ firms locate at x^R. By symmetry we assume $x_0 \in [0, 1/2]$ without loss of generality. Matsushima and Matsumura (2003) show that the optimal location is 1/2 when $\beta = 0$. Thus we consider the case where $\beta > 0$. Differentiating aggregate social surplus with respect to x_0 and then substituting x^L and x^R into it, we have:

$$\frac{\partial W}{\partial x_0} = \begin{cases} (2a - n + \beta n - 1)(1 - 2x_0)/2 & \text{if } 0 < \beta < 2/5 \\ (10a - 3n - 5)(1 - 2x_0)/10 & \text{if } 2/5 \leq \beta \leq 1. \end{cases} \tag{6}$$

Since we assume that a is sufficiently large ($a \geq (n+1)(1+\beta)/2)$), the first-order condition is satisfied if and only if $x_0 = 1/2$ (Note that the second order condition is satisfied). Hence, we have an equilibrium. ■

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