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The effects of specific commodity taxes on output and location of free entry oligopoly

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Abstract

This paper examines the impact of a specific commodity tax on output and the location decision of undifferentiated oligopolistic firms with free entry. It shows that (1) the optimum output and location of the oligopolistic firm is independent of the specific commodity tax if the demand function is linear (2) an increase in the specific commodity tax will increase (decrease) output per firm and move the plant location toward (away from) the output market if the demand function is concave (convex). These results are consistent with the conventional results based on the non-spatial setting. In the case in which the demand function is linear or concave, it shows that the number of firms and total output of oligopoly may increase. These results are significantly different from the conventional results based on non-spatial setting. It indicates that the effects of the specific tax on total output and the number of firms crucially depend upon transport costs and the location decisions of oligopolistic firms.

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1. Introduction

In his famous paper, *Commodity Taxation and Imperfect Competition* (1989), Besley investigated the effects of a commodity tax on output per firm, the number of firms and total output of undifferentiated oligopolistic firms with free entry. Under the assumptions that (1) firms are symmetric and identical; (2) firms produce a homogenous good and make Cournot-Nash conjectures about their rivals' production decisions; (3) firms are free to enter and leave the industry; (4) the sufficient second order conditions and the stability conditions are satisfied, Besley obtained the following propositions.

B1. *An increase in the specific commodity tax will increase (decrease) output per firm if the inverse demand function is concave (convex).* Besley (1989, p. 363).
B2. *An increase in the specific commodity tax will decrease the number of firms if the demand function is linear or concave*. Besley (1989, p. 363).
B3. *An increase in the specific commodity tax will decrease total output.* Besley (1989, p. 363).

These results are based on the non-spatial setting in which location and transport costs are negligible. However, the real economy is characterized by dispersion of consumers and producers over geographic space with trade between them always incurring transport costs. It would be interesting and important to investigate the effects of the specific commodity tax on output per firm, the number of firms and total output of undifferentiated oligopolistic firms in a spatial setting.

The purpose of this paper is to fill this gap. It explicitly incorporates oligopolistic market structure into the Weber triangle and examines the impact of a change in the specific commodity tax on output and the plant location of undifferentiated oligopolistic firms. It will be shown that the well-known Besley's B1 holds but B2 and B3 may not hold in the oligopolistic location model.

2. An Oligopolistic Location Model

Our analysis is based on the well-known Weber triangular model with the following assumptions.

(a) N firms employ two transportable inputs (m_1 and m_2) located at A and B to produce a homogenous product (Q) which is sold at the output market locating at C. The location triangle in Figure 1 illustrates the location problem of oligopolistic firms. In figure 1, the distance a and b and the angle γ are known; h is the distance between the plant location (E) and the output market (C); z_1 and z_2 are the distances of plant location (E) from A and B, respectively; θ is the angle between CA and CE.

(b) Firms make Cournot-Nash conjectures about their rivals' production and location decisions and enter the industry without any restrictions until there is no economic profit. Assume also that equilibra are symmetric. Thus, we can neglect the location dispersion of firms and focus on the impact of the specific commodity tax on the production and location decisions of a representative firm.

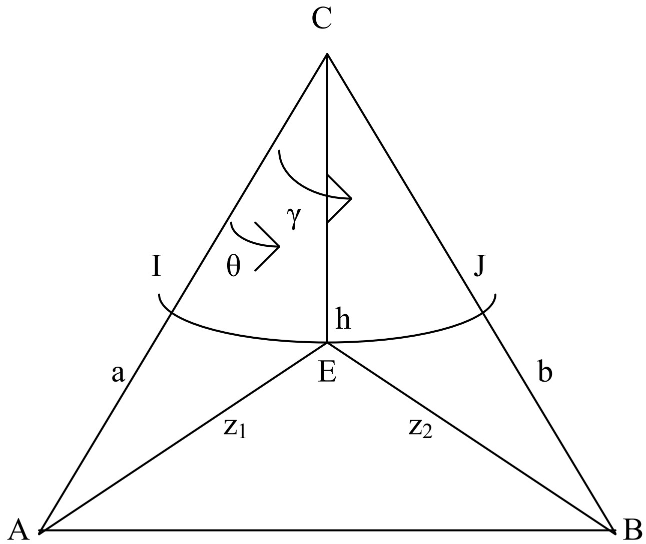

Figure 1. The Weber Triangle

(c) The production function is homothetic and can be specified as:

$$q = f(m_1, m_2) \tag{1}$$

with $f_{m1} \equiv \partial q/\partial m_1 > 0$, $f_{m2} \equiv \partial q/\partial m_2 > 0$, $f_{m1m1} \equiv \partial^2 q/\partial m_1^2 < 0$, and $f_{m2m2} \equiv \partial^2 q/\partial m_2^2 < 0$.

(d) The industry inverse demand function for output is given by

$$P = P(Q) \tag{2}$$

where $Q = \sum q^i$ is the market quantity demanded, $P_Q \equiv \partial P/\partial Q < 0$, $P_Q + qP_{QQ} < 0$, cf. HM (1992, p. 256). It should be noted that $\sum$ denotes $\sum_{i=1}^{N}$.

(e) The prices of inputs and output are evaluated at the plant location (E). The cost of purchasing inputs is the price of input at the source plus the freight cost, and the price of output is the market price minus the freight cost.

(f) Transportation rates are constant.

(g) The government imposes a specific tax which can be specified as:

$$T = tq \tag{3}$$

where t = the specific tax rate, $1 > t > 0$.

(h) The objective of each firm is to find the optimum location and production within the Weber triangle which maximizes the profit.

With these assumptions, the profit maximizing location problem of the representative firm is given by

$$\max \Pi = [P(Q)-rh]f(m_1, m_2) - (w_1+r_1z_1)m_1 - (w_2+r_2z_2)m_2 - tf(m_1, m_2) \tag{4}$$

where $z_1 = (a^2 + h^2 - 2ah\cos\theta)^{1/2}$, $z_2 = [b^2 + h^2 - 2bh\cos(\gamma-\theta)]^{1/2}$; w_1 and w_2 are the base prices of m_1 and m_2 at their sources A and B; r, r_1 and r_2 are constant transportation rates of q, m_1, m_2; z_1, z_2, and h are the distances from the plant location to the source location A, B and the market location C. It is worth mentioning that q, m_1, m_2, h and θ are choice variables and a, b, t, γ, w_1, w_2, r, r_1, r_2 are positive parameters.

Assuming that the oligopolistic firm treats q instead of m_1 and m_2 as a decision variable, we first derive the cost function by minimizing total cost subject to a given output at a given location,

$$\min L = (w_1+r_1z_1)m_1 - (w_2+r_2z_2)m_2 + \lambda[q - f(m_1, m_2)] \tag{5}$$

where λ is the Lagrange multiplier; q, h and θ are parameters. Using the standard comparative static analysis and the envelope theorem, we can show that the production function is homothetic if and only if the production cost function is separable in the sense that

$$C(q; h, \theta) = c(w_1+r_1z_1, w_2+r_2z_2)H(q) \tag{6}$$

where c is a function of the delivered prices of m_1 and m_2, e.g. Takayama (1993, Proposition 3.5., pp. 147-148). Hence, the average cost and marginal cost can be written as:

$$AC = C(q; h, \theta)/q = c(.)H(q)/q \tag{7}$$
$$MC = C_q = c(.)H_q \tag{8}$$

where $C_q \equiv \partial(q; h, \theta)/\partial q$ and $H_q \equiv dH(q)/dq$.

Following Hanoch (1975), from (7) and (8), we obtain the following relation:

$$H(q)/q > (=) < H_q \tag{9}$$

if the production function exhibits increasing (constant) or decreasing returns to scale.

Substituting the production cost function $C = C(q; h, \theta)$ into (4), we obtain the profit as a function of q, θ and h. The first-order condition for a maximum would be

$$\partial\Pi/\partial q = [(P + P_Qq) - rh] - c(.)H_q - t = 0 \tag{10}$$
$$\partial\Pi/\partial\theta = - c_\theta H(q) = 0 \tag{11}$$
$$\partial\Pi/\partial h = - rq - c_h H(q) = 0 \tag{12}$$

where $c_\theta \equiv \partial c(.)/\partial\theta$, $c_h \equiv \partial c(.)/\partial h$. Assume that the second-order conditions are satisfied and the possibility of the corner solution is excluded; cf. Kusumoto (1986) and Mai and Hwang (1992). We can solve (10)-(12) for q, θ and h when free entry is prohibited.

If free entry is allowed, each firm in the industry earns normal profit only. The following condition must be satisfied.

$$\Pi = [P(Nq) - rh]q - c(.)H(q) - tq = 0 \tag{13}$$

If there is an interior solution, we can solve equations (10) – (13) for q, θ, h and N in terms of t and v = (a, b, γ, w_1, w_2, r_1, r_2, r), where v is a vector of remaining parameters.

$$q = q(t, v), \quad \theta = \theta(t, v), \quad h = h(t, v), \quad N = N(t, v) \qquad (14)$$

The expressions for the partial derivatives such as ∂q/∂t, ∂θ/∂t, ∂h/∂t and ∂N/∂t can be obtained by applying the standard comparative static analysis. It is of interest to note that the production function must exhibit increasing returns to scale for (10) – (13) to have a solution as in (14). To see this, we divide both sides of equation (13) by q and obtain

$$[P(Nq) - rh] = [c(.)H(q)/q] - t \qquad (15)$$

Substituting (15) into (10), we obtain

$$P_Q q = c(.)[H_q(q) - H(q)/q] \qquad (16)$$

Since the left-hand side of (16) is negative, for the right-hand side of (16) to be negative, the production function must exhibit increasing returns to scale, i.e., $H(q)/q > H_q(q)$ (see also Hwang, Mai and Shieh, 2007) . It simply implies that in equilibrium all firms produce on the downward sloping part of the average cost curve under Cournot-Nash competition with free entry.

This completes our modeling of the basic framework for studying the effects of a specific tax on the oligopolistic firm's production and location decisions.

3. Effects of Specific Taxes on Production and Location Decisions

We are now in a position to examine the effects of a change in the specific tax rate on the optimum output and location. Totally differentiating equations (10)-(13) and applying Cramer's rule, we obtain the following results.

$$(\partial\theta/\partial t) = (-1/D_4)P_{QQ}q^3\Pi_{\theta h}c_h\{[H(q)/q] - H_q\} \qquad (17)$$
$$(\partial h/\partial t) = (1/D_4)P_{QQ}q^3\Pi_{\theta\theta}c_h\{[H(q)/q] - H_q\} \qquad (18)$$
$$(\partial q/\partial t) = (-D_2/D_4)P_{QQ}q^3 \qquad (19)$$
$$(\partial N/\partial t) = (q/D_4)(D_2\{[2P_Q + P_{QQ}q - c(.)H_{qq}] + (N - 1)P_{QQ}q\} - \Pi_{\theta\theta}\Pi_{qh}^2) \qquad (20)$$
$$(\partial Q/\partial t) = N(\partial q/\partial t) + q(\partial N/\partial t)$$
$$= (q/D_4)\{D_2q[2P_Q - c(.)H_{qq}] - \Pi_{\theta\theta}\Pi_{qh}^2\} \qquad (21)$$

where $\Pi_{\theta h} = -c_{h\theta}H(q)$, $\Pi_{\theta\theta} = -c_{\theta\theta}H(q)$, $\Pi_{qq} = (N + 1)P_Q + NP_{QQ}q - cH_{qq}$, $\Pi_q = P_Qq(N - 1)$, $\Pi_{qh} = c_h\{[H(q)/q] - H_{qq}\}$, $D_2 = \Pi_{\theta\theta}\Pi_{hh} - \Pi_{\theta h}^2$ and D_4 is the relevant Hessian determinant. It should be noted that $\Pi_{\theta\theta} < 0$, $D_2 > 0$ and $D_4 > 0$ by the stability conditions, $c_h < 0$ can be seen from equation (12) and $[H(q)/q] - H_q > 0$ is due to increasing returns to scale.

It is clear that the signs of (∂θ/∂t), (∂h/∂t) and (∂q/∂e) crucially depend upon the shape of market demand function. In the case where the market demand function is linear, i.e.,

$P_{QQ} = 0$. From (17) – (19), we obtain $(\partial q/\partial t) = 0$, $(\partial \theta/\partial t) = 0$ and $(\partial h/\partial t) = 0$. Thus, we can conclude that

Proposition 1. *The optimum output and location of an oligopolistic firm is independent of a change in the specific tax if the demand function is linear.*

The economic interpretation behind Proposition 1 is given as follows. A change in the specific tax does not change the slope of the demand curve at any output level but will increase the output price in equilibrium for the oligopolistic firms to break even. In the case where the demand function is linear, i.e., $P_{QQ} = 0$, a higher output price will not alter the slope of demand curve and so the required tangency between demand curve and average cost curve occurs at the same output level for each firm, i.e., $(\partial q/\partial t) = 0$. Since the output per firm remains unchanged, the optimum location will remain the same.

Next, we consider the case where the demand function is not linear, i.e., $P_{QQ} \neq 0$. Since the signs of P_{QQ} and $\Pi_{\theta h}$ can not a priori be determined, the signs of $(\partial q/\partial t)$, $(\partial \theta/\partial t)$ and $(\partial h/\partial t)$ in (17) - (19) are ambiguous. However, from (18) and (19), we can obtain

$$(\partial q/\partial t) < (>) \ 0, \text{ as } P_{QQ} > (<) \ 0 \tag{22}$$
$$(\partial h/\partial t) > (<) \ 0, \text{ as } P_{QQ} > (<) \ 0 \tag{23}$$

Thus, we can conclude that

Proposition 2. An increase in the specific tax will increase (decrease) the *output of an oligopolistic firm and will move its plant location closer to (farther away from) the CBD if the demand function is concave (convex).*

The impact of the specific commodity tax on the output per firm is consistent with B1. The economic intuition underlying Proposition 2 is given as follow. An increase in the specific tax rate does not change the slope of the demand curve at any output level but will increase the output price in equilibrium for firms to break even. In the case where the demand function is concave (i.e., $P_{QQ} < 0$), a higher output price decreases the absolute value of the slope of the demand curve and so the point of tangency between demand curve and average curve occurs at a larger output level for each firm. Since the production function exhibits increasing returns to scale, the quantity of inputs, m_1 and m_2, per unit of output declines, then the resources pull decreases while the market pull increases. As a result, the optimum location moves towards the CBD. In the case where the demand function is convex (i.e., $P_{QQ} > 0$), the opposite applies.

Next, we turn to the effect of a change in the specific tax on equilibrium number of undifferentiated oligopolistic firms. From equation (20), we can see the sign of $(\partial N/\partial t)$ can not be a priori determined because $[2P_Q + P_{QQ}q - c(.)H_{qq}]D_2 < 0$, $-\Pi_{\theta\theta}\Pi_{qh}^2 > 0$ and the sign of $(N-1)P_{QQ}q^2$ can be either positive or negative depending on the shape of demand function. We can show

$$(\partial N/\partial t) > (<) \ 0, \text{ as } -D_2\{[2P_Q + P_{QQ}q - c(.)H_{qq}] + (N - 1)P_{QQ}q\} > (<) -\Pi_{\theta\theta}\Pi_{qh}^2 \tag{24}$$

Thus, we can conclude that

Proposition 3. *An increase in the specific commodity tax may increase the number of undifferentiated firms even if the demand function is linear or concave.*

This result is significantly different from that of Besley (1989, p. 363) in the non-spatial setting. The different results are due to the location effect, $\Pi_{\theta\theta}\Pi_{qh}^2 = - c_{\theta\theta}H(q) \{c_h[H(q)/q]-H_q\}^2$. It shows that the firm will change its plant location after the change of the specific commodity tax. If the location effect dominates the output effect, the number of firms may increase.

Finally, we consider the effect of an increase in the commodity specific tax on the total output of oligopoly. From equation (21), we obtain

$$(\partial Q/\partial t) < (>) \ 0, \text{ as } - D_2 q[2P_Q - c(.)H_{qq}] > (<) - \Pi_{\theta\theta}\Pi_{qh}^2 \quad (25)$$

Thus, we can conclude that

Proposition 8. *An increase in the specific tax may increase total output of undifferentiated oligopoly.*

This result is also significantly different from that of Besley (1989, p. 363). Once again the different results are due to the location effect.

4. Concluding Remarks

We examine the impact of a specific commodity tax on the production and plant location decisions of undifferentiated oligopolistic firms with free entry. In the case where the demand function is linear, we show that an increase in the specific commodity tax does not change the location decision and output of an oligopolistic firm. In the case where the demand function is not linear, we show that an increase in the specific tax will cause each firm's output to rise (fall) and move the plant location closer to (farther away from) the output market if the demand function is concave (convex). These results indicate that B1 holds in the Weber triangular location model.

In the case where the demand function is linear or concave, we show that an increase in the specific commodity tax may increase the number of firms and total output of oligopoly. This result is significantly different from B2 and B3. It indicates that the location decision has very important influence on the impact of a change in the specific commodity tax on the number of firms and total output of oligopoly with free entry.

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