Economics Bulletin

Volume 29, Issue 4

Revisiting Nash wages negotiations in matching models

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Abstract

In labour economics theory, wage negotiations use to rely on a Symmetric Nash Bargaining Solution. The aim of this study is to show that this kind of solution may be not relevant. Indeed, in a matching model framework, the comparison with the Kalai-Smorodinsky Solution suggests that a reflection should systematically be made with respect to the negotiation power of each agent (a same ascertainment has been pointed out by McDonald and Solow (1981)). Finally, we characterize the Kalai-Smorodinsky in the job matching setting.

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Citation: Samir Amine and Pedro Lages dos santos and Sylvain Baumann and Fabrice Valognes, (2009) ''Revisiting Nash wages negotiations in matching models'', *Economics Bulletin*, Vol. 29 no.4 pp. 3203-3213.
Submitted: Dec 04 2009. **Published:** December 23, 2009.

1. Introduction

The search and matching model is the corner stone for the analysis of labour market. The job matching theory originating with Mortensen and Pissarides into the tradition of unemployment theory provides a benchmark model in labour economics. In fact, the equilibrium search and matching literature, coming from Diamond (1971) and (1982), Mortensen (1982), and Pissarides (2000), has branched out into different research programs. The equilibrium theory of unemployment is probably the best known for the analysis of labour markets.

In numerous papers dealing with the matching models, the Symmetric Nash Bargaining solution is usually applied. However, this kind of solution could not be appropriated in some cases and leads to move away from the labour market reality. Consequently, it could skew the analysis and the policy decisions. Experiments due to Siegal and Fouraker (1960), Nydegger and Owen (1974) also suggest that the Nash solution is an unreasonable model of pair-wise negotiations. The reason why is that players make interpersonal comparison of utility gains such as would be the case with for example the equal-gain model of Myerson (1977) but can not occur with the Nash solution because of the independence of irrelevant alternatives axiom.

Other solutions exist to solve bargaining problems: among them the Kalai-Smoro-dinsky solution (KS, thereafter) (Kalai-Smorodinsky 1975) or even the Equal-loss solution (Chun 1988). According to the selected solution, the interpretation (and thus some effects of public policies) can differ. Few authors applied the KS-solution to the labour market analysis. Gerber and Upmann (2006) analyze a classic bargaining problem between a labour union and an employers' federation through the Nash and KS solutions. Notably, they point out the effect of the reservation wage on the employment and on the wage determination. Indeed they conclude that a higher reservation wage leads to a lower employment level with the Nash Solution, whereas the KS-solution leads up to an ambiguity. Laroque and Salanié (2004) stress the effect of the minimum wage on the employment in the case of wage bargaining between firms and workers. They show that the KS solution does better than the Nash solution.

This paper aims at developing this kind of analysis applied to the matching issue. Actually, in a matching model, we show that the effects of public policy can be different according to the solution we take into consideration. Then, the comparison between the "usual" Nash solution and the Kalai-Smorodinsky solution suggests that a reflection should systematically be made about the choice of the solution applied (and, in particular, about the value of the negotiation power of each agent).

The rest of paper is organized as follows. The model and comparison of the bargaining solutions are presented in Section 2. Then, the quantitative analysis results are discussed in Section 3. Finally section 4 concludes the paper.

2. The Model

Using a matching model with standard hypothesis (Pissarides 2000), we consider an economy composed of a large exogenous number of workers and a large endogenous number of firms. Firms are supposed to be identical and offer a single job. The hypothesis of firm free-entry enables to maintain a fixed number of firms at the steady state. Agents are risk neutral and discount the future with the same rate of time preference denoted by r. The exogenous job destruction rate is s.

Frictions are present in the labour market which means that it takes time for firms with a vacant job to find a worker. Such frictions are represented by a constant-returns matching function $m(V, U)$, where U is the number of employable unemployed workers and V is the number of vacant jobs. This matching function (Pissarides 2000) is an homogenous function of degree 1, increasing in V and U. Instantaneous matching depends on the market tightness, noted $\theta = V/U$. The probability for a firm to meet an employable worker is given by:

$$q = \frac{m(V,U)}{V} = m(1, \frac{1}{\theta}) = q(\theta) \tag{1}$$

This probability is a decreasing function of θ. A rise in the number of vacancies leads to a negative impact on the rate to fill a job due to the congestion effect.

The probability for an employable worker to find a job is given by:

$$p = \frac{m(V,U)}{U} = \theta q(\theta) = p(\theta) \tag{2}$$

This hiring probability is increasing in θ. Indeed, a rise of vacancies implies more opportunities for workers to find a job.

2.1 Expected lifetime Utilities and Profits

According to the usual Belmann's equations, the expected utility of an employed worker, denoted U_1, depends on his current wage w and on the probability that he become unemployed (under the destruction rate s).

$$rU_1(w, \theta) = w - s(U_1 - d_1) \tag{3}$$

With respect to unemployed worker, his expected utility, noted d_1, depends on his current income and on the probability that he gets employed (under the hiring probability $p(\theta)$). We suppose that this income is only composed of unemployed benefits b.

$$rd_1(w, \theta) = b + p(U_1 - d_1) \tag{4}$$

Differentiating the worker utility with respect to w and θ (holding the level of U_1 constant) shows that the worker's indifference curves are downward sloping:

$$\left.\frac{dw}{d\theta}\right|_{U_1=u_1} = \frac{p'_\theta s(b-w)}{(r+p(\theta))(r+p(\theta)+s)} < 0 \tag{5}$$

The expected utility (profit) of firms depends on the probability that the job gets filled.
Concerning a filled job, the expected profit is composed of the net instantaneous income $(y - w)$ and the future profit with respect to the destruction rate s.

$$rU_2(w, \theta) = y - w - s(U_2 - d_2) \tag{6}$$

In regard to a vacant job, as long as this job is unfilled, firms have to invest c corresponding to the job creation and the search of a worker. Firms can expect to fill the job (and reach the corresponding expected profit) with a probability $q(\theta)$. The expected value for a vacant job d_2 is then given by:

$$rd_2(w, \theta) = -c + q(\theta)(U_2 - d_2) \tag{7}$$

The free-entry hypothesis implies that new jobs are created until the optimal value of a vacant job be equal to zero.

Differentiating firm's profit with respect to w and θ (holding the level of U_2 constant) shows that the firm's indifference curves are downward sloping:

$$\left.\frac{dw}{d\theta}\right|_{U_2=u_2} = \frac{q'_\theta s(y-w+c)}{(r+q(\theta))(r+q(\theta)+s)} < 0 \tag{8}$$

The Pareto-curve is defined as the set of all pair $(w,\ \theta)$ such that $U(w,\theta)$ is Pareto efficient. Hence, it is the set of all $(w,\ \theta)$ for which worker's and firm's indifference curves are tangent to each other, $i.e$ which satisfies:

$$\left.\frac{dw}{d\theta}\right|_{U_1=u_1} = \left.\frac{dw}{d\theta}\right|_{U_2=u_2}$$

$$\Longleftrightarrow \quad \frac{p'_\theta s(b-w)}{(r+p(\theta))(r+p(\theta)+s)} = \frac{q'_\theta s(y-w+c)}{(r+q(\theta))(r+q(\theta)+s)} \tag{9}$$

The differentiation with respect to θ and w points out that the wage w is decreasing with θ along the Pareto curve ($\partial w/\partial\theta < 0$).

2.2 Wage Bargaining and Surplus Sharing

Before determining the bargaining solutions, we have to present the axioms which will be used to characterize these solutions. We denote by S the set of the payoffs in the bargaining set, u_1 and u_2 the utility function for each agent, d the disagreement point (d_1 for agent 1 and d_2 for the second) and u_1^*, u_2^* the solutions. The set S is compact and convex. The solution is an application ϕ which combines a payoff vector $\phi(S,d) = (\phi_1(S,d), \phi_2(S,d)) = (u_1^*, u_2^*)$ with each bargaining problem (S,d).

- *(A1)* Individual rationality (IR): $u_1^* \geq d_1$ and $u_2^* \geq d_2$, i.e. $\phi(S,d) \geq d$.
- *(A2)* Pareto optimality (PO): For $u^* \in S$ and $\forall \hat{u} \in S$, if $\hat{u} \geq u^*$, then $\hat{u} = u^*$.
- *(A3)* Symmetry (SYM): If $d_1 = d_2$ and if $\{(u,v) : (v,u) \in S\} = S$, then $u^* = v^*$ if (S,d) is symmetric.
- *(A4)* Invariance with respect to linear utility transformations (ILUT): If T is obtained from S by a linear transformation, then the solution (u_1^*, u_2^*) will be transformed by the same function. If $T = \{(\alpha_1 u_1 + \beta_1, \alpha_2 u_2 + \beta_2) : (u_1, u_2) \in S\}$ and $h = (\alpha_1 d_1 + \beta_1, \alpha_2 d_2 + \beta_2)$, then $\phi(T,h) = (\alpha_1\phi_1(S,d) + \beta_1, \alpha_2\phi_2(S,d) + \beta_2)$.

- *(A5)* Independence of irrelevant alternatives (IIA): For all closed and convex set $T \subset S$, if $\phi(S,d) \in T$, then $\phi(T,d) = \phi(S,d)$.

The optimization program is given by:

$$\max_{p}(U_1 - d_1)(U_2 - d_2) \tag{10}$$

The difference between the Nash and the Kalai-Smorodinsky solutions concerns the fifth axiom: *the Independence of irrelevant alternatives.* This one is replaced by the monotonicity axiom.

- $(A5')$ Individual monotonicity (IM): considering two sets S and T with $S \subseteq T$ and the disagreement point of the two sets d, if (u_1^*, u_2^*) is the solution of (S,d) and if $(u_1'^*, u_2'^*)$ is the solution of (T,d), then $u_1'^* \geq u_1^*$ and $u_2'^* \geq u_2^*$.

Replacing IIA by this monotonicity axiom allows players to make interpersonal comparison of utility gains and hence more in accord with the literature on the job matching.

Theorem 1 (Kalai and Smorodinsky, 1975)**.** *The Kalai-Smorodinsky solution is the unique solution that satisfies IR, PO, SYM, ILUT and IM. The KS curve is given by the function ϕ^{KS} :*

$$\phi^{KS} = (U_2 - d_2)(U_1^{max} - d_1) - (U_2^{max} - d_2)(U_1 - d_1) = 0$$

KS enables to define the ideal point I corresponding to the maximum payoff (U_1^{max}, U_2^{max}) for each agent. However, this ideal point is not feasible. The negotiation process leads to a solution which goes away the least from this point. So KS have shown that if the bargaining set is compact and convex and if there is at least one point in the bargaining set which is strictly individually rational for both players, these axioms are satisfied by a unique point on its boundary.

2.2.1 The Nash solution

In accordance with usual matching models, surplus created by a firm/worker is divided between the two agents according to their respective bargaining strength. If β $(0 < \beta < 1)$ represents the workers bargaining strength, the optimization program is:

$$\text{Nash}:\max_{w,\theta}(U_1-d_1)^{\beta}(U_2-d_2)^{1-\beta}$$

Therefore, the global surplus, noted S, is divided between the two agents according to the Nash rule:

$$\text{Nash}:\begin{cases} U_1-d_1=\beta(U_1-d_1+U_2-d_2)=\beta S \\ U_2-d_2=(1-\beta)(U_1-d_1+U_2-d_2)=(1-\beta)S \end{cases}$$

$$\phi^N(w,\theta)=(1-\beta)\frac{w-b}{r+s+p(\theta)}=\beta\frac{y-w+c}{r+s+q(\theta)}\iff\frac{\dfrac{w-b}{r+s+p(\theta)}}{\dfrac{y-w+c}{r+s+q(\theta)}}=\frac{\beta}{1-\beta}\tag{11}$$

By differentiating this expression with respect to θ and w, we deduce that the wage w is increasing with θ, along the Nash curve.

2.2.2 The KS solution

In this section we characterize the ks solution for our framework. The theorem 1 defines the KS curve within the framework of matching model:

KS:$(U_2-U_2^{min})(U_1^{max}-U_1^{min})-(U_2^{max}-U_2^{min})(U_1-U_1^{min})=0$

$$\phi^{KS}(w,\theta)=\left(\frac{y-w+c}{r+s+q(\theta)}\right)\left(\frac{\hat{w}-b}{r+s+p(\hat{\theta})}\right)-\left(\frac{y-\tilde{w}+c}{r+s+q(\tilde{\theta})}\right)\left(\frac{w-b}{r+s+p(\theta)}\right)=0\tag{12}$$

As for the Nash curve, the KS curve gives an increasing relation between the wage and the market tightness. The intersection with the Pareto curve leads to the following solution:

$$\frac{\dfrac{\hat{w}-b}{r+s+p(\hat{\theta})}}{\dfrac{y-\tilde{w}+c}{r+s+q(\tilde{\theta})}}=\frac{\dfrac{q\prime_{\theta}}{r+q(\theta)}}{-\dfrac{p\prime_{\theta}}{r+p(\theta)}}\tag{13}$$

Each worker and each firm has a maximal payoff represented by an ideal point I. Concerning the firm, his ideal is to have a maximum profit resulting from a minimum wage $\tilde{w}$ payed to each worker (*i.e.* a wage equal to the unemployment benefits b, $\tilde{w}=b$). The ideal wage $\hat{w}$ for the worker is equal to his productivity ($\hat{w} = y$). In this case, the probability for a worker to find a job $p(\hat{\theta})$ and the probability for a firm to recruit a worker $q(\tilde{\theta})$ are supposed maximal ($p(\hat{\theta}) = 1$ and $q(\tilde{\theta}) = 1$).

We have an equality between $\dfrac{\dfrac{q\prime_\theta}{r+q(\theta)}}{-\dfrac{p\prime_\theta}{r+p(\theta)}}$ and $\dfrac{\dfrac{w-b}{r+s+p(\theta)}}{\dfrac{y-w+c}{r+s+q(\theta)}}$, which leads to an other expression for the KS solution. It enables us to compare with the Nash solution.

$$\frac{y-b}{y-b+c} = \frac{\dfrac{w-b}{r+s+p(\theta)}}{\dfrac{y-w+c}{r+s+q(\theta)}} \tag{14}$$

2.2.3 Comparison of the bargaining solutions

The Nash and KS curves are increasing. For a fixed θ, we can determine the wage according to the two bargaining solutions :

We denote by $\Psi(\theta) = \dfrac{r+s+q(\theta)}{r+s+p(\theta)}$.

$$w_N = \frac{y + c + b\Psi(\theta)\left(\dfrac{1-\beta}{\beta}\right)}{1 + \Psi(\theta)\left(\dfrac{1-\beta}{\beta}\right)} \tag{15}$$

$$w_{KS} = \frac{y + c + b\Psi(\theta)\left(\dfrac{y-b+c}{y-b}\right)}{1 + \Psi(\theta)\left(\dfrac{y-b+c}{y-b}\right)} \tag{16}$$

The wage resulting from the Nash solution is higher than the one from the KS solution under a condition:

$$w_N > w_{KS} \quad if \quad \frac{y-b+c}{y-b} > \frac{1-\beta}{\beta}$$

The figure 1 gives the position of the curves according to this two solutions. Thus, we deduce that the Nash solution is preferable for workers.

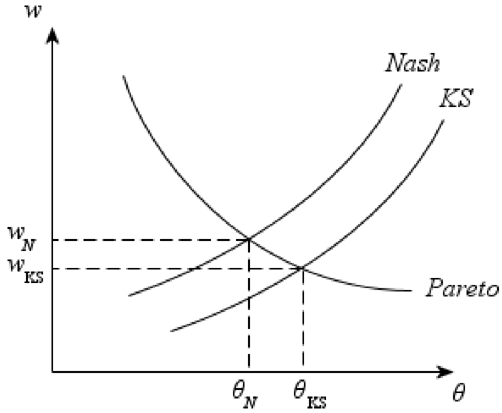

Figure 1: Nash and Kalai Smorodinsky Solutions

Proposition 1. *In the literature, the symmetric Nash solution, in which the negotiation power between the firm and the worker is equal, is usually applied. However, the KS solution points out that this hypothesis is not relevant if the cost of a vacant job is positive. Beside, the KS solution would enable to determinate the " real " negotiation power of each agent. This power is then stronger for the firm to the detriment of the worker.*

Proof. Considering the Nash solution, the negotiation power is given by $(1-\beta)/\beta$. In the literature, the value of β is equal to " 1/2 ", resulting in an equal negotiation power between the two agents. The KS bargaining solution leads to $(y-b+c)/(y-b)$. It is obvious that the negotiation power is unequal for a positive vacant job cost. By comparing these two expressions, we conclude that the value " 1/2 " is not appropriated and it brings an imbalance in the power struggle between the workers and the firms. Moreover, the two solutions coincide if $\beta = y - b$ and $1 - \beta = y - b + c$. □

3. Quantitative analysis

Now it would be interesting to pursue this analysis by focusing on the effects of the various variables on the equilibrium values. To this purpose, we use the following calibration. The matching function is represented by a Cobb-Douglas function: $M(V, U) = V^{1/2}U^{1/2}$, which gives $q(\theta) = \theta^{1/2}$. We retain the following standard parameters values : $\beta = 0.5; c = 0.3; r = 0.05; s = 0.15; y = 1$.

	b	c	r	s	y
w_N	++	-	-	-	+
w_{KS}	+	- -	- -	- -	++

Table 1: Impacts on the wage according to the bargaining solutions

Proposition 2. *Through the calibration of this model, we obtain that the parameters have the same impacts on the equilibrium wage, whatever the solution chosen. However, the variation of Kalai on the wage is stronger than in the Nash solution, except for unemployment benefits.*

4. Final remarks

Considering the choice of the bargaining solution as a secondary issue, most of the literature about matching models (among others) generally retains the Nash solution, without justifying this choice, and discussing its relevance (maybe because the mathematics involved are little more complicated than for the nash solution). However, other solutions can actually be applied, not without consequences. Indeed, by using the KS solution (we do believe that this solution is better if we consider negotiation process between a worker and a firm in the labour market) in a matching model, we show that the equilibrium as well as some effects of public policies (for example about unemployment benefits) are different. Without reconsidering the validity of works using the Nash solution, we conclude that the choice of the solution may actually be decisive and should therefore be subject to further and systematic analysis.

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