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### Preferences, market power and oligopolistic competition: an example

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#### Abstract

This paper studies the way market power operates under symmetric oligopoly equilibrium. Stressing the role of preferences and focusing on price manipulation, four results are obtained about asymptotic identifications (for degenerate preferences and large economies) and about welfare configurations.

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This paper is a piece of a more general research devoted to strategic interactions in general equilibrium.

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## 1. Introduction

The Cournotian oligopolistic competition has mainly been studied in the case of production economies under partial equilibrium analysis (Friedman (1993), Vives (1999)). Nevertheless, the idea of Cournot oligopolists trying to take advantage of their market power has been developed in a general equilibrium framework by Gabszewicz and Vial (1972) in an economy with production, and pursued by Codognato and Gabszewicz (1991), (1993) in pure exchange economies. The behaviors of agents are asymmetric, as ‘significant’ agents are price manipulators while ‘small’ agents remain price takers. Other concepts of oligopoly general equilibria can be developed, based on alternative ways to introduce strategic behavior (Gabszewicz and Michel (1997))<sup>1</sup>. In particular, an equilibrium concept for which *all* traders try to manipulate the price system may be defined, echoing the vision developed by Shapley and Shubik (1977).

In this note, we consider a simple two-good pure exchange economy with a finite number of traders similar to this analyzed in Gabszewicz and Michel (1997). About endowments, the market sizes are and the market shares are the same in each sector, but the market concentration may be different or not between the two sectors. About preferences, we assume an identical Cobb-Douglas function for every individual, the parameter  $\alpha$  being an index of the preference of good 1 relatively to good 2. We thus define the oligopolistic behavior as *effective* when the equilibrium Cournotian supply is smaller than the competitive equilibrium one. In addition, we define the oligopolistic behavior as *efficient* when the supply reduction indeed leads to a more favorable price and beyond to increased payoffs.

In this framework, we show that the Cournotian behavior is neither effective when the number of suppliers of the considered good is large (result 1), nor effective when the preferences are strongly unbalanced (result 2). The oligopolistic behavior is efficient only if the suppliers have over the other traders a net relative advantage. The latter is the result of the advantage based on the preferences and the one based on the numbers of traders (results 3 and 4). These results suggest that the strategic behavior may not hold when market power is equally distributed in economies with a finite number of traders.

## 2. The model

Let’s consider a pure exchange economy with two consumption goods (1 and 2) and  $m+n$  traders, indexed  $i$ ,  $i = 1, \dots, m+n$ . We assume the following Cobb-Douglas specification for the utility function of every trader:

$$U_i = x_{i1}^\alpha x_{i2}^{1-\alpha}, \quad 0 < \alpha < 1, \quad \forall i. \quad (1)$$

The structure of the initial endowments is assumed to be the same as in the case of the homogeneous oligopoly developed by Gabszewicz and Michel (1997):

$$\begin{aligned} \omega_i &= \left( \frac{1}{m}, 0 \right), \quad i = 1, \dots, m \\ \omega_i &= \left( 0, \frac{1}{n} \right), \quad i = m+1, \dots, m+n. \end{aligned} \quad (2)$$

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<sup>1</sup> These authors capture a large variety of market structures through a general notion of non cooperative equilibrium for a quantity setting oligopoly in pure exchange economies.

It is assumed that good 2 is taken as the *numéraire*, so  $p$  is the price of good 1 as expressed in units of good 2. Each trader  $i$  is an oligopolist who tries to manipulate the price by contracting his supply. We denote  $s_{i1}$  the pure strategy of agents  $i = 1, \dots, m$ , with  $s_{i1} \in [0, 1/m]$  and  $s_{i2}$  the pure strategy of agents  $i = 1, \dots, m$ , with  $s_{i2} \in [0, 1/n]$ . The strategy set of each trader may be written:

$$S_i = \left\{ s_{i1} \in \mathfrak{R}_+ \mid 0 \leq s_{i1} \leq \frac{1}{m} \right\}, \quad i = 1, \dots, m \quad (3)$$

$$S_i = \left\{ s_{i2} \in \mathfrak{R}_+ \mid 0 \leq s_{i2} \leq \frac{1}{n} \right\}, \quad i = m+1, \dots, m+n \quad (4)$$

The market clearing condition implies that the price must be  $p = \frac{\sum_{i=m+1}^{m+n} s_{i2}}{\sum_{i=1}^m s_{i1}} = \frac{s_2}{s_1}$ .

A *symmetric oligopoly equilibrium* is a  $(m+n)$ -tuple of strategies  $(\tilde{s}_{11}, \dots, \tilde{s}_{m1}, \tilde{s}_{m+12}, \dots, \tilde{s}_{m+n2})$ , with  $\tilde{s}_{i1} \in S_i$ ,  $i = 1, \dots, m$  and  $\tilde{s}_{i2} \in S_i$ ,  $i = m+1, \dots, m+n$ , and an allocation  $(\tilde{x}_1, \dots, \tilde{x}_m, \tilde{x}_{m+1}, \dots, \tilde{x}_{m+n}) \in \mathbb{R}_+^{2(m+n)}$ , such that (i)  $\tilde{x}_i = x_i(\tilde{s}_{i1}, \tilde{s}_{-i})$  and  $U_i(x_i(\tilde{s}_{i1}, \tilde{s}_{-i})) \geq U_i(x_i(s_{i1}, \tilde{s}_{-i}))$ ,  $\forall s_{i1}$  for  $i = 1, \dots, m$  and (ii)  $\tilde{x}_i = x_i(\tilde{s}_{i2}, \tilde{s}_{-i})$  and  $U_i(x_i(\tilde{s}_{i2}, \tilde{s}_{-i})) \geq U_i(x_i(s_{i2}, \tilde{s}_{-i}))$ ,  $\forall s_{i2}$  for  $i = m+1, \dots, m+n$ .

The non-cooperative equilibrium is associated with the resolution of the simultaneous programs:

$$\text{Arg max}_{\{s_{i1}\}} \left( \frac{1}{m} - s_{i1} \right)^\alpha \left( \frac{s_2}{s_1} s_{i1} \right)^{1-\alpha}, \quad i = 1, \dots, m \quad (5)$$

$$\text{Arg max}_{\{s_{i2}\}} \left( \frac{s_1}{s_2} s_{i2} \right)^\alpha \left( \frac{1}{n} - s_{i2} \right)^{1-\alpha}, \quad i = m+1, \dots, m+n, \quad (6)$$

which give the following optimal strategies:

$$\tilde{s}_{i1} = \frac{(1-\alpha)(m-1)}{m[m-(1-\alpha)]}, \quad i = 1, \dots, m \quad (7)$$

$$\tilde{s}_{i2} = \frac{\alpha(n-1)}{n(n-\alpha)}, \quad i = m+1, \dots, m+n. \quad (8)$$

We deduce the market price  $\tilde{p} = \frac{\sum_{i=m+1}^{m+n} \tilde{s}_{i2}}{\sum_{i=1}^m \tilde{s}_{i1}}$ , which is:

$$\tilde{p} = \left( \frac{\alpha}{1-\alpha} \right) \frac{(n-1)[m-(1-\alpha)]}{(n-\alpha)(m-1)}. \quad (9)$$

The individual allocations are:

$$(\tilde{x}_{i1}, \tilde{x}_{i2}) = \left( \frac{\alpha}{m-(1-\alpha)}, \frac{\alpha(n-1)}{m(n-\alpha)} \right), \quad i = 1, \dots, m \quad (10)$$

$$(\tilde{x}_{i1}, \tilde{x}_{i2}) = \left( \frac{(1-\alpha)(m-1)}{n[m-(1-\alpha)]}, \frac{1-\alpha}{n-\alpha} \right), \quad i = m+1, \dots, m+n \quad (11)$$

The utility levels reached may be written:

$$\tilde{U}_i = \frac{\alpha}{m} \left[ \frac{m}{m - (1 - \alpha)} \right]^\alpha \left( \frac{n-1}{n-\alpha} \right)^{1-\alpha}, \quad i = 1, \dots, m \quad (12)$$

$$\tilde{U}_h = \left( \frac{1-\alpha}{n} \right) \left[ \frac{m-1}{m - (1 - \alpha)} \right]^\alpha \left( \frac{n}{n-\alpha} \right)^{1-\alpha}, \quad i = m+1, \dots, m+n. \quad (13)$$

We now compute the competitive equilibrium. The individual plans come from a non-strategic maximization of the utility subject to the budget constraint:

$$\text{Arg max}_{\{z_{i1}\}} \left( \frac{1}{m} - z_{i1} \right)^\alpha (pz_{i1})^{1-\alpha}, \quad i = 1, \dots, m \quad (14)$$

$$\text{Arg max}_{\{z_{i2}\}} \left( \frac{1}{p} z_{i2} \right)^\alpha \left( \frac{1}{n} - z_{i2} \right)^{1-\alpha}, \quad i = m+1, \dots, m+n, \quad (15)$$

where  $z_{i1}$  and  $z_{i2}$  respectively represent the competitive supply of good 1 by trader  $i$ ,  $i = 1, \dots, m$ , and the competitive supply of good 2 by trader  $i$ ,  $i = m+1, \dots, m+n$ . From (14) and (15), we deduce the competitive plans:

$$z_{i1} = \frac{1-\alpha}{m} \quad \text{and} \quad (x_{i1}, x_{i2}) = \left( \frac{\alpha}{m}, (1-\alpha) \frac{p}{m} \right), \quad i = 1, \dots, m \quad (16)$$

$$z_{i2} = \frac{\alpha}{n} \quad \text{and} \quad (x_{i1}, x_{i2}) = \left( \frac{\alpha}{np}, (1-\alpha) \frac{1}{n} \right), \quad i = m+1, \dots, m+n. \quad (17)$$

The competitive equilibrium price  $p^*$  is solves  $\sum_{i=1}^m z_{i1} = \sum_{i=m+1}^{m+n} z_{i2}$ , which yields:

$$p^* = \frac{\alpha}{1-\alpha}. \quad (18)$$

We deduce the competitive equilibrium allocations:

$$(x_{h1}^*, x_{h2}^*) = \left( \frac{\alpha}{m}, \frac{\alpha}{m} \right), \quad i = 1, \dots, m \quad (19)$$

$$(x_{h1}^*, x_{h2}^*) = \left( \frac{1-\alpha}{n}, \frac{1-\alpha}{n} \right), \quad i = m+1, \dots, m+n. \quad (20)$$

The utility levels reached by each type of agents are respectively:

$$U_i^* = \frac{\alpha}{m}, \quad i = 1, \dots, m \quad (21)$$

$$U_i^* = \frac{1-\alpha}{n}, \quad i = m+1, \dots, m+n. \quad (22)$$

**Result 1.** *The symmetric oligopoly equilibrium coincides with the competitive equilibrium when the economy becomes large.*

*Proof.* Immediate from (7), (8), (16) and (17):  $\lim_{m \rightarrow \infty} \tilde{s}_{i1} = z_{i1}^*$ ,  $\forall i = 1, \dots, m$  and  $\lim_{n \rightarrow \infty} \tilde{s}_{i2} = z_{i2}^*$ ,  $\forall i = m+1, \dots, m+n$ . And from (9)-(11), (18)-(20) and the utility levels, we have:  $\lim_{\substack{m \rightarrow \infty \\ n \rightarrow \infty}} \tilde{p} = p^*$ ,  $\lim_{\substack{m \rightarrow \infty \\ n \rightarrow \infty}} (\tilde{x}_{i1}, \tilde{x}_{i2}) = (x_{i1}^*, x_{i2}^*)$ ,  $\forall i$  and  $\lim_{\substack{m \rightarrow \infty \\ n \rightarrow \infty}} \tilde{U}_i = U_i^*$ ,  $\forall i$ .

This result explains as follows: the oligopolistic behavior tends to the competitive one when the sector becomes large.

**Result 2.** *The oligopolistic behavior tends to the competitive one when the supplied good becomes strongly depreciated.*

*Proof.* Immediate from (7), (8), (16) and (17):  $\lim_{\alpha \rightarrow 0} \tilde{s}_{i1} = z_{i1}^*$ ,  $\forall i = 1, \dots, m$  and  $\lim_{\alpha \rightarrow 1} \tilde{s}_{i2} = z_{i2}^*$ ,  $\forall i = m+1, \dots, m+n$ .

When the good supplied by the traders of one sector is strongly depreciated, adopting a strategic behavior is no longer effective and is eventually equivalent to adopting a parametric behavior. As the traders of the other sector supply a strongly valued good, their strategic behavior is effective and their price manipulation works well (as long as their sector is not large):  $\lim_{\alpha \rightarrow 0} (\tilde{p}/p^*) = (n-1)/n < 1$  (but goes to 1 as  $n$  grows) and  $\lim_{\alpha \rightarrow 1} (\tilde{p}/p^*) = m/(m-1) > 1$  (but goes to 1 as  $m$  grows).

**Result 3.** *The oligopoly equilibrium is Pareto dominated by the competitive one when the relative advantages of the two types of traders offset each other.*

*Proof.* The absence of a net relative advantage<sup>2</sup> on any side of the exchange might be translated by  $\tilde{p} = p^*$ . Then, we want to know whether or not  $\tilde{U}_i < U_i^*$  for  $i = 1, \dots, m$  implies  $\tilde{U}_i < U_i^*$  for  $i = m+1, \dots, m+n$  (and conversely). The price equality  $\tilde{p} = p^*$  is equivalent to  $1 = [(n-1)/(n-\alpha)] \cdot [m - (1-\alpha)] / (m-1)$ . So, we have  $\tilde{U}_i < U_i^*$  for  $i = 1, \dots, m$  if and only if  $(m-1)^{1-\alpha} m^\alpha < m - (1-\alpha)$ . This leads to prove that  $[1 + 1/(m-1)]^\alpha < 1 + [\alpha/(m-1)]$ . Define  $\Gamma(x) \equiv \alpha \log(1+x) - \log(1+\alpha x)$ , where  $x = 1/(m-1)$ , with  $0 < x \leq 1$ . We must verify that  $\Gamma(x) < 0$  when  $x \in ]0, 1]$ . As  $\Gamma(0) = 0$  and  $\Gamma'(x) = \alpha^2(x-1)/[(1+\alpha)(1+\alpha x)] < 0$ , we have  $\Gamma(x) < 0$ ,  $\forall x \in ]0, 1]$ . The argument is similar for  $i = m+1, \dots, m+n$ . *QED.*

**Result 4.** *There is no Pareto domination between the oligopoly equilibrium and the competitive equilibrium when the relative advantages of the two types of traders do not offset each other.*

*Proof.* The absence of Pareto domination between the two equilibria means that  $\tilde{U}_i > U_i^*$  for  $i = 1, \dots, m$  and  $\tilde{U}_i < U_i^*$  for  $i = m+1, \dots, m+n$  (or conversely). Little algebra shows that these two inequalities require  $(1-1/m)^{1-\alpha} < (1-1/m)^\alpha$ . If  $\alpha = 1/2$  (absence of relative advantage due to preferences), this condition stands if  $m < n$  (relative advantage due to the endowments in favor of the  $m$  first traders). If  $m = n$ , this condition stands if  $\alpha > 1/2$ . The argument is similar for  $\tilde{U}_i < U_i^*$  for  $i = 1, \dots, m$  and  $\tilde{U}_i > U_i^*$  for  $i = m+1, \dots, m+n$ . *QED.*

### 3. Conclusion

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<sup>2</sup> For instance, the first  $m$  agents detain a relative advantage based on preferences when  $\alpha > 1/2$ ; and they detain one based on the numbers of agents when  $m < n$ .

It is well known that the oligopolistic behavior does not work when the number of traders is too large (result 1). Another condition of its effectiveness is the minimum preference valuation of the supplied good (result 2). We then put forward that when effective, this Cournotian behavior is efficient only if it is supported by a net relative advantage (results 3 and 4).

The Pareto domination of the competitive equilibrium over the oligopolistic equilibrium (see result 3) suggests a further inquiry dedicated to the relevant field of perfect competition, which might not only concern economies where agents have no market power, but also economies where agents have an equivalent market power.

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