Economics Bulletin

Volume 30, Issue 1

Non-linear technological progress and the substitutability of energy for capital: An application using the translog cost function.

David Clive Broadstock
Surrey Energy Economics Centre

Abstract

This paper analyses the production process of three industries over three separate time periods using datasets taken form Berndt and Wood (1975, 1979), Hunt (1984a, 1986) and Norsworthy and Harper (1981). In their initial paper Berndt and Wood failed to explore the alternative options available to them to represent technological progress, a deficiency noted by Hunt (1986) who tested for alternative representations of technology (inter alia) using the Berndt and Wood data. This paper extends this line of reasoning/research by allowing technological progress to take more flexible non-linear forms using a polynomial deterministic trend model. The results reveal that 'non-linear trend' models are generally preferred to 'linear trend' or 'no trend' models hence raising a question over the validity of assumptions used in much previous empirical research. Further the results reveal that the different assumptions lead to different results for the energy-capital elasticity of substitution.

I am grateful for comments and questions received at the 9th annual conference of the International Association of Energy Economists (IAEE), Florence June 2007. Also I would like to thank Lester C. Hunt (SEEC) for comments received on an earlier draft of this paper. I alone am responsible for all omissions and errors.
Citation: David Clive Broadstock, (2010) "Non-linear technological progress and the substitutability of energy for capital: An application using the translog cost function.", *Economics Bulletin*, Vol. 30 no.1 pp. 84-93.
Submitted: Jul 24 2009. **Published:** January 11, 2010.

1. Introduction

Many hundreds of papers have studied economic productivity, of which around 100 have looked directly at the possibility to substitute capital consumption for energy consumption, as comprehensively summarised in a review of empirical studies of the capital-energy elasticity of substitution (σ_{KE}) by Broadstock *et al* (2007). Of those particular studies featuring energy as a factor of production approximately 35-40% did not attempt to control for the effects of technological progress. Koetse *et al* (2007) further indicated in a meta-study[1] of elasticities of substitution between capital and energy that the effects of controlling for technological progress can significantly change the value of the measured elasticities. As such there is a need for production analysts to be mindful of the manner in which they consider the empirical measurement of technological progress effects and that their assumptions can influence the empirical results.

Using a Translog model of production Berndt and Wood (1975, 1979) generated the seminal results within the energy literature to place an empirical value on the elasticity of substitution between energy and capital [for US Manufacturing]. Hunt (1986) extended these results,[2] also comparing results to those found using data for the UK [industrial sector] to test for the role of technological progress in production with the inclusion of non-neutral (or factor augmenting) technological progress, achieved by the inclusion of a deterministic linear trend. For the Berndt and Wood data this was rejected, whilst conversely for the UK, linear technological progress was statistically preferred.

However the assumptions within these, and all known translog applications in the energy literature is that technology, if accounted for at all, is treated as a linear function of time, either constantly increasing or constantly decreasing depending on the econometric results. The present paper therefore provides a case study based on the data used in these previous empirical studies (and one other) to test the hypothesis that *Technological progress is not a linear function of time* in the context of a Translog cost function. The empirical elasticity of substitution between energy and capital (σ_{KE}) is then derived to observe the policy implications when applying different representations of technology. This is pertinent due to the prominent role its understanding plays in the development of sustainability conscious policy measures, and the importance such measures have within wider analytical models such as general equilibrium systems.

The order of the paper is as follows; the next section outlines the estimation methodology, outlining the approaches to defining the linear and non-linear technological progress terms followed by a brief description on how the elasticities of substitution are derived from these models. Section 3 provides a short note on the data used in this study, while section 4 presents the results of the analysis, focussing upon the empirical shape of the underlying effects of technological progress and subsequent effects on the elasticity of substitution. Concluding remarks are then offered in section 5.

---

[1] The Study by Koetse *et al* (2007) is a more formal meta study than by Broadstock *et al* (2007), however due to the constraints imposed by the statistical pre-requisites of formal meta analysis, the study by Koetse *et al* (2007) covered only a subset of those studies reviewed by Broadstock *et al* (2007). Given their different approaches to analysing the literature, these two papers should be considered complementary, and where possible read in conjunction with each other.

[2] Although this was not the only, or first piece empirically reviewing the results of earlier Berndt and Wood.

2. Methodology

This section discusses the key methodological considerations, as well as the empirical methods used to account for non-linear technological progress. The most commonly used functional form for empirically estimating production functions is the Translog specification originally due to Christensen *et al* (1973), which offers increased flexibility over other forms.[3] The research hypothesis is tested in the context of a general production function featuring four factor inputs, namely ***K**apital*, ***L**abour*, ***E**nergy* and ***M**aterials*. i.e. *y=f(K,L,E,M)* where *y*=output. The exposition of the Translog function (including Hicks neutral and linear factor augmenting trends) is well defined in the literature and so not discussed here.

When accounting for technological progress linear trend approaches are a useful and accessible start point for empirical modelling, however they represent a pre-defined assumption that returns to technology are *constantly* increasing or decreasing over time. However, such an assumption is just one of an infinite class of assumptions that could be made regarding the shape of the underlying technological progress. The remainder of this section outlines an alternative empirical approach which offers a more realistic representation that technological progress can take non-linear forms.

The method applied to allow for the non-linear trend is an ϕ^{th} order static polynomial trend function of a standard deterministic representation of technological progress i.e. for factor *i,* $T_i = \sum \beta_{t_i^1} t^1 + ... + \beta_{t_i^\phi} t^\Phi$ where *t* is the standard deterministic time trend raised to the power ϕ with ($0 < \phi < \infty$) and $(\beta_{t_i^1},...,\beta_{t_i^\phi})$ are parameters to be estimated. Capturing the technological progress in this way is more realistic in that it allows for the, highly plausible, event that returns to technology are not *constantly* increasing or decreasing over time, and that both increases and decreases could occur in the sample period. Denoting S_i as the cost share of factor *i* and $\ln p$ as the natural logarithm of the price of factor *i* then the specific form of the share equations are written as;

$$S_K = \alpha_K + \gamma_{KK} \ln p_K + \gamma_{KL} \ln p_L + \gamma_{KE} \ln p_E + \gamma_{KM} \ln p_M + (\beta_{t_K^1} t^1 + ... + \beta_{t_K^\phi} t^\Phi)$$

$$S_L = \alpha_L + \gamma_{LK} \ln p_K + \gamma_{LL} \ln p_L + \gamma_{LE} \ln p_E + \gamma_{LM} \ln p_M + (\beta_{t_L^1} t^1 + ... + \beta_{t_L^\phi} t^\Phi)$$

$$S_E = \alpha_E + \gamma_{EK} \ln p_K + \gamma_{EL} \ln p_L + \gamma_{EE} \ln p_E + \gamma_{EM} \ln p_M + (\beta_{t_E^1} t^1 + ... + \beta_{t_E^\phi} t^\Phi)$$

$$S_M = \alpha_M + \gamma_{MK} \ln p_K + \gamma_{ML} \ln p_L + \gamma_{ME} \ln p_E + \gamma_{MM} \ln p_M + (\beta_{t_M^1} t^1 + ... + \beta_{t_M^\phi} t^\Phi)$$

With $t^2 = ... = t^\Phi = 0$, this model reduces to the factor augmenting linear trend model, further restricting $t^1 = 0$ leads to the Hicks neutral representation of technology. For convenience the polynomial order is restricted to $\phi=4$, although could be set to any number (depending on the number of available degrees of freedom), and tested down accordingly. In accord with many previous empirical studies including for instance Hunt (1984, 1986), the model is estimated via Zellner's Seemingly Unrelated Regression, which among other things allows for the parameter restrictions, often necessary to ensure a well behaved production function, to be set prior to estimation

---

[3] Alternative flexible functional forms do exist, such as the generalised Leontief due to Diewert (1971), though these are not considered in the present application due to their relative lack of empirical implementation.

and parameter. The details of these restrictions are well expounded in the empirical literature, and so are not discussed here.

As discussed, after estimation the econometric results will be used to derive the empirical elasticity of substitution between capital and energy. The most common measure of substitution used in empirical work (see Broadstock *et al*, 2007) is the Allen Elasticity of Substitution (AES) which can be written in cross-price and own-price forms respectively as;

$$AES_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j}\ ;\ AES_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2}$$

However this measure has been shown ***not*** to be an accurate measure of the curvature of the production iso-surface see Blackorby and Russell (1981) who alternatively propose the use of the Morishima (1967) elasticity. As a result the AES will be evaluated for information (and for general consistency with earlier studies) but will be compared with the empirical value of the Morishima Elasticity of Substitution (MES) which is defined, in terms of the AES, as;

$$MES_{ij} = S_j (AES_{ij} - AES_{jj})$$

It should be noted that as $\gamma_{ij} = \gamma_{ji}$ i.e. the production functions parameters are symmetric, then $AES_{ij}=AES_{ji}$ i.e. also symmetric. However $MES_{ij} \neq MES_{ij}$ i.e. is not symmetric given its formulation, even though the parameters of the production function are. Thompson (2006) provides a more recent account of the substitutability debate outlining the merits of measuring cross-price elasticities (CPE) rather than substitution elasticities, therefore these are also presented. These are defined for the Translog function as;

$$CPE_{ij} = \frac{(\hat{\beta}_{ij} + S_i S_j)}{S_i}$$

The next introduces the datasets which these elasticities will be derived for with the following section providing the empirical results.

3. The Data

The original data from the following papers are used for the empirical analysis; Berndt and Wood (1975, 1979) - US Manufacturing 1947-1971, Hunt (1984a, 1986)[4] - UK Industry 1960-1980 and Norsworthy and Harper (1981) - US Manufacturing 1958-1977. The specific details of these datasets and their sources are explained in further detail in each of the respective papers. For the remainder of the paper these data are referred to as BW (1975, 1979), Hunt (1984, 1986) and NH (1981) respectively.

4. Results

The results of the econometric analysis are cumulatively presented in Table I, the key parameter estimates, and Table II, which presents the empirical technological progress trends. The derived elasticities of substitution and accompanying discussion are presented later in Table III. The non linear trend approach, the most general specification, is compared to the more restricted linear trend and no trend models using Wald tests. The results of these tests (also given in Table I) identify (i) whether

[4] Hunt's (1984, 1986) data for the UK contains no information on materials, though this is not the only empirical work in this area of literature which estimates a ***KLE*** rather than ***KLEM*** function.

accounting for technological progress benefits model performance and (ii) if this is better done using non-linear trends rather than linear. Given that this is not a full general to specific econometric exercise and that more general production specifications could be applied, these tests are indicative of potential avenues for further research, rather than providing conclusive answers.

The results reveal that the alternative representations for technological progress impart an observable impact upon the estimated parameters. For instance the three alternative model specifications applied to the BW (1975, 1979) data produce results for γ_{MM} that range from +0.09 to +0.20. Of greater interest is the impact of the alternative methods upon γ_{KL} from the Hunt (1984, 1986) data which range from -0.04 to +0.04, thus indicating that changes in sign and magnitude can arise, dependent on the assumption made. It can therefore be inferred that there is some potential bias being imparted on the estimated parameters by not appropriately accounting for the effects of technological progress. There does not appear to be any clear pattern as to whether these apparent biases are systematically over/under-estimating the production functions main parameters.

		SK				**SL**			**SE**		**SM**
		γ_{KK}	γ_{KL}	γ_{KE}	γ_{KM}	γ_{LL}	γ_{LE}	γ_{LM}	γ_{EE}	γ_{EM}	γ_{MM}
BW (1975)	***Hicks***	0.03	0.00	–0.01	–0.02	0.08	0.00	–0.07	0.02	0.00	0.09
		(0.006)	(0.003)	(0.003)	(0.010)	(0.007)	(0.002)	(0.011)	(0.005)	(0.009)	(0.023)
	Linear	0.04	0.02	0.00	–0.04	0.12	0.03	–0.14	0.01	–0.04	0.21
		(0.006)	(0.010)	(0.006)	(0.015)	(0.053)	(0.020)	(0.053)	(0.012)	(0.021)	(0.067)
	Non-linear	0.04	0.01	–0.01	–0.04	0.13	0.03	–0.15	0.02	–0.05	0.20
		(0.007)	(0.012)	(0.006)	(0.017)	(0.012)	(0.020)	(0.051)	(0.011)	(0.019)	(0.067)
Wald restrictions test:				***From Non-linear to Linear***					***From Non-linear to no trend***		
				Unable to reject*					Reject		
Hunt (1986)	***Hicks***	–0.02	0.04	–0.01	–	–0.03	–0.01	–	0.02	–	–
		(0.005)	(0.008)	(0.004)		(0.014)	(0.006)		(0.003)		
	Linear	0.03	–0.04	0.01	–	0.09	–0.05	–	0.04	–	–
		(0.009)	(0.007)	(0.005)		(0.009)	(0.004)		(0.003)		
	Non-linear	0.02	–0.04	0.01	–	0.09	–0.06	–	0.04	–	–
		(0.012)	(0.012)	(0.004)		(0.014)	(0.006)		(0.004)		
Wald restrictions test:				***From Non-linear to Linear***					***From Non-linear to no trend***		
				Reject					Reject		
NH (1981)	***Hicks***	0.07	–0.01	0.00	–0.05	–0.01	0.00	0.02	0.02	–0.02	0.06
		(0.003)	(0.005)	(0.001)	(0.006)	(0.030)	(0.001)	(0.033)	(0.001)	(0.002)	(0.036)
	Linear	0.07	–0.02	0.00	–0.05	0.20	0.01	–0.19	0.02	–0.02	0.27
		(0.003)	(0.005)	(0.001)	(0.006)	(0.020)	(0.004)	(0.018)	(0.002)	(0.005)	(0.020)
	Non-linear	0.07	–0.03	0.00	–0.03	0.20	–0.01	–0.15	0.00	0.01	0.17
		(0.006)	(0.007)	(0.002)	(0.009)	(0.017)	(0.003)	(0.016)	(0.002)	(0.004)	(0.022)
Wald restrictions test:				***From Non-linear to Linear***					***From Non-linear to no trend***		
				Reject					Reject		

(i) Standard errors in parentheses
(ii) Rejection of the Wald test result implies maintained non-linear trend model outperforms specification being tested against
(iii) * p-value=0.052 therefore marginal rejection

Table I: Translog Cost Function – Key Parameters

The Wald test results indicate that, with the marginal exception of the BW (1975, 1979) data the non-linear trend model is preferred. The linear/no trend models fail to represent the dynamics of technological progress which are seemingly present within the model. Over the sample period it has been demonstrated, in Table II, that there are periods in which the effects of technological progress are factor saving and others where it may be factor using (at least in relative terms). The linear approach to modelling technological progress is only capable of representing either (i) constant growth in factor saving or (ii) constant decline.

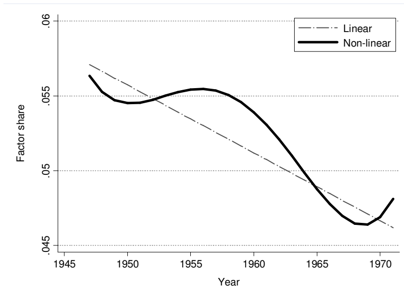
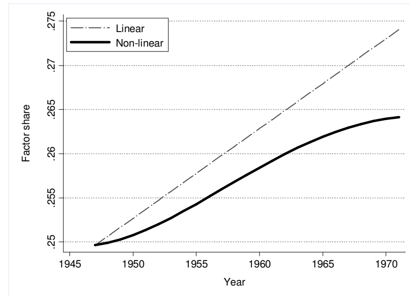
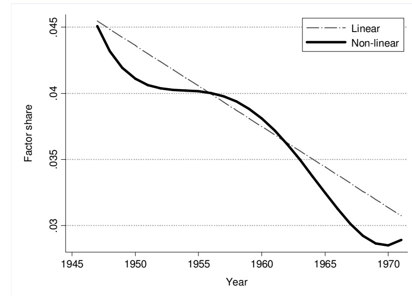
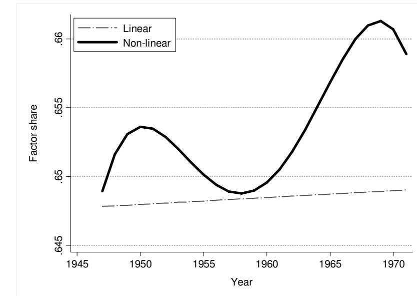
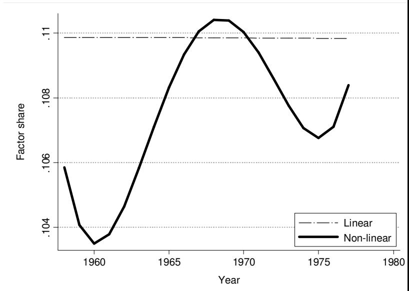
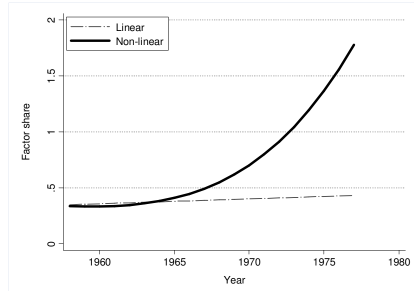
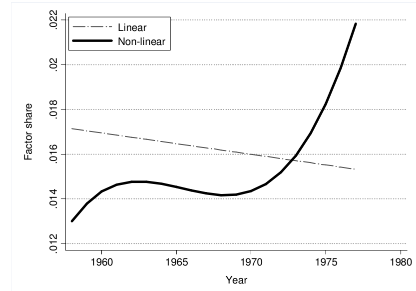
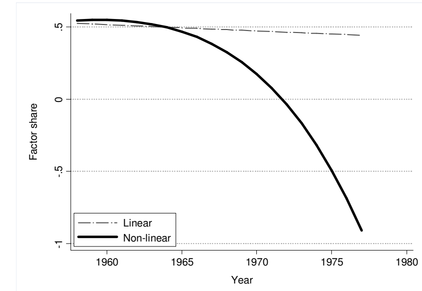
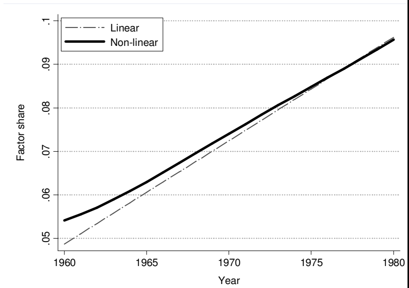
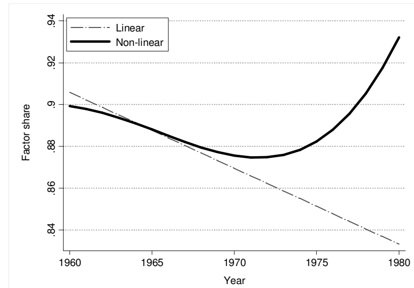
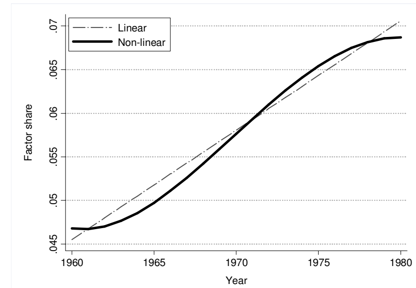
	Capital	Labour	Energy	Materials
Berndt & Wood				
Norsworthy & Harper				
Hunt				

Table II: Factor Augmenting Technological progress on Energy and capital;

With respect to the general implications for energy-capital substitutability, the results in Table III fail to identify any clear patterns[5] as to whether they are complements or substitutes, though this is not entirely unexpected given the unique nature of each of the datasets. The Morishima elasticity favours substitutability between capital and energy (and vice versa), while conversely the more widely used Allen-Uzawa elasticity tends to suggest substitutability between capital and energy but complementarity between energy and capital. However as already alluded to there are methodological concerns with the Allen-Uzawa, and so it is merely presented here to highlight the implications of incorrectly using this measure.

The different assumptions over the form of the underlying technological progress are seen (in Table III) to result in different values for the elasticity of substitution, mostly in terms of magnitude, though sometimes in terms of sign. The increased flexibility in the trend function seems to enhance the robustness (through tighter standard errors) of the results.

Form of Technical progress	**BW (1975)**		**Hunt (1986)**		**NH (1981)**	
	σ_{KE}	σ_{EK}	σ_{KE}	σ_{EK}	σ_{KE}	σ_{EK}
Allen-Uzawa elasticities						
Hicks	–3.33 (0.446)		–1.66 (0.437)		–0.55 (0.166)	
Linear	–1.04 (0.211)		2.68 (0.276)		–0.36 (0.146)	
Non-linear	–2.17 (0.327)		3.30 (0.378)		–1.25 (0.241)	
Morishima elasticities						
Hicks	0.39 (0.032)	0.11 (0.059)	0.47 (0.046)	1.08 (0.026)	–0.33 (0.201)	0.18 (0.068)
Linear	0.49 (0.027)	0.10 (0.059)	0.46 (0.058)	0.70 (0.039)	–0.44 (0.220)	0.21 (0.069)
Non-linear	0.51 (0.027)	0.13 (0.060)	0.47 (0.059)	0.89 (0.026)	–0.46 (0.219)	0.14 (0.056)
Cross price elasticities						
Hicks	–0.15 (0.017)	–0.18 (0.016)	–0.11 (0.022)	–0.13 (0.026)	–0.01 (0.002)	–0.06 (0.018)
Linear	–0.05 (0.008)	–0.06 (0.009)	0.17 (0.009)	0.21 (0.011)	–0.01 (0.002)	–0.04 (0.016)
Non-linear	–0.10 (0.012)	–0.11 (0.012)	0.21 (0.013)	0.26 (0.014)	–0.02 (0.003)	–0.14 (0.030)

(i) standard errors in parentheses
(ii) if $\sigma > 0$ then substitutes
(iii) if $\sigma < 0$ then complements

Table III: Allen-Uzawa, Morishima and Cross Price Elasticities

5. Discussion and Conclusions.

This paper has provided two useful contributions for empirical analysis of productivity. First and foremost it has provided a simple extension to the assumptions regarding the linearity of technological progress in translog cost functions. From the econometric results it is concluded that non-linearity is a valid extension to the model and although this application has been quite focussed in terms of the choice of

[5] Even though some of the Translog parameters show little variation, as for instance in the Norsworthy and Harper data, which shows four identical σ_{KE} values. This is because of the addition and multiplication by factor shares and/or the inclusion of own-price parameters in the calculation of the final elasticities, as in the MES.

functional form (i.e. the translog), the form chosen is arguably arbitrary, as the key principles of potential non-linearities apply to alternative forms also. There is a vast array of techniques available to model technology in a non-linear fashion and these should be explored further and also in the context of alternative functional forms such as the CES.

The second contribution of this paper has been to re-emphasise another important aspect of empirical research of productivity, that of elasticities of substitution. The Allen-Uzawa elasticity has been shown to be an incorrect metric for the surface curvature of an n-dimensional isoquant (See Blackorby and Russell, 1981). Put simply it is not fit for purpose as a measure of factor substitutability. The empirical results further highlight that it produces substantially different values when compared to measures such as the Morishima and cross price elasticities, which have been proved to be theoretically more accurate measures. The Allen-Uzawa elasticity has been the most widely used empirical measure of substitutability between factors of production, however in accord with other authors over the years it must be concluded that it should not be used for this purpose. Further it should be noted that previous studies that have used this measure may therefore be less informative than previously conceived.

Further research is required on both of the aspects covered in this paper. The theoretical issues surrounding substitution elasticities need resolution to ensure that analysts are able to provide a consistent way of analysing the various aspects of productivity. Arguably the cross price elasticity should take precedent, however this needs to be confirmed, or not as the case may be, theoretically.

Bibliography

Allen. R, (1938), *Mathematical analysis for economists*, MacMillan, London.

Baumol. W, (1972), *Economic theory and operations analysis*, Prentice Hall, UK.

Berndt. E and Christensen. L, (1971), The Translog function and the substitution of equipment, structures, and labour in US manufacturing 1929-68, Journal of Econometrics, **1**, 81-114.

Berndt. E and Wood. D, (1975), Technology, prices, and the derived demand for energy, *The Review of Economics and Statistics*, **57**(3), 259-268.

Berndt. E and Wood. D, (1979), Engineering and econometric interpretations of energy-capital complemetarity, *The American Economic Review*, **69**(3), 342-354.

Blackorby. C and Russell. R, (1981), The Morishima elasticity of substitution: symmetry, constancy, separability, and its relation to the Hicks and Allen elasticities, *The Review of Economic Studies*, **48**(1), 147-158.

Broadstock, D. C.**,** Hunt, L. C. & Sorrell, S. (2007) 'Elasticity of substitution studies', UK Energy Research Centre (UKERC) Review of Evidence for the Rebound Effect, Technical Report 3, October, Ref: UKERC/WP/TPA/2007/011.

Christensen. L, Jorgenson. D and Lau. L, (1973), Transcendental logarithmic production frontiers, *The Review of Economics and Statistics*, **55**, 28-45.

Diewert. W, (1971), An Application of Shephard Duality Theorem: A Generalized Leontief Production Function, *Journal of Political Economy*, **79**(3), 481-507.

Frondel. M and Schmidt. C, (2002), The capital-energy controversy: an artefact of cost shares?, *The Energy Journal*, **23**(2), 53-79.

Hicks. J. R, (1932), *The theory of wages*, Macmillan, London.

Hunt. L, (1984a), Energy and capital: Substitutes or complements? Some results for the UK industrial sector, *Applied Economics*, **16**, 783-789.

Hunt. L, (1984b), The energy crisis of the early 1970's and the capital-labour-energy relationship in the U.K. industrial sector, *Kashmir Economic Review*, **1**(2), 29-38.

Hunt. L, (1986), Energy and capital: substitutes or complements? A note on the importance of testing for non-neutral technical progress, *Applied Economics*, **18**, 729-735.

McNown. R, Pourgerami. A and von Hirschhausen. C, (1991), Input substitution in manufacturing for three LDCs: Translog estimates and policy implications, *Applied Economics*, **23**, 209-218.

Morishima. M, (1967), A few suggestions on the theory of elasticity, *Kezai Hyoron (Economic Review)*, **16**, 144-150.

Koetse. M, de Groot. H, and Florax. R, (2007), Capital-energy substitution and shifts in factor demand: A meta-analysis, *Energy Economics*, doi: 10.1016/j.eneco.2007.06.006

Norsworthy. J, and Harper. M, (1981) Dynamic Models of Energy Substitution in U.S. Manufacturing. In: Berndt ER, Field BC (Eds), *Modeling and Measuring Natural Resource Substitution*, MIT Press: Cambridge, Massachusetts, pp 177-208.

Thompson. H, (2006), The applied theory of energy substitution in production, *Energy Economics*, **28**, 410-425.

Turnovsky. M, Folie. M, and Ulph. A, (1982), Factor substitutability in Australian manufacturing with emphasis on energy inputs, *Economic Record*, **58**(160), 61-73

Uzawa. H, (1962), Production functions with constant elasticities of substitution, *The Review of Economic Studies*, 29, 291-299.