



Volume 30, Issue 2

Spatial competition among multiple platforms

Toshiki Kodera

Graduate School of Economics, Nagoya University

Abstract

We study spatial competition in two-sided markets, in which platforms engage in price competition in a circular city. After analyzing the pricing and profits of the unique symmetric equilibrium for a given number of platforms, we derive the number of platforms under free entry and compare it with the social optimum. We consider the case with or without a price restriction. In contrast to the excess entry result in Salop's (1979) model, the number of platforms is smaller than the social optimum if a minimum price binds, and if cross-group network effects are sufficiently large for a group of agents.

1 Introduction

There are markets in which intermediate service providers, or *platforms*, are required for two groups of agents to transact with an opponent. These markets are *two-sided markets*. Examples of two-sided markets include the magazine market (readers and advertisers), the classifieds markets (i.e., yellow pages) (users and firms), the nightclub market (men and women), and the telecommunications market (callers and receivers), among others.¹ In these markets, a platform attracts groups of agents and promotes transactions in order to generate a surplus. Central issues include how platforms behave and how the results affect welfare.

Competition that is limited to two platforms has been studied previously (see Armstrong 2006, Armstrong and Wright 2007 and Anderson and Coate 2005). If a potential entrant finds it profitable to enter, entry should occur, and it is natural to consider entry and competition among many platforms.² Accordingly, we investigate competition among three or more platforms by adopting Salop's (1979) circular city model, which adopts a game theory framework. First, platforms simultaneously decide whether to enter, and entering platforms are symmetrically settled in a circle. In the second stage, those platforms set membership prices for two groups of agents, sellers and buyers, who are uniformly located on the circle. In the third stage, sellers and buyers simultaneously choose whether to join, at most, one platform or not. The agent's payoff consists of membership fees, transportation costs, and utility from *cross-group network effects*. Cross-group network effects are defined as the positive externality from the number of the other group's agents who join the same platform.³ We derive a unique symmetric subgame perfect equilibrium for a given number of platforms. We discuss how the equilibrium price for buyers and sellers is affected by cross-group network effects of sellers and buyers, respectively. If cross-group network effects of sellers are larger than that of buyers, platforms are in greater price compe-

¹Free magazines and newspapers have been growing in Japan. About 40 percent of free magazine and newspaper operations were launched in the 2000s (http://www.jafna.or.jp/freepaper/freepaper_3.html). Magazines provide readers with employment information, such as job openings. Therefore, magazines connect readers and firms that need workers.

²With the exceptions of price restriction and multihoming, Armstrong and Wright (2007) are the same as Armstrong (2006). Anderson and Coate (2005) investigate the competition between two broadcasting platforms with negative cross-group network effects. They discuss the conditions in which one or two platforms are in the market.

³Cross-group network effects are evident in many two-sided markets. In magazines, readers appreciate a greater number of job openings, whereas firms that post job openings value a magazine's employment classifieds if it has a larger reader base.

tition for buyers and extract a surplus from sellers, because platforms can attract many sellers by obtaining one buyer. After analyzing the equilibrium, we derive the number of platforms under conditions of free entry. Free entry in two-sided markets has never been discussed. Compared with the social optimum, there are an excess number of entering platforms. This result is an extension of Salop (1979) and Navon et al. (1995).

Platforms set a *negative* equilibrium price for buyers, for example, if the cross-group network effects of sellers are sufficiently large. However, setting negative prices may not be feasible. We thus consider the case in which price is restricted by a lower boundary, such as the case of non-negative prices.⁴ This restriction is considered in Armstrong and Wright (2007) and Anderson and Coate (2005). To see how price restriction affects equilibrium behavior, consider a seller's price that is bound by a minimum price. This restriction limits each platform's ability to attract sellers by lowering prices. This induces platforms to reduce prices for buyers in order to attract sellers through cross-group network effects. Therefore, the equilibrium price for sellers (buyers) is higher (lower) than that without the restriction. The effect on profits is ambiguous. However, whether each platform's profit increases depends on the relative size of the profit increase from sellers and the profit decrease from buyers.

With price restrictions, we also derive the number of platforms under free entry. We find that the number of platforms is smaller than the social optimum, if the minimum price binds and cross-group network effects are sufficiently large for a group of agents.⁵ Under these conditions, platform profits are considerably reduced, because an increase of profit from one group is smaller than a decrease of profit from the other group. An entering platform can earn smaller profits and thus has fewer incentives to enter, therefore resulting in a lower propensity to enter.

In section 2, we set up the model of two-sided markets with many platforms and derive equilibria given the number of platforms. Section 3 analyzes the consequences of free entry. Section 4 concludes.

⁴In this model, we can also consider maximum prices. An example of a maximum price is a price cap. Regulators impose price caps on telecommunications companies to provide incentives for cost reduction (see Laffont and Tirole 2000).

⁵In addition, if the maximum price binds and cross-group network effects are sufficiently large for sellers, excess entry is generated, because the cross-group network effects intensify profits from buyers.

2 Model

There are $m \geq 3$ homogeneous platforms. Each platform is located equidistant from each other on the unit circle. There are two groups of agents, sellers (S) and buyers (B). Each agent is distributed uniformly on the circle and needs to join a platform to meet an agent from the other side. In this model, assume that each agent joins one platform. To join the platform, each agent incurs transportation costs per unit of distance $t_k > 0$, $k = S, B$. An agent who joins the platform obtains cross-group network benefits $b_k > 0$ multiplied by the number of *other side* agents who join the same platform. The utility of an agent who is located at distance $x_k \in [0, 1/m]$ from platform i is:

$$u_k^i = a_k + b_k n_j^i - p_k^i - t_k x_k,$$

where p_k^i is the price to join platform i and n_j^i is the number of agents in group j , $j \neq k$. The agent from group k receives sufficiently large common benefits a_k when that agent joins a platform.

Given that each agent participates in a platform, an agent in group k is indifferent between joining platform i and joining neighbor platform $i + 1$ if $p_k^i + t_k x_k - b_k n_j^i = p_k^{i+1} + t_k(1/m - x_k) - b_k n_j^{i+1}$. Note that there is another neighbor platform $i - 1$ and the associated indifference condition holds. Suppose that price \tilde{p}_k is set by platforms other than i , and each platform other than i , $i + 1$, and $i - 1$ has $1/m$ market share. The number of agents k who join platform $i + 1$ is therefore: $n_k^{i+1} = (1/m - x_k) + 1/2m$. Group k 's demands for platform i are:⁶

$$D_k^i(p_k^i, \tilde{p}_k, n_j^i, n_j^{i+1}) \equiv n_k^i = \frac{1}{m} + \frac{4t_j(\tilde{p}_k - p_k^i) + 6b_k(\tilde{p}_j - p_j^i)}{4t_k t_j - 9b_k b_j}. \quad (1)$$

The profit of platform i is:

$$\pi^i = (p_S^i - c_S)n_S^i + (p_B^i - c_B)n_B^i - f, \quad (2)$$

where $c_k > 0$ is constant marginal cost and $f > 0$ is the fixed cost associated with setting up a platform. Substituting (2) with (1), and maximizing (2) with respect to p_k^i :

$$\frac{\partial \pi^i}{\partial p_k^i} = \frac{1}{m} + \frac{4t_j(\tilde{p}_k + c_k - 2p_k^i) + 6b_k(\tilde{p}_j - p_j^i) - 6b_j(p_j^i - c_j)}{4t_k t_j - 9b_k b_j} = 0.$$

We assume the following inequality to satisfy the second-order condition,⁷

⁶If there are two platforms in the market, demand for platform i becomes $1/2 + [t_j(p_k^2 - p_k^1) + 2b_k(p_j^2 - p_j^1)] / (t_k t_j - 4b_k b_j)$ due to the fact that the competing platforms at the two marginal agents are the same.

⁷This is similar to Armstrong (2006), $(2t_S + 2t_B)^2 - 16t_S t_B = (2t_S - 2t_B)^2 \geq 0$, $(2t_S +$

$16t_S t_B > 9(b_S + b_B)^2$. In a symmetric equilibrium, each platform charges the same price for each group $p_k^i = \tilde{p}_k = p_k$, rewriting the first-order conditions,

$$p_k = c_k + \frac{t_k}{m} - \frac{3b_j}{2t_j} \left(p_j - c_j + \frac{3b_k}{2m} \right). \quad (3)$$

The following proposition derives from these conditions.

PROPOSITION 1 *If each agent contracts with a platform, there is a symmetric equilibrium. Symmetric equilibrium prices and the profit are:*

$$p_S = c_S + \frac{t_S}{m} - \frac{3b_B}{2m}, \quad p_B = c_B + \frac{t_B}{m} - \frac{3b_S}{2m}, \quad (4)$$

$$\pi = \frac{2t_S + 2t_B - 3b_S - 3b_B}{2m^2} - f. \quad (5)$$

The equilibrium prices (4) and those in Armstrong (2006) share similar features.⁸ However, there are two big differences between his model and ours, which are the number of marginal agents and the degree of competition stemming from the number of platforms. To see the differences between these models, let us rescale the circumference of the circle in order to have each platform locates at one unit of length from the neighboring platform. Therefore, the circumference is equal to the number of platforms.⁹ Now, consider the case of two platforms on the circle, and suppose platform 1 obtains a seller from platform 2.¹⁰ Then, by the cross-group network effects, the utility of each buyer in platform 1 increases by b_B but that in platform 2 decreases by the same amount. The same utility change occurs in Armstrong, but its implication about pricing is different, since each platform has two marginal buyers in our model. The influence of cross-group network effects on our equilibrium prices with two platforms is twice as much as that on Armstrong. Next, consider the case of three or more platforms on the circle, and suppose platform i obtains a seller from platform $i + 1$. Platform i competes with the *two* neighboring platforms, $i + 1$ and $i - 1$, on the circle. The utility change for buyers between platform i and $i + 1$ is similar to that in two-platform case. However, the utility change of buyers between platform i and $i - 1$ is half, since the number of sellers in platform $i - 1$ is unchanged. This is why the influence of cross-group network effects on our equilibrium

$2t_B)^2 \geq 16t_S t_B > 9(b_S + b_B)^2$. Therefore, $2t_S + 2t_B - 3b_S - 3b_B > 0$. $16t_S t_B - 36b_S b_B > 9(b_S + b_B)^2 - 36b_S b_B$, $4(4t_S t_B - 9b_S b_B) > 9(b_S - b_B)^2 \geq 0$. Therefore, $4t_S t_B - 9b_S b_B > 0$.

⁸Armstrong shows the equilibrium price of agent k as $p_k = c_k + t_k - b_j$, in his model of Hotelling-type two-platform competition.

⁹Making this adjustment for (4), new equilibrium prices become $p_k = c_k + t_k - 3b_j/2$.

¹⁰In two-platform competition with rescaling, the equilibrium prices are $p_k = c_k + t_k - 2b_j$, due to the difference in the demand function. See footnote 6.

prices with three or more platforms is one and a half times as much as that on Armstrong.

If cross-group network effects disappear from our model, a platform faces two independent circular cities. The equilibrium price and profit are the same as Salop's (1979) model,¹¹ which are $p_k = c_k + t_k/m$ and $\pi = (t_S + t_B)/m^2 - f$. Compared with (4) and (5), the equilibrium prices and profit in two-sided markets are decreased by the cross-group network effects. Cross-group network effects promote competition among platforms.¹²

We consider another pricing mechanism, in which platforms maximize profit when a price for one group is restricted by a minimum (or maximum) price, \hat{p}_k . A typical example of minimum prices is a non-negative price. In free magazines or yellow pages, levying charges for readers is not feasible.¹³ The following proposition derives from these conditions.

PROPOSITION 2 *When the price of a seller is restricted,¹⁴ symmetric equilibrium prices and the profit of platform are:*

$$p_S = \hat{p}_S, \quad p_B = c_B + \frac{t_B}{m} - \frac{3b_S}{2t_S} \left(\hat{p}_S - c_S + \frac{3b_B}{2m} \right), \quad (6)$$

$$\hat{\pi} = \frac{4t_S t_B - 9b_S b_B - 2(2t_S - 3b_S)(c_S - \hat{p}_S)m}{4t_S m^2} - f. \quad (7)$$

In Proposition 1, if platform i collects a large profit from buyers by obtaining a seller, platform i sets a negative price for sellers and a high price for buyers. Suppose a seller's price is bound by a minimum price. Each platform can not attract sellers by lowering prices for this restriction. Platforms reduce prices for buyers in order to attract sellers through cross-group network effects. Whether each platform's profit increases depends on the relative size of the profit increase from sellers and the profit decrease from buyers.

In equilibrium, market shares of platform i are $1/m$. We therefore compare profits with and without restriction, by comparing the sum of equilibrium prices. We define buyer price with a restriction, (6), as \bar{p}_B . Subtracting the sum of equilibrium prices without the restriction from that with the restriction equals $(\hat{p}_S + \bar{p}_B) - (p_S + p_B) = (1 - 3b_S/2t_S)(\hat{p}_S - p_S)$. If the number of obtainable sellers when a platform obtains a buyer is larger than one, profits with the minimum (maximum) price restriction are smaller (respectively,

¹¹Navon et al. (1995) investigates Salop's (1979) model by using one group of agents with network effects.

¹²The equilibrium is not influenced by the common benefit as in Salop (1979).

¹³Rysman (2004) shows an empirical analysis for the yellow page market.

¹⁴We focus on a restriction on sellers. We can discuss a restriction on buyers accordingly.

larger) than that without the restriction. Otherwise, profits with the minimum (maximum) price restriction are larger (respectively, smaller) than that without the restriction.

We focus on the case in which the number of obtainable sellers when a platform obtains a buyer is larger than one. When sellers are bound by a minimum price, platform profits from sellers increase. On the other hand, platforms actively compete for buyers because a platform obtains more than one seller when the platform obtains a buyer. A decrease in profits from buyers is larger than an increase in profits from sellers. Therefore, platform profits with the restriction are smaller than that without the restriction.¹⁵

Consider how profit responds to a change in the restriction of prices, $\partial\hat{\pi}/\partial\hat{p}_S = (2t_S - 3b_S)/2t_S m$. Therefore, when cross-group network effects of sellers are larger than transportation costs of sellers, platform profits decrease with the restriction of prices.

3 Free entry

Each potential platform decides whether to enter in the first stage. If a platform enters the market, it obtains at least zero profit. In this section, we primarily consider the case in which the price for sellers is restricted, but we discuss the case without the restriction at the end of this section. Let \hat{m} denote the number of platforms under free entry.¹⁶

The socially optimal number of platforms is defined by the number of platforms to maximize social welfare, which is:

$$W = a_S - c_S + \frac{b_S + b_B}{m} - 2m \left(\int_0^{1/2m} t_S x_S dx_S + \int_0^{1/2m} t_B x_B dx_B \right) + a_B - c_B - mf.$$

Note that entry dilutes contributions of the cross-group network effects to the social welfare. Maximizing the social welfare with respect to m , $\partial W/\partial m = (t_S + t_B - 4b_S - 4b_B)/4m^2 - f$. When $t_S + t_B - 4b_S - 4b_B > 0$, the socially optimal number of platforms m^* is: $m^* = [(t_S + t_B - 4b_S - 4b_B)/4f]^{1/2}$. The socially optimal number of platforms increases with transportation costs and

¹⁵When sellers are bound by a maximum price, platform profits from sellers decrease, whereas platforms attract additional sellers. Platforms can levy additional fees on buyers by adapting to the growth of sellers. Cross-group network effects of sellers intensify profits from buyers. Under these conditions, an increase in platform profits from buyers is larger than a decrease in platform profits from sellers. Therefore, profits with the restriction are larger than that without the restriction.

¹⁶Note that \hat{m} has real roots because $4t_S t_B - 9b_S b_B > 0$. We neglect to discuss the case in which the number of entrants becomes an integer.

decreases with cross-group network effects and fixed costs.¹⁷

Consider how many platforms can enter the market when the price is not restricted. Using (5), the number of platforms under free entry is $m = [(2t_S + 2t_B - 3b_S - 3b_B)/2f]^{1/2} > m^*$. Therefore, free entry yields excess entry, because an entering platform harms the profit of other platforms. This result is an extension of Salop (1979) and Navon et al. (1995).

We now consider the influence of cross-group network effects on excess entry. From the above formula, it is straightforward that the numbers of firms in the social optimum and under free entry, m^* and m , both decrease in the cross-group network effects, and thus those numbers themselves provide poor information on the influence. For a better evaluation on the influence, we propose the ratio¹⁸ m/m^* . Importantly, this ratio is independent of the entry cost f , so that we can always adjust the entry cost to normalize m^* without affecting the ratio. Direct calculation shows the ratio is increasing in b 's; the cross-group network effects intensify the degree of excess entry.¹⁹ This is because, in addition to the well-known business stealing effect identified in Salop, negative externalities of entry on the incumbents' profits due to diluting the cross-group network effects make entry further socially undesirable. Accordingly, the degree of excess entry in our model is severer than in Salop.²⁰

When the price is restricted, to compare the number of platforms under free entry with the social optimum, denoted by Δ , the difference between the profit of platforms with a price restriction and the marginal social benefit is:

$$\Delta \equiv \frac{\partial W}{\partial m} - \hat{\pi} = -\frac{3t_S + 3t_B - 2b_S - 2b_B}{4m^2} - \frac{2t_S - 3b_S}{2t_S m}(\hat{p}_S - p_S). \quad (8)$$

The first term of the right-hand side of (8) is comprised of the increase in social welfare when a platform enters the market, as well as a portion of profits unaffected by the restriction. Note $16t_S t_B > 9(b_S + b_B)^2$, $3t_S + 3t_B - 2b_S - 2b_B > 0$. The second term of the right-hand side of (8) denotes the difference between profits with the price restriction and without the price restriction. Therefore, the following proposition is:

PROPOSITION 3 *Under free entry, if the minimum or maximum price binds, when $\Delta > 0$, the number of platforms is smaller than the social op-*

¹⁷If $t_S + t_B - 4b_S - 4b_B < 0$, note that the socially optimal number of platforms is one. However, we focus on the case where $t_S + t_B - 4b_S - 4b_B > 0$.

¹⁸The ratio is $[2(2t_S + 2t_B - 3b_S - 3b_B)/(t_S + t_B - 4b_S - 4b_B)]^{1/2} > 2$.

¹⁹Differentiating the ratio with respect to the cross-group network effects, $5(t_S + t_B)/(t_S + t_B - 4b_S - 4b_B)[2(2t_S + 2t_B - 3b_S - 3b_B)(t_S + t_B - 4b_S - 4b_B)]^{1/2} > 0$.

²⁰We can integrate Salop's model into ours by setting $t = t_S + t_B$ and $b_S = b_B = 0$. The ratio in Salop is 2.

imum. When $\Delta < 0$, the number of platforms is larger than the social optimum.

proof: When the marginal social benefit is positive at the number of platforms under free entry $\partial W/\partial m - f > 0$, additional entry is needed to improve social welfare. But an extra platform does not enter because $\hat{\pi} - f < 0$ under $\Delta > 0$. Excess entry result can be proven accordingly.

When the price is not bound by the restriction (for example a minimum price), we have excess entry result. However, we show that too few platforms enter the market under conditions of the minimum price. When the price for sellers is bound by the minimum price, the profit from sellers increase and the profit from buyers decrease. When the profit increase from sellers is smaller than the profit decrease from buyers, the number of platforms with the restriction decreases compared with the number of platforms with restriction. If the cross-group network effects of sellers are sufficiently large, platforms earn considerably low profits because platforms actively compete for buyers. An entering platform cannot sufficiently steal business from other platforms. Therefore, the number of entering platforms is smaller than the social optimum.²¹

If we set the minimum or maximum price on Salop (1979), there is the case where too few firms enter the market. However, the restriction is limited to a price cap. The minimum price increases the number of entrants and enforces excess entry, because the minimum price relaxes price competition. Meanwhile, when the price for sellers is bound by the minimum price, we show that too few platforms enter the market compared with the social optimum. Fierce price competition for buyers brings the profit down, if the cross-group network effects of sellers are sufficiently large.

4 Conclusion

We investigate a model of multiple platforms competition in a circular city. If a minimum (maximum) price binds and the cross-group network effects are sufficiently large for a group of agents, the profits of a platform with a price restriction is lower (higher) than that without a price restriction. As a result, the number of platforms is smaller (respectively, larger) than the social optimum.

²¹If the price for sellers are bound by the maximum price, profits from sellers decrease, whereas profits from buyers increase. If the cross-group network effects of sellers are sufficiently small, the profit increase from buyers is very few because competition for buyers without the restriction is inactive. Therefore, the number of platforms is smaller than the social optimum.

In our model, platforms charge subscription fees to agents. However, platforms may use more complicated pricing rules. Shopping malls (buyers and shops) demonstrate an example. Buyers usually do not pay a price to go into a shopping mall. On the other hand, sellers pay the shopping mall to set up shops. Shopping malls aggressively compete on prices charged to sellers when buyers value the number of sellers. However, shopping malls may charge a price per transaction. Extending our model to include per-transaction prices would be an interesting topic for future research.

We take a preliminary step in analyzing multiple platform competition when agents join only one platform. In reality, agents join multiple platforms in order to interact with more customers. For example, firms may post multiple magazines in order to attract more potential employees, and workers may subscribe to multiple magazines in order to find a better job. Armstrong (2006) and Armstrong and Wright (2007) consider the multi-home problem in their two-platform competition model. However, there are some difficulties in extending our model to the multi-home problem. The multi-home problem should thus be undertaken in future research.

References

- [1] Anderson, S. and S. Coate (2005) “Market Provision of Broadcasting: A Welfare Analysis” *Review of Economic Studies* **72**, 947-972
- [2] Armstrong, M. (2006) “Competition in Two-Sided Markets” *Rand Journal of Economics* **37**, 668-691
- [3] Armstrong, M. and J. Wright (2007) “Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts” *Economic Theory* **32**, 353-380
- [4] Laffont, J. and J. Tirole (2000) *Competition in Telecommunications*, Cambridge: MIT Press
- [5] Navon, A., O. Shy, and J. F. Thisse (1995) “Product Differentiation in the Presence of Positive and Negative Network Effects” CEPR Discussion Paper No.1306, December
- [6] Rysman, M. (2004) “Competition Between Networks: A Study of the Market for Yellow Pages” *Review of Economic Studies* **71**, 483-512
- [7] Salop, S. C. (1979) “Monopolistic Competition with outside goods” *Bell Journal of Economics* **10**, 141-156