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### Optimal size of central government and agglomeration

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#### Abstract

Though the central government uses neither a transfer nor a regional allocation policy, it can affect the distribution of the population. This paper analyzes the optimal government policy and examines whether or not the government should take into account agglomeration without a regional redistribution policy. The optimal size of central government depends on the degree of increasing returns in the private and the public sector. When the central government shows a much lower degree of increasing returns in contrast to the private sector, it should decrease the provision of the public good. As a result, the central government limits agglomeration. If the central government does not consider its effect on agglomeration, it is too large in size, and it causes too much agglomeration.

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## 1. Introduction

The public sector affects the population distribution and the agglomeration of the population. Roos (2004) shows that the competition of local governments restrains agglomeration. Burbidge and Myers (1994) utilize the local government's transfer policy to control migration. Moreover, the national or central government may lead to agglomeration. Riou (2006) analyzes the transfer policy to offset agglomeration. When the central government uses neither a transfer nor a regional allocation policy, should it consider the distribution of the population?

The purpose of this paper is to investigate the size of government when its scale affects the agglomeration of the population. Pirttilä and Tuomala (2005) analyze the size of the public sector when the government uses public production as a redistribution policy. But they do not examine regional agglomeration. Dascher (2002), without examining the optimal policy of the government, considers that the government affects the distribution of the population through the labor market. Following Dascher's model, this paper analyzes the optimal size of government. In this optimal policy, I examine whether or not the government should take into account agglomeration without a regional redistribution policy.

## 2. The model

The economy is composed of regions 1 and 2. In each region, one kind of consumption good is produced under conditions of perfect competition. This economy has two kinds of consumption goods. These goods can be traded across regions and are produced from intermediate goods that can not be traded across regions. The total population in the economy is  $\bar{L} = L_1 + L_2$ , where  $L_i$  is the population of a region  $i$  ( $i = 1, 2$ ). Each individual supplies one unit of labor inelastically. Individuals can migrate among regions without cost.

The central government provides public goods that can be consumed by all individuals in the economy. This pure public good yields the same effect across both regions. The

government locates in one region to produce the public good by using intermediate goods in that region. To supply the public good, the government collects an income tax from all individuals. The tax rate is uniformly applicable across individuals. The government does not use transfers to reduce regional inequality. In the following, I assume that the central government is located in region 1.

The individual in region  $i$  has the utility function  $U^i$  defined by

$$U^i = (x_1^i x_2^i)^{\frac{1}{2}} G$$

where  $x_j^i$  is the amount of consumption goods  $j$  ( $j=1,2$ ), and  $G$  is the public good provided by the government. In this economy, individuals can consume the public good wherever they locate. Since each individual supplies one unit of labor, that individual's income is the labor's wage. Then the individual's budget constraint in region  $i$  is

$$P_1 x_1^i + P_2 x_2^i = (1 - t) w_i$$

where  $P_j$  ( $j = 1, 2$ ) is the price of consumption goods,  $w_i$  is region  $i$ 's labor wage, and  $t$  is the government's tax rate that is equal in both regions. Individuals are free to migrate to the region where the utility is higher. Therefore, in an equilibrium, the utility is equal across regions.

The consumption good  $i$  is produced only in region  $i$  ( $i=1,2$ ) with intermediate goods as the inputs. The production function is as follows:

$$X_i = \left( \int_0^{N_i} q_n^\rho dn \right)^{\frac{1}{\rho}}, \quad 0 < \rho < 1 \quad (1)$$

where  $q_n$  is the intermediate good  $n$ ,  $N_i$  is the number of intermediate goods produced in region  $i$ .  $\rho$  is the parameter of substitution. This production function provides increasing returns to scale, and the degree of increasing returns is higher when  $\rho$  is smaller. Using the first-order condition for profit maximization, aggregate private demand of the intermediate good  $k$   $q_k^d$  is

$$q_k^d = \frac{p_k^{\frac{1}{\rho-1}}}{\left[ \int_0^{N_i} p_n^{\frac{\rho}{\rho-1}} dn \right]^{\frac{1}{\rho}}} X_i = \frac{p_k^{\frac{1}{\rho-1}}}{B_i^{\frac{1}{\rho-1}}} X_i \quad (2)$$

where  $B_i = \left[ \int_0^{N_i} p_n \frac{\rho}{\rho-1} dn \right]^{\frac{\rho-1}{\rho}}$  is a price index.

The intermediate goods sector has the structure of monopolistic competition. In this sector, each good is produced by a single producer. Each producer uses labor as the input. The amount of labor input to produce  $q_k$  units of the intermediate good  $k$  is

$$L_{q_k} = f + bq_k \quad (k \in [0, N_i])$$

where  $f$  is the fixed labor input and  $b$  is the marginal labor input. Each producer acts as a monopolist. The producer faces the private demand for the intermediate good (2) and takes as given the price index  $B_i$  and the total amount of the consumption good  $X_i$ . Moreover, for simplifying the analysis, I assume that the producer operates as if the government's demand is zero. When this assumption is relaxed, the equilibrium populations of each region do not change if the tax rate is constant. With regard to government behavior, though the optimal tax rate changes, the main results of the study remain unchanged.

Following Dascher (2002), the central government produces the public good  $G$  in region 1. In its production of the public good, the government uses intermediate goods. Because intermediate goods are not tradable across regions, only region 1 supplies intermediate goods that are used by the government. The production function of the public good is as follows:

$$G = \left( \int_0^{N_1} g_n^\tau dn \right)^{\frac{1}{\tau}}, \quad 0 < \tau < 1 \quad (3)$$

where  $g_n$  is the amount of the government's intermediate good  $n$  input.  $\tau$  is the parameter of substitution. Like the consumption good, the production function of the public good yields increasing returns to scale. Since the government collects income tax to cover the cost of public good production, the government's budget constraint is written:

$$\int_0^{N_1} p_n g_n dn = t(w_1 L_1 + w_2 L_2) \quad (4)$$

From this model, when the tax rate is constant, the equilibrium populations of regions

1 and 2 are written as

$$L_1 = \frac{1+t}{2}\bar{L} \quad (5)$$

$$L_2 = \frac{1-t}{2}\bar{L} \quad (6)$$

(5) and (6) show that the presence of the government causes agglomeration. To produce intermediate goods for the government, region 1 needs a larger labor force. Moreover, except for the public sector, each region is symmetric. If the government did not exist, the population would be equally distributed between regions and no agglomeration would arise. Thus, the equilibrium population in region 1 is larger than that in region 2.

The government maximizes individual utility because it is equalized in the equilibrium. To analyze the government behavior, I derive the indirect utility function when the tax rate is constant. The indirect utility function of the individual in region  $i$  is given as

$$V^i = (1-t)(t\bar{L})N_1^{\frac{1}{\tau}-1}\frac{\rho}{b}\left[\frac{w_i}{P_1}\frac{w_i}{P_2}\right]^{\frac{1}{2}}\left[\frac{1}{2}\right] \quad (7)$$

If the government does not take into account the influence of its action on the distribution of the population and the market equilibrium, the objective function is (7).

If the government takes into account the effect of its behavior on the distribution of the population, the government's objective function is different. In that case, the objective function (7) is rewritten as follows:

$$V^i = t(1-t)^{\frac{1+\rho}{2\rho}}(1+t)^{\frac{1}{\tau}+\frac{1-3\rho}{2\rho}}\bar{L}\left[\frac{\bar{L}}{2}\right]^{\frac{1}{\tau}+\frac{1-2\rho}{\rho}}\left[\frac{1-\rho}{f}\right]^{\frac{1}{\tau}+\frac{1-2\rho}{\rho}}\left[\frac{\rho}{b}\right]^2\left[\frac{1}{2}\right] \quad (8)$$

The government maximizes the utility (8) when it knows the effect on the population.

### 3. Agglomeration and government size

This section shows the regional distribution of the population when the central government behaves so as to maximize social welfare. In order to analyze whether or not the government should consider the agglomeration, I first explain the optimal case in which

the government perceives the influence of its action on the population. Next, I show the case in which the government does not take that effect into account. I then compare the regional distributions of the population in each case.

First, I explain the optimal case where the government considers the effect of its behavior on the population and the market equilibrium. The government maximizes the individual utility (8) with the use of the tax rate as a policy instrument. To solve the utility maximizing problem, I obtain the optimal tax rate as follows

$$t^* = \frac{\frac{1}{\tau} - 2 + \sqrt{\left(\frac{1}{\tau}\right)^2 + 4\left(\frac{1}{\rho} + 1\right)}}{2\left(\frac{1}{\tau} + \frac{1}{\rho}\right)} \quad (9)$$

Next, I analyze the case in which the government neglects its influence on the population and the market equilibrium. In that case, the government chooses the tax rate  $t$  in order to maximize the individual utility expressed by (7) where the government takes the prices of each good as given. Then, the tax rate that maximizes the utility (7) is

$$t^n = \frac{1}{2} \quad (10)$$

In the case of neglect, the government chooses the tax rate  $t^n = 1/2$  and the populations of each region are determined by (5) and (6). I compare region 1's population to the optimal case. Among these populations, if the optimal tax rate  $t^*$  is put over  $t^n$ , region 1 has a smaller population than the optimal case, but if  $t^*$  is put below  $t^n$ , the population of region 1 is larger.

(9) shows that  $t^*$  is determined by  $\tau$  and  $\rho$ . If  $\tau$  is equal to  $\rho$ , then  $t^* = t^n = 1/2$ . This implies that if the public and private sectors have the same production technology, the tax rate is  $1/2$  regardless of whether or not the government considers its effect on market equilibrium. Then, the populations of each region are not changed by the government behavior. These results are summarized as follows.

**Proposition 1** If the degree of increasing returns to scale in public sector production is equal to that in private sector production ( $\tau = \rho$ ), the central government need not take account of its effect on the population.

When the production of public goods increases more than the production of consumption goods in the case of expanding the number of intermediate goods,  $\tau$  is smaller than  $\rho$ . Then the tax rate  $t^*$  is greater than  $1/2 = t^n$ . When the production of consumption goods increases more than the production of public goods ( $\tau > \rho$ ), the tax rate  $t^*$  is smaller than  $1/2 = t^n$ . These results are summarized as follows.

**Proposition 2** Assume that  $\tau < \rho$  ( $\tau > \rho$ ). If the central government ignores its effect on the population, the government chooses the lower (higher) tax rate in contrast to the optimal case. Then, the government causes too little (much) agglomeration.

The above proposition indicates that the government should decrease the extent of agglomeration when it has a lower degree of increasing returns in contrast to the private sector. In the optimal policy, the government with a lower level of productivity attempts to decrease public goods because the production of such goods yields less benefit than that of private goods. The government then sets a lower tax rate. With a smaller government, the region in which the government is located has a smaller demand for labor, thus restraining agglomeration. In that case, the government without a regional redistribution policy limits the regional inequality of the population. If the central government does not consider its effect on the population, such behavior does not arise. As a consequence, too much agglomeration arises in contrast to the optimal case.

To determine the optimal size of the government, I should compare the production function of the public and private sectors. Previous studies estimate that the private sector performs more efficiently than the public sector when they produce the same good. For example, Christoffersen, Paldam and Würtz (2007) examine the cleaning of Danish schools and conclude that the private sector reduces the cost compared to the public sector because the private sector utilizes economies of scale more efficiently. In this paper, when the private goods are similar to the public goods, these results mean that the degree of increasing returns to scale in the public sector production is lower than in private sector production ( $\tau > \rho$ ). Then, the government should decrease its size

to increase the private sector and restrain the regional inequality of population. If the central government does not consider its effect on the population, it is too large in size, and it causes too much agglomeration. Even if the government uses neither the transfer nor the regional allocation policy, it should consider its effect on the agglomeration.

#### 4. Conclusion

This paper analyzed whether or not the government should take into account agglomeration without a regional redistribution policy. If the degree of increasing returns to scale in public sector production is equal to that in private sector production, the central government need not take account of its effect on the population. But when the government has a lower degree of increasing returns in contrast to the private sector, it should take into account agglomeration. If the central government does not consider its effect on the population, its size is too large and it causes too much agglomeration.

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