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On the power of modified Kapetanios-Snell-Shin (KSS) tests

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Abstract

This paper investigates if the recursive detrending method that works well for linear unit root tests also provides good outcomes for nonlinear unit root tests. It is found that the method improves the power of the nonlinear test when only a non-trending mean needs to be removed. The test, however, is no longer performing well if removal of a deterministic trend is required.

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1. Introduction

Since the seminal work of Dickey and Fuller (1979), a large body of literature has come out considering the test of the unit root hypothesis. While issues such as robustness to serial correlation, heteroskedasticity, and structural change have been well explored, perhaps the most prominent aspect of this literature is the development of tests that aspire to improve the power properties of the Dickey-Fuller (DF) test. It is a well-known fact that the OLS mean adjustment (detrending) used in the DF test has a detrimental effect on the test's power properties. Elliot et al. (1996, ESR) investigate the issue of efficient detrending and derive a more powerful test by utilizing GLS detrending (see also Hwang and Schmidt (1996)). Alternatively, So and Shin (1999) and Shin and So (2001) suggest a recursive detrending method based test that they show is as powerful as the GLS-based test.

There has also been increasing concern that the DF test, which is derived under a linear setting, may fail to reject the null of a unit root when applied to non-linear but stationary time series. As a response to the concern, a range of unit root tests have been developed under a variety of nonlinear frameworks (see, for example, Enders and Granger (1998), Kapetanios, et al. (2003, KSS), Bec, et al. (2004), and Sollis (2009)). Among them, KSS (2003) propose a unit-root test using an auxiliary regression model that approximates the exponential smooth transition autoregressive (ESTAR) process using the Taylor series. As shown in KSS (2003), the test is more powerful than the DF test under the alternative of a globally stationary ESTAR process. Like the usual DF test, the KSS test uses OLS-detrending and its power may be improved if more efficient detrending methods were used. Kapetanios and Shin (2008) use GLS-detrending method and demonstrate that the GLS-based test is more powerful than the OLS-based test. Yet, to the best of our knowledge, there is no study that examines if recursive detrending also works for the KSS test. In this paper we aim to fill this gap in the literature.

The paper proceeds as follows. Section 2 reviews the KSS test with different detrending methods. Section 3 presents results from Monte Carlo experiments. Section 4 gives an empirical example. Section 5 concludes.

2. The KSS test

Let y_t, t=1,2,…,T, be an observed time series that can be decomposed into a deterministic part d_t and a stochastic (mean-adjusted) part x_t:

$$y_t = d_t + x_t \,. \tag{1}$$

For the deterministic part, we consider two cases: Case A (level): $d_t = \alpha$; Case B (trend): $d_t = \alpha + \beta t$. For the stochastic part, we consider the following ESTAR model

$$\Delta x_t = \gamma x_{t-1}\left\{1-\exp\left(-\theta x_{t-1}^2\right)\right\} + \varepsilon_t \,, \tag{2}$$

where ε_t ~iid(0,σ^2) and $-2 < \gamma < 0$. Given γ, when $\theta = 0$, x_t contains a unit root (so does y_t) while when $\theta > 0$, x_t is globally stationary (so that y_t is either level-stationary or trend-stationary) and the speed of mean reversion is determined by θ. Testing for a unit root in y_t

can be done by examining the null hypothesis of $H_0: \theta = 0$ (unit root) against the alternative hypothesis $H_1: \theta > 0$ (stationary).

Obviously, testing the null hypothesis is not feasible since γ is unidentified under the null. To overcome the problem, KSS apply a first-order Taylor series expansion to derive the following auxiliary equation:

$$\Delta x_t = \delta x_{t-1}^3 + \varepsilon_t, \tag{3}$$

where $\Delta x_t = x_t - x_{t-1}$ and $\delta = -\gamma\theta$, and suggest a t-type test for $\delta = 0$ against $\delta<0$ as

$$t = \hat{\delta} / s.e.(\hat{\delta}), \tag{4}$$

where $\hat{\delta}$ is the OLS estimate of δ and $s.e.(\hat{\delta})$ is the standard error of $\hat{\delta}$. Notice that x_t represents the unobserved deviation from the deterministic part d_t of y_t in (1). To make the t test in (4) feasible, d_t needs to be removed from the series first. KSS suggest using OLS residuals from the regression of y_t on d_t. Like the usual OLS-based DF test that also uses the OLS residuals, the OLS-based KSS test is lack of power. To improve the power in the context of linear unit root tests, ESR (1996) derive a more powerful DF test based on GLS residuals. Following ESR, Kapetanios and Shin (2008) suggest a modified KSS test with GLS residuals: $y_t - \tilde{d}_t$, with $\tilde{d}_t$ obtained from the regression of $y_t^{\bar{\rho}}$ on $d_t^{\bar{\rho}}$, where $y_t^{\bar{\rho}} = \{y_1, y_2 - \bar{\rho} y_1, ..., y_T - \bar{\rho} y_{T-1}\}$ and $z_t^{\bar{\rho}} = \{z_1, z_2 - \bar{\rho} z_1, ..., z_T - \bar{\rho} z_{T-1}\}$ with $z_t = 1$ (Case A) or $(1,t)'$ (Case B). Kapetanios and Shin (2008) recommend setting $\bar{\rho} = -17.5$ and confirm with simulation that the GLS detrending-based KSS test is more powerful than its OLS counterpart.

The deterministic part can also be removed through recursive detrending. So and Shin (1999) and Shin and So (2001) first suggest this approach in a linear unit root testing context and find the resulting test dominates the OLS-based DF test and is as powerful as the GLS-based test. In this paper, we are of interest to examine if the recursive-detrending approach will also be useful for the KSS test. For Case A, we use

$$y_t - \frac{1}{t-1}\sum_{i=1}^{t-1} y_i \quad \text{and} \quad y_{t-1} - \frac{1}{t-1}\sum_{i=1}^{t-1} y_i, \tag{5}$$

respectively for x_t and x_{t-1} in (3), for $t \geq 2$. In (5), y_t and y_{t-1} are both demeaned using lagged recursive mean $(t-1)^{-1}\sum_{i=1}^{t-1} y_i$, as in So and Shin (1999) and Shin and So (2001). For the trending case (Case B), there are various discussions of the problem regarding invariant transformation – see Taylor (2002), Phillips et al. (2004), and Rodrigues (2006). In this paper, following Phillips et al (2004), we use

$$\begin{gathered} y_t - \frac{1}{T-t+1}\left(y_T - y_{t-1}\right) + \frac{2}{t-1}\sum_{i=1}^{t-1} y_i - \frac{6}{t(t-1)}\sum_{i=1}^{t-1} i\, y_i \\ y_t + \frac{2}{t-1}\sum_{i=1}^{t-1} y_i - \frac{6}{t(t-1)}\sum_{i=1}^{t-1} i\, y_i \end{gathered} \tag{6}$$

in place of x_t and x_{t-1} in (3), respectively, for $t \geq 2$. It is worth noting that we have also tried other recursive-detrending methods suggested in Taylor (2004) and Rodrigues (2006) and found that the test based on the detrending method of Phillips et al (2004) is more powerful than the others. Critical values of the KSS statistics have been tabulated via simulations at T=50, 100, 200, 500, 1,000 and 2,000 with 50,000 replications, and presented in Table 1.

3. Monte Carol Results

In this section we report the results of Monte Carlo simulations designed to investigate the power of the KSS test when different methods of mean adjustment are considered. Simulations are performed in GAUSS for two sets of experiments: (1) stationary AR(1) processes (2) globally stationary ESTAR processes. We construct the model $y_t = d_t + x_t$, where

$$x_t = \rho x_{t-1} + \varepsilon_t \tag{7}$$

for the AR(1) model and

$$x_t = x_{t-1} + \gamma x_{t-1}\left\{1 - \exp\left(-\theta x_{t-1}^2\right)\right\} + \varepsilon_t \tag{8}$$

for the ESTAR model with $\varepsilon_t \sim N(0,1)$ and t=1,2,…, T. In line with the literature, we set $d_t = 0$ since all the tests considered are similar. For the AR processes, we consider ρ=0.99, 0.95, 0.9, 0.85. For the ESTAR processes, we consider γ=-0.1, -0.5, -1 and θ=0.01, 0.05, 0.01. The nominal size is set at 5% using the critical values in Table 1. The results are calculated using 5,000 replications at sample sizes T=100, 200, and 500. Following Kapetanios and Shin (2008), for all experiments the first 200 initial observations are discarded to minimize initial effects. We report the result in Table 1 and 2 and denote the KSS tests based on OLS, GLS and recursive detrending as KSS, KSSg and KSSr, respectively.

Table 2 gives the empirical power of the KSS test against the stationary AR(1) processes. Among the three tests, it is clearly shown that the recursive detrending-based test has the lowest power in both cases. Comparing the two modified tests the recursive detrending-based test appears to be somewhat more powerful than the GLS-based test in Case A (the only exception is when $\rho = 0.99$ and T=500). For example, at T=200, the rejection frequency is 0.764 for KSSr and 0.703 for KSSg when $\rho = 0.9$ and 0.378 for KSSr and 0.354 for KSSg when $\rho = 0.95$. For Case B, the recursive detrending-based test still dominates the GLS-based test when $\rho = 0.85$ or 0.9 but does not when $\rho = 0.95$ or 0.99. For example, at T=200, the rejection frequency is 0.516 for KSSr and 0.477 for KSSg when $\rho = 0.9$ but 0.194 for KSSr and 0.198 for KSSg when $\rho = 0.95$. Overall, the recursive detrending-based test seems to perform well to reject linear stationary alternatives.

Table 3 gives the empirical power of the KSS test against the stationary ESTAR processes. For Case A, as expected, the OLS-based test has the lowest power among the three tests and the two modified tests seem to have very similar power. For example, at T=200, the rejection frequency is 0.433 for KSS, 0.534 for KSSr and 0.555 for KSSg when $(\gamma, \theta) = (-0.1, 0.05)$ and

0.542 for KSS, 0.666 for KSSr and 0.658 for KSSg when $(\gamma,\theta)=(-0.1,0.1)$. For the trending case (Case B), the GLS-based test is still more powerful than the others. Surprisingly, the recursive detrending-based test is no longer working well – as a matter of fact, in several occasions (when θ is small, in particular) it can be even less powerful than the OLS-based test. For example, at T=200 with $(\gamma,\theta)=(-0.5,0.01)$, the rejection rate is 0.429 for KSS and 0.524 for KSSg but only 0.370 for KSSr.

Overall, it appears to be a good idea applying the recursive detrending-based KSS test to series without a deterministic trend as the test is more powerful than its OLS and GLS counterparts. However, it might not if removal of a deterministic trend is required.

4. Application to inflation rates

We apply the three KSS tests to quarterly CPI inflation rates of three countries (US, UK and Australia). The data are taken from the International Financial Statistics, covering the period 1958(1)-2007(4) with 200 observations. To accommodate serially correlated errors, we follow Kapetanios and Shin (2008) and augment with 4 lags to match quarterly frequency of the data. We assume there is no trend in the data (Case A) and report three sets of empirical results – the whole period and two sub-periods (1958(1)-1982(4) and 1983(1)-2007(4)) with 100 observations each – in Table 4.

The unit root null hypothesis is rejected for US and UK inflation rates in most cases but not for Australia. The rejection tends to be stronger when modified tests (KSSg and KSSr) are used. Also, there are occasions (i.e., US and UK during 1983(1)-2007(4)) that KSSr is able to reject the null while KSSg is not, a result that is presumably due to the better power performance of KSSr.

5. Conclusion

In this paper, we study the power performance of a modified KSS test by means of simulation. We show that the recursive detrending-based KSS test is more powerful than its OLS and GLS counterparts in the case that only a non-trending mean needs to removed. The recursive test, however, is no longer performing well if adjustment for a deterministic trend is required.

References

Bec, F., M.B. Salem and M. Carrasco (2004) "Tests for unit-root versus threshold specification with an application to the purchasing power parity relationship" *Journal of Business & Economic Statistics* **22**, 382-395.

Dickey, D. and W. Fuller (1979) "Distribution of the estimators for autoregressive time series with a unit root" *Journal of the American Statistical Association* **74**, 427-431.

Enders W. and C.W.J. Granger (1998) "Unit-root tests and asymmetric adjustment with an example using the term structure of interest rates" *Journal of Business & Economic Statistics* **16**, 304-311.

Elliott G., T.J. Rothenberg and J.H. Stock (1996) "Efficient tests for an autoregressive unit root" *Econometrica* **64**, 813-836.

Kapetanios, G., A. Snell and Y. Shin (2003) "Testing for unit root in the nonlinear STAR framework" *Journal of Econometrics* **112**, 359-379.

Kapetanios, G. and Y. Shin (2008) "GLS detrending-based unit root tests in nonlinear STAR and SETAR frameworks" *Economics Letters* **100**, 377-380.

Rodrigues, P.M.M. (2006) "Properties of recursive trend-adjusted unit root test" *Economics Letters* **91**, 413-419.

Phillips, P.C.B., J.Y. Park and Y. Chang (2004) "Nonlinear instrumental variable estimation of an autoregression" *Journal of Econometrics* **118**, 219-249.

Hwang, J. and P. Schmidt (1996) "Alternative methods of detrending and the power of unit root tests" *Journal of Econometrics* **71**, 227-248.

Shin, D. and B. So (2001) "Recursive mean adjustment for unit root tests" *Journal of Time Series Analysis* **5**, 595-612.

So, B.S. and D. Shin (1999) "Recursive mean adjustment in time series inferences" *Statistics and Probability Letters* **43**, 65-73.

Sollis, R. (2009) "A simple unit root test against asymmetric STAR nonlinearity with an application to real exchange rates in Nordic countries" *Economic Modelling* **26**, 118-125.

Taylor, A.M.R. (2002) "Regression-based unit root tests with recursive mean adjustment for seasonal and nonseasonal time series" *Journal of Business & Economic Statistics* **20**, 269-281.

Table 1: Critical Values of Modified KSS tests

(a) Case A: Level

	KSS			KSSr			KSSg		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
50	-2.66	-2.95	-3.55	-1.77	-2.10	-2.70	-2.59	-2.86	-3.46
100	-2.65	-2.94	-3.51	-1.79	-2.12	-2.73	-2.44	-2.72	-3.26
200	-2.64	-2.94	-3.50	-1.81	-2.13	-2.73	-2.26	-2.53	-3.08
500	-2.65	-2.93	-3.51	-1.82	-2.13	-2.73	-2.10	-2.37	-2.92
1000	-2.66	-2.93	-3.47	-1.82	-2.14	-2.74	-2.01	-2.33	-2.90
2000	-2.65	-2.93	-3.49	-1.82	-2.14	-2.74	-1.97	-2.23	-2.82

(b) Case B: Trend

	KSS			KSSr			KSSg		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
50	-3.14	-3.45	-4.12	-1.65	-1.97	-2.60	-2.93	-3.24	-3.87
100	-3.13	-3.42	-3.98	-1.70	-2.01	-2.62	-2.81	-3.09	-3.65
200	-3.13	-3.41	-3.97	-1.71	-2.01	-2.63	-2.74	-3.01	-3.56
500	-3.12	-3.40	-3.95	-1.72	-2.03	-2.65	-2.68	-2.97	-3.58
1000	-3.13	-3.40	-3.93	-1.72	-2.03	-2.64	-2.66	-2.95	-3.52
2000	-3.13	-3.39	-3.93	-1.72	-2.03	-2.63	-2.65	-2.93	-3.50

Table 2: Empirical Power of Modified KSS tests against AR(1)

(a) Case A: level

	KSS			KSSr			KSSg		
AR	T=100	T=200	T=500	T=100	T=200	T=500	T=100	T=200	T=500
0.99	0.059	0.067	0.117	0.066	0.085	0.163	0.061	0.084	0.170
0.95	0.120	0.263	0.736	0.164	0.378	0.874	0.145	0.354	0.846
0.90	0.249	0.599	0.978	0.363	0.764	0.994	0.295	0.703	0.992
0.85	0.434	0.837	0.996	0.599	0.934	1.000	0.497	0.902	0.999

(b) Case B: trend

	KSS			KSSr			KSSg		
AR	T=100	T=200	T=500	T=100	T=200	T=500	T=100	T=200	T=500
0.99	0.053	0.055	0.078	0.053	0.060	0.084	0.055	0.057	0.090
0.95	0.080	0.153	0.548	0.089	0.194	0.671	0.092	0.198	0.614
0.90	0.152	0.403	0.929	0.188	0.516	0.978	0.184	0.477	0.930
0.85	0.264	0.668	0.988	0.333	0.798	0.998	0.314	0.734	0.990

Table 3: Empirical Power of Modified KSS tests against ESTAR

(a) Case A: level

		KSS			KSSr			KSSg		
γ	θ	T=100	T=200	T=500	T=100	T=200	T=500	T=100	T=200	T=500
	0.01	0.090	0.160	0.698	0.121	0.210	0.772	0.108	0.239	0.731
-0.1	0.05	0.147	0.433	0.978	0.201	0.534	0.991	0.183	0.555	0.981
	0.10	0.189	0.542	0.984	0.258	0.666	0.995	0.233	0.658	0.990
	0.01	0.242	0.735	0.999	0.307	0.785	1.000	0.309	0.787	0.999
-0.5	0.05	0.773	0.996	1.000	0.827	1.000	1.000	0.813	0.994	1.000
	0.10	0.940	1.000	1.000	0.961	1.000	1.000	0.943	1.000	1.000
	0.01	0.489	0.953	1.000	0.550	0.964	1.000	0.566	0.944	1.000
-1.0	0.05	0.980	1.000	1.000	0.987	1.000	1.000	0.980	1.000	1.000
	0.10	0.999	1.000	1.000	0.999	1.000	1.000	0.998	1.000	1.000

(b) Case B: trend

		KSS			KSSr			KSSg		
γ	θ	T=100	T=200	T=500	T=100	T=200	T=500	T=100	T=200	T=500
	0.01	0.062	0.095	0.403	0.075	0.106	0.344	0.071	0.122	0.484
-0.1	0.05	0.094	0.230	0.890	0.099	0.238	0.855	0.107	0.300	0.868
	0.10	0.111	0.317	0.940	0.114	0.336	0.942	0.143	0.390	0.918
	0.01	0.147	0.439	0.993	0.138	0.370	0.982	0.186	0.524	0.962
-0.5	0.05	0.528	0.977	1.000	0.455	0.946	1.000	0.604	0.963	1.000
	0.10	0.783	0.998	1.000	0.710	0.994	1.000	0.834	0.991	1.000
	0.01	0.280	0.808	1.000	0.223	0.696	1.000	0.346	0.821	0.997
-1.0	0.05	0.900	1.000	1.000	0.812	0.998	1.000	0.913	0.998	1.000
	0.10	0.990	1.000	1.000	0.970	1.000	1.000	0.983	1.000	1.000

Table 4: KSS test results for CPI Inflation (Case A: level)

Time Period	Country	KSS	KSSr	KSSg
1958(1)-2007(4)	US	-2.38	-2.11*	-2.30*
	UK	-3.17**	-2.39**	-3.07**
	Australia	-2.35	-1.54	-2.11
1958(1)-1982(4)	US	-3.18**	-2.76***	-3.08**
	UK	-2.68*	-1.87*	-2.59*
	Australia	-2.30	-1.39	-2.14
1983(1)-2007(4)	US	-2.17	-2.01*	-2.38
	UK	-2.25	-2.12**	-2.35
	Australia	-1.31	-0.26	-1.21

Note: * denotes rejection at 10% significance level, ** denote rejection at 5% level, and *** denote rejection at 1% level. Critical values from Table 1(a) for T=100 and 200 are used.