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Bank insolvency risk and aggregate Z-score measures: a caveat

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Abstract

We demonstrate that a popular approach to constructing (weighted) mean-based aggregate bank insolvency risk measures is inherently biased; we also suggest an alternative approach that avoids this problem.

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1. Introduction

A commonly used risk measure reflecting a bank's probability of insolvency is the Z-score; it is most often attributed to Boyd and Graham (1986), Hannan and Hanweck (1988) and Boyd et al. (1993), although its roots can be traced back as far as Roy (1952). Its use in banking and financial stability related studies is widespread due to its relative simplicity and the fact that it can be calculated relying solely on accounting information.

Such Z-score measures are now also being applied at aggregate, i.e. sectoral, regional or country, rather than individual bank levels, using various approaches. Uhde and Heimeshoff (2009) and Demirguc-Kunt and Detragiache (2010) aggregate individual bank balance sheets before calculating aggregate Z-scores, while Beck et al. (2010) use the median of individually calculated Z-scores instead; Houston et al. (2010), on the other hand, use the (weighted) mean of individually calculated Z-score measures as their aggregate insolvency risk measure.

The purpose of this note is to demonstrate that aggregate bank insolvency risk measures that are constructed using the (weighted) mean of individually calculated Z-score measures are inherently biased; we also propose an alternative approach to their construction that avoids this problem.

2. Aggregate Z-score measures: a caveat

Let us first recapitulate the probabilistic rationale for the use of Z-score measures in the following

Lemma. *Define bank insolvency as a state where* $(car + roa) \leq 0$*, with car the bank's capital-asset ratio and roa its return on assets. Then, if roa is a normally distributed random variable such that* $roa \sim N(\mu_{roa}, \sigma^2_{roa})$*, Boyd and Graham (1986) noted that the probability of insolvency can be given as*

$$p(roa \leq -car) = p(\frac{roa - \mu_{roa}}{\sigma_{roa}} \leq -Z) = \Phi(-Z) \tag{1}$$

where the Z-score is defined as $Z \equiv \frac{car + \mu_{roa}}{\sigma_{roa}} > 0$*, and* $\Phi(\cdot)$ *is the cumulative distribution function (CDF) of the standard normal distribution* $N(0, 1)$*.*

We can then directly state the subsequent

Corollary 1. *A (weighted) average of Z-scores* $\sum_{i=1}^{N} w_i Z_i$ *(with* $\sum_{i=1}^{N} w_i = 1$*) gives a downwardly biased measure of the (weighted) average probability of insolvency* $\sum_{i=1}^{N} w_i p_i$*, as it holds that* $\sum_{i=1}^{N} w_i p_i = \sum_{i=1}^{N} w_i \Phi(-Z_i) > \Phi(-\sum_{i=1}^{N} w_i Z_i)$ *from Equation (1).*

Proof. Noting that $\Phi''(-Z) = (2\pi)^{-\frac{1}{2}} e^{-\frac{Z^2}{2}} Z > 0$ for $Z > 0$, this is a direct consequence of Jensen's inequality (see Feller, 1971, p. 153), which states that if $u(x)$ is a strictly convex function, then $E\left[u\left(X\right)\right] > u\left(E\left[X\right]\right)$ provided that the expectations exist.

Clearly, Corollary 1 represents an important caveat for the construction of aggregate bank insolvency risk measures in stressing the inherent bias introduced if they are calculated using a (weighted) average of Z-score measures. However, it also points towards a straightforward solution to this potential bias problem, which we state in

Corollary 2. *When constructing (weighted) mean-based aggregate insolvency risk measures, these should be computed as (weighted) averages of the probabilities of insolvency, as implied in the normally distributed case by Equation (1), i.e. using* $\sum_{i=1}^{N} w_i p_i = \sum_{i=1}^{N} w_i \Phi(-Z_i)$.

Proof. This follows from Corollary 1.

Aggregate bank insolvency risk measures constructed in this manner can then be appropriately transformed for use in regression analysis, e.g. using a probit[1] and/or logarithmic transformation; (implicitly) applying such nonlinear transformations before aggregation leads to a biased measure of aggregate insolvency risk because of Jensen's inequality.

3. Conclusion

We showed that a popular approach to constructing (weighted) mean-based aggregate bank insolvency risk measures is inherently biased because of Jensen's inequality; we also proposed a straightforward alternative approach that avoids this problem.

References

Beck, T., A. Demirguc-Kunt, and R. Levine (2010) "Financial institutions and markets across countries and over time: the updated financial development and structure database" *The World Bank Economic Review 24*(1), 77–92.

Boyd, J. H. and S. L. Graham (1986) "Risk, regulation, and bank holding company expansion into nonbanking" *Quarterly Review - Federal Reserve Bank of Minneapolis 10*(2), 2–17.

Boyd, J. H., S. L. Graham, and R. S. Hewitt (1993) "Bank holding company mergers with nonbank financial firms: effects on the risk of failure" *Journal of Banking & Finance 17*(1), 43–63.

Demirguc-Kunt, A. and E. Detragiache (2010) "Basel core principles and bank soundness: Does compliance matter?" *Journal of Financial Stability, in press*.

Feller, W. (1971) *Probability Theory and its Applications, vol. II,* New York, NY: John Wiley & Sons.

Hannan, T. H. and G. A. Hanweck (1988) "Bank insolvency risk and the market for large certificates of deposit" *Journal of Money, Credit and Banking 20*(2), 203–211.

Houston, J. F., C. Lin, P. Lin, and Y. Ma (2010) "Creditor rights, information sharing, and bank risk taking" *Journal of Financial Economics 96*(3), 485–512.

Roy, A. D. (1952) "Safety first and the holding of assets" *Econometrica 20*(3), 431–449.

Uhde, A. and U. Heimeshoff (2009) "Consolidation in banking and financial stability in Europe: empirical evidence" *Journal of Banking & Finance 33*(7), 1299–1311.

[1] The probit function is the inverse CDF $\Phi^{-1}(\cdot)$ of the standard normal distribution; thus $\Phi^{-1}(p) = \Phi^{-1}(\Phi(-Z)) = -Z$, giving the (negative) Z-score.