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Search externalities with crowding-out effects

Bruno Decreuse
GREQAM and University of Aix-Marseilles

Abstract

We consider a static search model with two types of workers, Nash bargaining, and free entry of firms. The matching function is specified so as cross-type congestion effects are asymmetric. Skilled workers create congestion effects for all, while unskilled workers do not affect the odds of employment for the skilled. An increase in the share of skilled workers has two effects on the welfare of the unskilled: a negative crowding-out effect, and a positive labor demand effect. The former (latter) effect dominates whenever the skill differential is small (large).

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1 Introduction

This paper examines the following market situation. The labor market is frictional, good workers are preferred to bad workers in the recruitment process, and the total number of available jobs depends on the profitability of employment relationships. Do good workers hurt bad workers in this environment? Gautier (2002) addresses this question in a random matching model. Gautier's model features skilled and unskilled workers who compete for simple jobs. Once employed in such a job, skilled workers go on searching. Skilled workers have two effects on job profitability. First, they are more productive, which increases profitability though rent-sharing. Second, they quit jobs at faster rate, which lowers profitability. Whenever the former effect dominates the latter, the skilled proportion boosts job creation. In turn, the rise in job availability benefits to everyone as random matching means that workers equally share job opportunities irrespective of talent.[1]

The key feature of Gautier's model is that the economic position of the unskilled improves with the skilled proportion whenever firms prefer to hire skilled workers, that is whenever skilled workers are good and unskilled workers are bad. Then, the model ambiguity is due to the fact that 'skilled' does not necessarily mean 'good' once accounted for higher quit rates among the skilled. Gautier's result is due to the random search assumption. In random search models, congestion effects are symmetric and the matching process is non-discriminant. Improving job availability must be good for all as no groups of workers can disproportionately capture the new jobs. The purpose of the current note is to revisit this relationship between the good proportion and the job opportunities of the bad. In particular, I get rid off the random search assumption.

I consider a very stylized static search model with two types of workers, free entry of firms, and Nash bargaining. As there is a single type of jobs, all workers seek for the same jobs and skilled workers are unambiguously good for potential employers. Cross-type congestion effects are asymmetric. Skilled workers create congestion effects for all. But unskilled workers do not affect the odds of employment for the skilled. I show that an increase in the skilled proportion has two effects on the welfare of the unskilled. On the one hand, the positive *labor demand effect*, whereby the total number of advertised jobs increases with the skilled proportion. This effect is very similar to Gautier. On the other hand, the negative *crowding-out effect*, according to which the unskilled probability of getting a job decreases at given number of jobs per job-seeker. I show that the crowding-out effect is stronger than the labor demand effect if and only if the productivity differential between skill groups is lower than a threshold value.

The crowding-out effect highlighted in this paper differs from Acemoglu (1999) and Rosen and Wasmer (2005), in which holding a vacancy features an option value. In these papers, an increase in the skilled proportion may have a negative impact on the unskilled because firms reject unskilled applications to make a better match in the future (Acemoglu), or because unskilled wage goes down through wage bargaining (Rosen and Wasmer). These papers make predictions that deeply differ from mine. An increase in the skill differential magnifies the labor demand effect in my model, thereby improving

[1] See also Dolado et al (2009) who provide a matching model with on-the-job search and worker-firm heterogeneity.

the unskilled economic situation. A similar increase improves the value of a vacancy in Acemoglu and Rosen and Wasmer, thereby deteriorating unskilled welfare.

The closest paper is Shi (2002) who considers a directed search model. There are two types of jobs and two types of workers. Firms announce wages, workers observe wage offers and send a single application. Firms can select among the pool of applicants, which lead them to favor skilled workers. Shi shows that there are two types of equilibria: a pooling equilibrium in which high-tech and low-tech firms coexist, and a separating equilibrium in which there are only high-tech firms. The separating equilibrium is very close to the equilibrium I study in this paper (the main difference relies on wage determination). However, Shi only studies comparative statics for the pooling equilibrium, but does not examine the properties of the separating equilibrium. I conjecture that one could reach a similar result to mine in this case.[2]

The remaining of the paper is organized in two parts. Section 2 introduces the model, while section 3 shows the results.

2 The model

I present a static matching model with two types of workers, a single type of job, Nash bargaining over match surplus, and endogenous supply of jobs. The key assumption relates to the matching technology: there is a unique search market, yet the unskilled do not create congestion effects for the skilled.

There are two types of agents seeking a job: n_1 skilled and n_2 unskilled workers. Agents differ with respect to the amount of output they produce if employed. Type-i agents produce $2y_i$. Let $y_2 = (1-\rho)\, y_1$, where $\rho/(1-\rho) \geq 0$ is the skill premium.

There is a large number of firms, each endowed with a single job slot which can be either active or inactive. Each active job costs $cy_1 > 0$, $c \in (0,1)$, and needs to be filled before production starts. Inactive jobs cost nothing.

Active jobs and job-seekers meet each other on the search market. Once a worker is hired, she starts producing. The wage is bargained along symmetric Nash bargaining. As the model is static and there is no unemployment income, this implies that each party receives half output. Output sharing implies that profit increases with skill. Firms prefer to hire skilled workers as a result.

I now present the matching side of the model. Usually, the matching technology gives the number of hires of each type as a function of the numbers of job-seekers and vacancies. Formally, let M_i denote type-i hires, and m_i be type-i-specific matching technology. Then, $M_i = m_i\,(n_1, n_2, v)$, $i = 1, 2$. The matching technology generally features congestion effects, as expanding individuals or jobs reduces the matching odds for individuals or jobs of the same type. The crucial point then is whether the matching technology displays cross-type congestion effects. When the search place is perfectly segmented by skill, we have $M_i = m_i\,(n_i, v_i)$, with $v_1 + v_2 = v$. In this case, workers of a given type do not create congestion effects on workers of the other type. When search is random, we have

[2] There are a number of directed search models with heterogeneous firms/workers (see for instance Shi, 2001, Lang and Manove, 2003, and Shimer, 2005). These models raise important issues, but do not focus on the impact of changes in the composition of worker types on the extent of mismatch and the crowding-out of lower-skilled workers.

$M_i = m(n_1 + n_2, v)\, n_i / (n_1 + n_2)$. Cross-type effects are symmetric: at given number of jobs, an increase in the number of type-i workers reduces the matching probability for both types. Workers are equally likely to get a job and the number of hires accruing to type-i workers is proportional to their share among the job-seekers. This is the case analyzed by Gautier (2002).

The matching technology I consider features asymmetric cross-type congestion effects. As in the random matching model, the aggregate number of hires M is determined by a function whose inputs are the total number of job-seekers $n_1 + n_2$ on the one hand, and the number of active jobs v on the other hand:

$$M \equiv \min \{m(n_1 + n_2, v), n_1 + n_2, v\} \tag{1}$$

The technology m is strictly increasing and strictly concave in each argument, and has constant returns to scale. Hereafter, I only focus on market situations where $M = m(n_1 + n_2, v)$.

Unlike random matching, matches are not equally shared between workers' types. As firms prefer skilled workers, the unskilled do not create congestion for the skilled. Therefore,

$$M_1 = m(n_1, v) \tag{2}$$

The unskilled get the residual number of hires:

$$M_2 = m(n_1 + n_2, v) - m(n_1, v) \tag{3}$$

The strict concavity of m guarantees M_2 is strictly decreasing in n_1 *at given number of jobs*. Therefore, the skilled crowd-out the unskilled. Of course, by construction $M_1 + M_2 = M$, which ensures that the number of jobs formed has constant returns to scale vis-à-vis job-seekers and vacancies.

The directed search model with worker heterogeneity is a particular case of the technology I consider. This model provides an explicit scenario based on coordination frictions. In this scenario, job-seekers observe available jobs, and each worker sends an application to one of the v jobs. If the job receives a single application, the worker is employed. If the job receives several applications, three cases may happen. If the worker is skilled, the probability that s/he is hired is one divided by the number of skilled who applied for the job. If the worker is unskilled and at least one skilled worker applied, the probability is zero. If the worker is unskilled and no skilled applied, the probability is one divided by the number of unskilled who applied. The probability that a given vacancy stays unfilled (respectively, not filled with a skilled worker) is $(1 - 1/v)^{n_1+n_2}$ (resp. $(1 - 1/v)^{n_1}$). Thus, the number of (resp. skilled) hires is $v\left[1 - (1 - 1/v)^{n_1+n_2}\right]$ (resp. $v\left[1 - (1 - 1/v)^{n_1}\right]$. As v, n_1 and n_2 are sufficiently large, $M = m(n_1 + n_2, v) = v\left[1 - \exp(-(n_1 + n_2)/v)\right]$ (resp. $M_1 = m(n_1, v) = v\left[1 - \exp(-n_1/v)\right]$, the urn-ball matching technology. Unskilled hires can be computed residually, i.e. $M_2 = M - M_1$.

Let μ_i denote type-i workers' probability of getting a job, and η_i be firms' probability of recruiting a type-i individual. The number of hires is equiprobably distributed within each side of the market. Hence,

$$\mu_1 \equiv \frac{m(n_1,v)}{n_1} \equiv m(1,\theta/x) \tag{4}$$

$$\mu_2 \equiv \frac{m(n_1+n_2,v)-m(n_1,v)}{n_2} \equiv \frac{m(1,\theta)-xm(1,\theta/x)}{1-x} \tag{5}$$

$$\eta_1 \equiv \frac{m(n_1,v)}{v} \equiv \frac{m(1,\theta/x)}{\theta/x} \tag{6}$$

$$\eta_2 \equiv \frac{m(n_1+n_2,v)-m(n_1,v)}{v} \equiv \frac{m(1,\theta)-xm(1,\theta/x)}{\theta} \tag{7}$$

where $x \equiv n_1/(n_1+n_2)$ is the share of skilled agents and $\theta \equiv v/(n_1+n_2)$ is the market tightness. Matching probabilities depend on market tightness θ and number of vacancies per skilled worker θ/x. For instance, the skilled matching probability μ_1 only depends on the number of jobs per skilled workers. Skilled workers create congestion effects for each other, while their employment prospects do not respond to changes in the number of unskilled workers. Conversely, the unskilled are hurt by the presence of skilled workers. The matching probability μ_2 is therefore decreasing in the skilled proportion x at given tightness θ.

Let w_i denote the expected utility of type-i workers, and π be firms' expected profit:

$$w_i = \mu_i y_i \tag{8}$$

$$\pi = \eta_1 y_1 + \eta_2 y_2 - c y_1 \tag{9}$$

Finally, the number of active jobs obeys the free-entry condition $\pi = 0$.

3 Results

I solve the model and analyze the impacts of demographic changes on the welfare of each skill group. The main result is the non-monotonic relationship between the share of skilled workers and the welfare of unskilled individuals. Namely, there exists a unique productivity differential above which the unskilled matching probability increases with the skilled proportion, and below which it decreases.

Consider the following function $\psi(z) \equiv m(1,z)/z$, that is the recruitment probability. It is strictly decreasing in z. From equations (6), (7) and (9), and constant returns to scale in the matching technology, solving reduces to finding market tightness θ such that

$$\psi(\theta/x)\rho + \psi(\theta)(1-\rho) = c \tag{10}$$

The properties of the function ψ imply uniqueness whenever there exists an equilibrium, which we assume.[3] Equilibrium tightness decreases with the skill premium $\rho/(1-\rho)$, and increases with the skilled proportion x.

[3] This involves additional restrictions on the matching technology m and the job creation cost c. One must check that matching probabilities are well defined, i.e. lower than one. This is so whenever $m(1,\theta/x) < 1$, and $m(1,\theta)/\theta < 1$. These conditions are always satisfied with the urn-ball technology. In this case, there exists an equilibrium if and only if $0 < c < 1$.

What are the effects of x on the welfare of skilled and unskilled workers? To answer this question, I need to compute how x alters skill-specific matching probabilities. Consider skilled workers. From (4), I have to find how changes in x alter the equilibrium ratio $z \equiv \theta/x$. Using (10), I get

$$\psi(z)\rho + \psi(xz)(1-\rho) = c \tag{11}$$

The variable z is strictly decreasing in x. Therefore, the welfare of skilled workers decreases with their share in the workforce. This result is typical of directed search models with mismatch (see Shi, 2002, for instance). This differs from random matching models where the skilled actually benefit from an increase in their proportion.

We now turn to the unskilled. From (5),

$$x\frac{d\mu_2/dx}{\mu_2} = \underbrace{x\frac{\partial\mu_2/\partial x}{\mu_2}}_{\substack{\text{crowding-out effect}\\ <0}} + \underbrace{\left(\theta\frac{\partial\mu_2/\partial\theta}{\mu_2}\right)\left(x\frac{d\theta/dx}{\theta}\right)}_{\substack{\text{labor demand effect}\\ >0}} \tag{12}$$

with

$$x\frac{\partial\mu_2/\partial x}{\mu_2} = \frac{x}{1-x}\left[1 - \frac{m(1,\theta/x)}{\mu_2}(1-\alpha(\theta/x))\right] < 0 \tag{13}$$

$$\theta\frac{\partial\mu_2/\partial\theta}{\mu_2} = \frac{\alpha(\theta)}{1-x}\frac{m(1,\theta)}{\mu_2}\left[1 - x\frac{\alpha(\theta/x)}{\alpha(\theta)}\frac{m(1,\theta/x)}{m(1,\theta)}\right] > 0 \tag{14}$$

$$x\frac{d\theta/dx}{\theta} = \frac{\rho}{\rho + (1-\rho)x\frac{(1-\alpha(\theta))m(1,\theta)}{(1-\alpha(\theta/x))m(1,\theta/x)}} > 0 \tag{15}$$

where $\alpha(\theta) = \theta m_2(1,\theta)/m(1,\theta) \in (0,1)$ is the elasticity of the matching technology with respect to the number of vacancies.

A change in the skilled proportion x has two conflicting effects on job opportunities for the unskilled. On the one hand, there is a negative *crowding-out effect*. This is a partial equilibrium effect due to the fact that the skilled are favored by the matching process. On the other hand, there is a positive *labor demand* effect. It is a general equilibrium effect whereby the total number of advertised jobs increases with the share of skilled workers. Indeed, job creation is driven by profitability, and rent-sharing implies job profitability increases with skill level. It follows equilibrium tightness positively responds to the skilled share.

Which of these two effects is the largest? The answer depends on the skill premium ρ. The labor demand effect is proportional to $\varepsilon_{\theta,x} \equiv x(d\theta/dx)/\theta$, the elasticity of equilibrium tightness with respect to the skilled proportion. In turn, the magnitude of this elasticity increases with ρ:

$$\frac{d\varepsilon_{\theta,x}}{d\rho} = x\left(\frac{\varepsilon_{\theta,x}}{\rho}\right)^2 \frac{(1-\alpha(\theta))m(1,\theta)}{(1-\alpha(\theta/x))m(1,\theta/x)} > 0 \tag{16}$$

The higher the skill differential, the higher the labor demand effect. Suppose first $\rho = 0$. Then, all workers are equally productive. It follows that $\varepsilon_{\theta,x} = 0$ and equilibrium tightness does not respond to changes in the composition of the workforce. The labor demand effect vanishes as a result. Then, the crowding-out effect implies that the welfare of unskilled workers strictly decreases with the skilled proportion. As ρ increases, the elasticity of equilibrium tightness with the skilled proportion increases too, and the magnitude of the labor demand effect rises. In the limit case where ρ tends to 1, we have

$$\begin{aligned} x \frac{d\mu_2/dx}{\mu_2}\bigg|_{\rho=1} &= \frac{x}{1-x}\left[1 - \frac{(1-\alpha(\theta/x))\, m(1,\theta/x)}{\mu_2}\right] \\ &\quad + \frac{\alpha(\theta)}{1-x}\frac{m(1,\theta)}{\mu_2}\left[1 - x\frac{\alpha(\theta/x)}{\alpha(\theta)}\frac{m(1,\theta/x)}{m(1,\theta)}\right] \end{aligned} \tag{17}$$

which has the sign of

$$\phi(x) = m(1,\theta)\left[x + \alpha(\theta) - \alpha(\theta)\, x\right] - x m(1,\theta/x) \tag{18}$$

But $\phi(0) > 0$, $\phi(1) = 0$ and $\phi'(x) = (1-\alpha(\theta))\, m(1,\theta) - (1-\alpha(\theta/x))\, m(1,\theta/x) < 0$ because m is strictly concave. Therefore, we have $x\,(d\mu_2/dx)/\mu_2|_{\rho=1} > 0$. It follows that there exists a unique skill differential $\widehat{\rho} \in (0,1)$ such that $x\,(d\mu_2/dx)/\mu_2 > 0$ if and only if $\rho > \widehat{\rho}$. In words, the labor demand effect dominates the crowding-out effect whenever the skill differential is sufficiently large. This result holds for a very large class of matching technologies, including the Cobb-Douglas function as well as the urn-ball technology.

4 Conclusion

I consider a static matching model with two types of workers, Nash bargaining, and free entry of firms. The matching function is specified so as cross-type congestion effects are asymmetric. Skilled workers create congestion effects for all, while unskilled workers do not affect the odds of employment for the skilled. An increase in the skilled proportion has two effects on the welfare of the unskilled: a negative crowding-out effect, and a positive labor demand effect. The former (latter) effect dominates whenever the skill differential is small (large).

The model only requires there are good and bad workers from employers' perspective. However, good workers are not necessarily the most productive. The assumption that there is a single type of jobs helps in this respect. There are a couple of papers with several job types, e.g. Marimon and Zilibotti (1999), Shimer and Smith (2000), Shimer (2005), Gautier and Teulings (2006), Gautier et al (2010). In those settings, match surplus typically depends on the distance between worker and job types and usually it is not the case (if production technology is sufficiently supermodular, i.e. log or root supermodular) that firms prefer the most productive workers. The reason is that better workers have a higher outside option and they require compensation for this in the form of higher wages.

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