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Coalition Stability with Heterogeneous Agents

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Abstract

We analyze coalition formation with heterogeneous agents based on an individual stability concept. Defining exchanging and refractory agents, we give existence and enlargement conditions for coalitions with heterogeneous agents. Using the concept of exchanging agents we give necessary conditions for internal stability and show that refraction is a sufficient condition for the failure of an enlargement of the coalition.

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1. Introduction

This paper proposes an extension of the concept of coalition stability introduced by d'Aspremont *et al.* (1983) in a non-cooperative framework. With that concept, a coalition is considered to be stable if none of its members has an incentive to withdraw (this is known as internal stability) and none of the non-members has an incentive to participate in the coalition (this is known as external stability). This concept has been used to study, among other issues, the formation of cartels (d'Aspremont *et al.*, 1983; Donsimoni *et al.*, 1986; d'Aspremont and Gabszewicz, 1986, and others), international environmental agreements (Carraro and Siniscalco, 1993; Barrett, 1994, and others) or trade unions (e.g, Baldwin, 1995). However, these papers typically assume that all agents are identical. This implies that the payoffs obtained by all members of the coalition are identical and, in addition, that the payoff obtained by the agents outside of the coalition are also identical. In fact, the payoffs obtained depend only on the size of the coalition and not on its composition. Thus, if an agent benefits from joining the coalition, no member of the coalition will find it beneficial to deviate. This is, however, not valid any more in the asymmetrical case: an initially externally unstable coalition (in d'Aspremont *et al.'s* (1983) sense**)** can be internally unstable when it integrates a new agent from the fringe.

The goal of this article is to propose an adaptation of these concepts to contexts where agents are heterogeneous. We propose to define external stability based on the property of internal stability of a coalition with one more member. That is, we say that a coalition is externally stable if any marginal enlargement would lead to an internally unstable coalition. Thus, we give conditions under which, after the enlargement, a decrease in the individual payoff of a former member of the coalition is a sufficient condition for the instability of the extended coalition (*i.e.* for the failure of the enlargement). We also define two sets of agents on which to focus the analysis to study the stability of a given coalition: the "exchanging" and the "refractory agents". The former "envy" a position outside of the coalition to which they belong, that is, they would like to be replaced within the coalition. We show that this set is related to difficulties in finding members willing to participate in the coalition, due to free-rider behaviors (incentives to prefer that others participate in the coalition instead of participating yourself). On the other hand, the refractory agents would not like to see a particular agent joining their coalition. We show that in an heterogeneous context, refraction does not imply deviation, making the connections between refraction and deviation interesting while analyzing the external stability of coalitions. As shown below, these subsets also prove to be useful to explain traditional concerns in industrial economics.

There are a number of solution concepts proposed to determine the size of a stable coalition in the presence of heterogeneous agents. Among others, the quasi-hybrid equilibrium (Zhao, 1992), the largest consistent set (Chwe, 1994), the equilibrium binding agreements (Ray and Vohra, 1997), the farsightedly stable set (Herings *et al.*, 2010) or the optimal sharing rule (Weikard, 2009). A first difference among our approach, and more generally the framework of d'Aspremont *et al*. (1983), and most of these approaches is that we are in a fully non-cooperative framework while these papers combine cooperative and non-cooperative concepts (see Xue (1998)). Another difference is that we remain in a static framework and that we do not assume that agents are perfectly farsighted (as in Chwe (1994) or in Mauleon and Vannetelbosch (2004); see Xue (1998) for a discussion of different types of farsightedness). Farsightedness is a nice feature, but assuming it with no-limit is probably a strong assumption in many settings (our framework implies limited farsightedness). Weikard (2009) is closely related to our paper since he works directly with the stability concept of d'Aspremont *et al.* (1983), although he keeps the original definition and adds the assumption that payoffs are shared using an "optimal sharing rule" (he obtains results for coalitions with negative spillovers and for a particular class of coalitions with positive spillovers). Finally, our analysis shares with d'Aspremont *et al.* (1983) what is probably its main strength and what explains that it is still popular in applications: it is simple. In addition, by defining the "exchanging" and the "refractory" sets we allow the analyst to focus on the main players that will drive the stability of a given coalition.

2. The framework

Let N = {1..., n} be all the agents, assumed to be heterogeneous, bargaining to coordinate their behavior. Let $C_p = \{1, \ldots, p\}$ be the subset of p agents forming a cooperating coalition ($C_p \subset N$) and $F_q = \{p+1, \ldots, n\}$ the complementary in N. We call the latter set a fringe of q = n-p agents. The partition $[C_p, F_q]$ defines a coalition structure. We suppose that for any coalition structure $[C_p, F_q]$, a system of payoffs exists that uniquely determines a payoff $\pi_i(C_p)$ for each agent $i \in N$.

We call an agent $i \in C_p$ a deviating member of the coalition if i obtains a higher payoff in the coalition structure $[C_p - \{i\}, F_q \cup \{i\}]$ than in $[C_p, F_q]$, *i.e* $\pi_i(C_p) < \pi_i(C_p - \{i\})$. Internal and external stability of a coalition is analyzed considering only individual moves from the coalition towards the fringe, or from the fringe towards the coalition. The set of all deviating members is denoted by:

$$D(C_p) = \left\{ i \in C_p,\ \pi_i(C_p) < \pi_i(C_p - \{i\}) \right\}.$$

With homogeneous agents (as in d'Aspremont *et al.* (1983)), a coalition is internally stable if its size cannot decrease, and it is externally stable if its size cannot increase. The study of the stability of a coalition consists only in the determination of the 'stable' size of the coalition. As long as this size remains constant, one can change the identity of its members without having any effect on its internal or external stability.

With heterogeneous agents, a coalition of any given size can be stable or unstable according to the identity of its members. In an asymmetrical framework, the incentive to join or leave a coalition can indeed be different from an agent to another. Thus, we define the stability of the coalition C_p using the sets $D(C_p)$ and $D(C_p \cup \{j\})$, where j indicates an agent of the fringe:

Definition 1.

The cooperating coalition C_p is stable if:

1- For each $i \in C_p$, $\pi_i(C_p - \{i\}) \leq \pi_i(C_p)$ (*i.e.* $D(C_p) = \varnothing$) and

2- for each $j \in F_q$, $\pi_k(C_p \cup \{j\}) < \pi_k((C_p \cup \{j\}) - \{k\})$ (*i.e.* for any agent $j \in F_q$: $D(C_p \cup \{j\}) \neq \varnothing$).

Condition (1) is the traditional definition of internal stability. Condition (2) defines external stability of a coalition by the internal instability of any coalition enlarged by one agent.

By including (2) in the definition, the concepts of stability defined by d'Aspremont *et al.* (1983) is applicable to asymmetrical contexts, as described in the introduction. This new definition of external stability takes into account two possibilities:

(a) The entry in the coalition is not advantageous for any agent of the fringe. This is the case analyzed for identical agents by d'Aspremont *et al*. (1983).

(b) The entry in the coalition is advantageous for an agent of the fringe, but involves the defection of an agent initially belonging to the coalition.

Now the members of a coalition have the possibility of leaving it after the adhesion of a new member. Thus, a coalition is externally stable when all the possible negotiations for its enlargement failed**.**

Definition (1) describes an open membership game, *i.e.* each player is free to join and to leave the coalition without the consensus of the other members of the coalition (d'Aspremont *et al*., 1983; Yi and Shin, 2000). Each player announces a message, and all the players who announce the same message form a coalition. Thus, this membership rule implies that a coalition accepts any new player who wants to join it.

However, to analyze the external stability of a coalition with heterogeneous agents we need to take into account the characteristics of q sets $D(C_p \cup \{j\})$ (j = p+1, …, n). We have to compare: (a) the payoff that any player $j \in F_q$ obtains in $[C_p, F_q]$ with the payoff he obtains in $[C_p \cup \{j\}, F_q - \{j\}]$ and, (b) the payoff that any player $i \in C_p$ gets in $[C_p \cup \{j\}, F_q - \{j\}]$ compared to $[(C_p - \{i\}) \cup \{j\}, (F_q - \{j\}) \cup \{i\}]$. Let us note that if a coalition is stable with d'Aspremont *et al*.'s (1983) concept, it is also stable using definition (1), whilst the reciprocal is not true.

3. Exchanging and internal stability

The possible existence of multiple equilibria in the preceding game, which would imply several stable coalitions, may complicate coalition formation analysis. This multiplicity can take two forms: (a) several stable cooperating coalitions of different sizes, or (b) several stable cooperating coalitions of the same size.

Existence of several stable coalitions with the same number of participants implies interchangeability of certain members of a stable coalition with certain members of the fringe. However, a member of a coalition is not necessarily interested in changing his place with a member of the fringe. To discuss this issue in more detail, we define the following set (for any coalition C_p of size p and for any agent j member of the fringe F_q):

$$S_j(C_p) = \left\{ i \in C_p,\ \pi_i(C_p) < \pi_i[(C_p \cup \{j\}) - \{i\}] \right\}.$$

This set contains all the agents of the cooperating coalition who would prefer to exchange their place with agent j of the fringe. We call them “exchanging” agents with j. This exchanging phenomenon implies that there was an incentive for agent i not to become a candidate to the formation of the p size coalition when C_{p-1} had already been formed. It also implies difficulties for the formation of the coalition of size p in a former phase.

To illustrate this concept, let us consider the following game Γ. This game has four players (subscripted i = 1, 2, 3, 4) and is represented by the payoff matrix shown in Table (1), where α, β and δ are real numbers.

Table (1). Game Γ [1]

Structure	Payoff			
	π_1	π_2	π_3	π_4
[{1}, {2}, {3}, {4}]	0	0	0	0
[{1,2},3,4]	1	α	2	1
[{1,3},2,4]	1	β	4	2
[{1,4},2,3]	1	β	1	1
[{2,4},1,3]	1/2	1	1	1
[{2,3},1,4]	δ	1	3	3
[{1,2,3},4]	3/2	2	3	4

Notation [{1,2},3,4] means that players 1 and 2 form a coalition while players 3 and 4 stay as singletons. Columns π_1 , …, π_4 represent the respective payoffs of each of the four players according to the given coalition structures. This game is very schematic, but with small variations it can represent the formation of a cartel, the adoption of an international environmental agreement or the creation of a trade union (see the references mentioned in the introduction). For an analysis using directly a more detailed version of the game presented here see Daidj and Hammoudi (2010) for the video sector or Giraud-Héraud *et al*. (1998) for international trade.

We can characterize the set $S_3(\{1,2\})$ as follows:

$$S_3(\{1,2\}) = \begin{cases} \{1\} & \text{if } \alpha>\beta \text{ and } \delta>1 \\ \{1,2\} & \text{if } \alpha<\beta \text{ and } \delta>1 \\ \{2\} & \text{if } \alpha<\beta \text{ and } \delta<1 \\ \varnothing & \text{if } \alpha>\beta \text{ and } \delta<1. \end{cases}$$

The exchanging concept formalizes the idea that a member of the coalition could wish to exchange his place with an unspecified agent of the fringe, and that this wish constitutes a sign of deviation. However, it is not sure that this incentive for exchanging would automatically imply an incentive to deviate. If we assume $0<\alpha<\beta$ and $\delta>1$ in game Γ, $S_3(\{1,2\}) = \{1,2\} \neq \varnothing$ and $S_3(\{1,2\}) \cap D(\{1,2\}) = \varnothing$. The issue raised here is the connection between the incentive to exchange and the incentive to deviate, *i.e* the impact of non-vacuity of $S_j(C_p)$ on the internal stability of C_p.

While a potential deviator compares his payoff inside the coalition with his payoff outside (without any further modification of the structure $[C_p, F_q]$), the exchanger compares his payoff inside and outside by keeping the size p of the coalition unchanged.

Let us define:

$$O_j(C_p) = \left\{ i \in C_p,\ \pi_i(C_p - \{i\}) < \pi_i((C_p - \{i\}) \cup \{j\}) \right\}.$$

This set contains the members which would prefer to see agent j joining the coalition if they decide unilaterally to leave the coalition. By using this set, proposition 1 shows the connections between exchanging and deviating:

Proposition 1.

For any agent $j \in F_q$ *,, the following properties are satisfied:*

[1] The game used is incomplete because the remaining cases (not stated) are not relevant for the discussion.

$$O_j(C_p) \cap D(C_p) \subset S_j(C_p) \quad (1)$$

$$S_j(C_p) \setminus O_j(C_p) \subset D(C_p). \quad (2)$$

Proof. *Directly from the notation in (1) and (2).*

To understand the relevance of this proposition, we have to compare it to Stigler's (1950) argument on intrinsic instability of collusions, given the higher payoffs obtained by agents which are not involved in the process. Let us suppose that an agent i of the fringe joins C_{p-1}, and call the new coalition of size p: $C_{p-1} \cup \{i\}$ $(= C_p)$. For an agent $i \in C_p$ and for an agent $j \in F_q$, we now consider the properties:

$$\pi_i(C_p) < \pi_j(F_q) \quad (P1)$$

$$\pi_i(C_p) < \pi_i[(C_p \cup \{j\}) - \{i\}]. \quad (P2)$$

Property (P1) means that for a given size of the coalition, the payoff of an agent of the fringe is higher than the payoff of a member of the coalition. This property can be seen as a first interpretation of Stigler's remark (1950). It constitutes the main cause of collusion failure when a great number of identical agents exist in the economy, so that an agent of the coalition can get the payoff obtained in the fringe. However**,** if there are a limited number of agents, a marginal variation in the size of the coalition (or of the fringe) is likely to modify significantly the outcome of the game. This is why we propose to reason with an invariant size of the coalition so as to analyze Stigler's conjecture in a context where the number of players is not excessive.

Property (P2) simply states that agent i is an exchanger with agent j of the fringe. In a homogeneous model, the sets C_p and $S_j(C_p)$ coincide for any agent j of the fringe if and only if property (P1) is satisfied by all agents in the game.

Proposition (1) can be seen as the condition validating Stigler's argument, since we characterize situations where the difficulties observed during the coalition formation predetermine its instability. Assertion (1) characterizes cases where the deviation of a member of the coalition is not related to the exchanging phenomena. Thus, if there is a member i who is deviating from C_p and who is not an exchanger with a member j of the fringe, then i is not part of $O_j(C_p)$. Assertion (2) gives a sufficient condition for the instability of a coalition that had problems in its formation process ($S_j(C_p) \setminus (S_j(C_p) \cap O_j(C_p)) \neq \emptyset$). Under this condition, the exchangers with one or more agents of the fringe will be potential deviators. They are thus an intrinsic cause of internal instability of the coalition. We can now formulate the following corollary, which provides another interpretation of assertion (2):

Corollary 1.

If a cooperating coalition C_p *is internally stable, then:* $\forall j \in F_q \;\; S_j(C_p) \subseteq O_j(C_p)$.

Proof.

If C_p *is internally stable,* $D(C_p) = \emptyset$.

Thus, according to (ii), *we have:* $S_j(C_p) \setminus (S_j(C_p) \cap O_j(C_p)) = \emptyset$.

Hence: $\{ i, \; i \in S_j(C_p), \; i \in S_j(C_p) \cap O_j(C_p)\} = \emptyset$, *which implies:* $S_j(C_p) \subseteq O_j(C_p)$. ■

Corollary (1) gives a necessary condition for internal stability of a coalition that had formation problems, in the sense of a lack of candidates. A coalition including exchanging agents with an external agent j is internally stable only if these agents belong to the set

$O_j(C_p)$. Hence, $O_j(C_p)$ is at the heart of the discussion on the deviating signals likely to be produced by exchanging agents[2].

Let us characterize (in game Γ) the set $O_3(\{1,2\})$:

$$O_3(\{1,2\}) = \begin{cases} \{1\} & \text{if } \beta < 0 \text{ and } \delta > 0 \\ \{1,2\} & \text{if } \beta > 0 \text{ and } \delta > 0 \\ \{2\} & \text{if } \beta > 0 \text{ and } \delta < 0 \\ \varnothing & \text{if } \beta < 0 \text{ and } \delta < 0. \end{cases}$$

If $\beta > 0$ and $\delta < 0$, set $O_3(\{1,2\})$ does not contain agent 1: the cooperation of agent 3 with agent 2 generates a negative externality on agent 1, who remained in the fringe. On the other hand, in this case we have $O_4(\{1,2\}) = \{1,2\}$. In other words, when agent 1 leaves coalition {1,2}, he prefers coalition {2,3} not to be formed, but he finds the formation of coalition {2,4} advantageous.

In homogeneous models, the characterization of $O_j(C_p)$ is trivial. For any coalition C_p:

$$[\,\forall j \in F_q \;\; O_j(C_p) = C_p\,] \Leftrightarrow [\,\pi^F(p-1) < \pi^F(p)\,].$$

Either $O_j(C_p)$ is empty, either it contains all the members of the coalition. In particular, if the welfare of the fringe is an increasing function of the size p of the coalition, we have systematically $O_j(C_p) = C_p$ for any coalition and any member j of the coalition. In addition, if property (P1) is checked (and all the agents of the coalition are exchangers), assertion (1) tells us that if $O_j(C_p) \equiv \varnothing$, then the coalition is internally unstable. Hence, in the case of identical agents whose payoffs are higher in the fringe than in the coalition, and if an internally stable coalition exists, an increase in the size of the coalition is always beneficial to agents who have remained independent. That is the result obtained by Barrett (1994) in his analysis of international environmental agreements.

4. Refraction and external stability

Let us consider henceforth the possibilities of enlargement of a coalition to new members. With heterogeneous agents, the entry of a new member in the cooperating coalition does not necessarily increase the payoff of all the former members. However, those who see their payoff decrease will not necessarily deviate within an extended coalition. In practice, the effect of this entry on the payoffs of the members of the coalition can vary in a more or less significant way, according to the characteristics of its members and of the newcomers.

We call a 'refractory member' to the entrance of $j \in F_q$ in the coalition, any agent $i \in C_p$ who gets a higher payoff in $[C_p, F_q]$ than in $[C_p \cup \{j\}, F_q - \{j\}]$. We note $R_j(C_p)$ the set of refractory members to the entrance[3] of $j \ni F_q$:

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[2] The set $O_j(C_p)$ can be empty in an economic sector where the formation of a coalition implies weakening competitors (e.g. when a cartel increases competition due to changes in firms' efficiency). If the formation of a coalition implies technology-sharing which tends to reduce costs, the intensification of competition resulting from cooperation could prove disastrous for firms which have remained independent. In the sequential analysis of coalition formation proposed by Bloch (1995), the non-cooperative equilibrium generates a reduction in profits for the firms that did not join the R&D coalition (this property holds for quantity and for price competition).

$$R_j(C_p) = \{ i \in C_p,\ \pi_i(C_p \cup \{j\}) < \pi_i(C_p) \}.$$

This set of refractory members was not taken into account in the definition of external stability (see definition (1)). The distinction between stability of the extended coalition and refraction is based on the assumption that the former members do not have the right to choose the future components of the coalition. Thus, our approach to the enlargement of the cooperating coalition neglects the idea of optimality in the choice of allies, since players do not choose their partners explicitly. The counterpart is that their decision to cooperate does not imply any commitment: the players may leave the coalition at any time. Refraction blocks the extension of the coalition if it goes together with an effective incentive for deviation.

Proposition (2) shows how the characterization of the exchangers set makes it possible to establish direct connections between refraction and deviation in the extended coalition.

Proposition 2

For any agent $j \in F_q$ *, we have:*

$$S_j(C_p) \cap R_j(C_p) \subset D(C_p \cup \{j\}) \quad (3)$$

$$[C_p \setminus S_j(C_p)] \cap D(C_p \cup \{j\}) \subset R_j(C_p)\,. \quad (4)$$

Proof. *Directly from the writing of conditions (3) and (4).*

To have a refractory agent i blocking the enlargement of the coalition to an agent j, we need only him to be an exchanger with j (assertion (3)). The refractory agent can threaten the coalition by announcing his deviation after the enlargement, and this threat is credible since it is the best action he can take. Assertion (4) gives a sufficient condition to ensure that a deviating agent of the extended coalition is a refractory member to the entry of an agent j of the fringe. For this, we only need that the agent is not an exchanger with j. Thus, the relation between refractory and external agents allows us to deduce external stability of a coalition.

Corollary 2.

If $\forall j \in F_q\ \ S_j(C_p) \cap R_j(C_p) \neq \varnothing$ *, then coalition* C_p *is externally stable.*

Corollary (2) is a sufficient condition for external stability of a coalition, based on the existence of members that are both not candidates to its formation and refractory to its enlargement. In other words, a coalition that is extended to an agent j of the fringe, in spite of the refraction of certain members, is a coalition whose members are happy to belong to it (in the sense that they are not willing to exchange their place with j).

Let us come back to game Γ. If $\alpha > 2$, $\beta < 2$ and $\delta < 3/2$, the coalition {1,2} is externally unstable. Indeed, it can integrate agent 3 $(D(\{1,2,3\})=\varnothing)$ even though agent 2 is refractory to its enlargement. Corollary (2) tells us that this enlargement is possible in spite of the refraction of agent 2 because this agent is not an exchanger with agent 3 of the fringe[4]. One can also show that the reciprocal corollary is not true. To see that, set $2<\beta<\alpha$ and $\delta < 1$. In this case, coalition {1,2} cannot be enlarged by the integration of agent 3, although $S_3(\{1,2\}) \cap R_3(\{1,2\}) = \varnothing$ since $S_3(\{1,2\}) = \varnothing$.

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[3] In game Γ, if $\alpha > 2$, agent 2 is the only refractory to the entry of agent 3 in the coalition {1,2 }. On the other hand, if $\alpha < 2$, the set $R_3(\{1,2\})$ is empty.

[4] Which is the case, since $S_3(\{1,2\}) = \{1\}$ for $1 < \delta < 3/2$ and $S_3(\{1,2\}) = \varnothing$ if $\delta < 1$, and hence $2 \notin S_3(\{1,2\})$.

Thus, our analysis of external stability in the presence of heterogeneous agents explicitly poses the problem of refraction compared to external stability. In terms of the general theory of coalition formation, the concept of individually stable equilibrium of Greenberg (1977) is closely related to the criterion of refraction as long as the set of achievable payoff vectors for the coalition is restricted to $[\pi_i(C_p)_{i\in C_p}]$. However, this criterion reduces the independence of the agents belonging to the cooperating coalition. Indeed, this approach gives the members an exogenous right of veto, in addition to their basic right to choose their individual action (*i.e.* destabilization of the coalition by deviation). This is in contradiction with the concept of coalition itself. Thus, we neglect this possibility.

With identical agents, if one member of the coalition is refractory to new membership, all the members are. Moreover, if property (P1) is satisfied, the refraction is enough to block the extension of the coalition ($R_j(C_p) \subset D(C_p \cup \{j\})$), since $\pi^C(p) < \pi^F(p)$. External stability of the coalition results from the incentive of the members of the fringe to join it. This last point explains why the concepts of refraction and deviation have not appeared in the literature as independent factors of analysis of external stability.

Proposition (1) and (2) enable us to characterize the cases for which stable coalitions can be established in spite of non-refraction of its members to its extension. The answer to this question highlights the role of $O_j(C_p)$ ($j \in F_q$). We can write the following corollary:

Corollary 3.
If C_p *is stable and for* $j \in F_q$, $R_j(C_p) = \varnothing$, *then* $O_j(C_p) = C_p$.

That is, if for any agent j of the fringe, $O_j(C_p) = C_p$ and $R_j(C_p) \neq \varnothing$, coalition C_p is necessarily unstable. In other words, given the negative external effects of concentration on outsiders, refraction is an indicator of the inexistence of stable coalitions.

5. Conclusion

We have adapted the concept of internal and external stability to a framework with heterogeneous agents. We have also defined two sets of agents on which to focus the analysis to study the stability of a given coalition: the “exchanging” and the “refractory agents”. Analyzing the properties of these sets in an homogeneous and in heterogeneous framework, we have shown that these sets highlight the difficulties encountered in adapting the classical concept of stability to an asymmetric context.

Two fundamental points come out from our analysis. First, we have shown that the existence of heterogeneous agents imposes an individualized analysis of their incentives to join the fringe. We have highlighted that the stability of a cooperating coalition cannot be analyzed studying only the ex-post possibilities of deviation of its members, since their propensity to exchange is also relevant. The definition of the exchanging agents’ set constitutes a reasonable interpretation of Stigler’s (1950) argument on the destabilization of a given coalition. In addition, our approach implies reconsidering the traditional concept of external stability. With exchanging agents, refraction can be seen as a sufficient condition for the failure of an enlargement. Hence, problems associated with the formation of a coalition can have direct impacts not only on its internal stability but also on the future options of enlargement.

Second, our paper offers some conclusions on the issue of the enlargement of a coalition by means of monetary transfers between its members. Under the assumption of sovereign agents, a non-deviating refractory agent cannot be opposed to the enlargement of the coalition. If this agent is deviating, the instability of the coalition is unavoidable (except when monetary transfers on behalf of the non-deviating members are possible). With identical agents, this stabilization by transfers is always impossible since all the members of an unstable coalition are systematically deviating and therefore cannot finance the transfers. The presence of heterogeneous agents shows the possibility for different results in this direction, since in an unstable coalition there may be a group of agents who do not deviate either within the coalition or within the extended coalition. The proposed game Γ shows the existence of a type of agent who does not deviate, whatever the size of the coalition. Without ex-ante transfers, the different payoffs inside the coalition can imply that certain agents, particularly favored, are never encouraged to leave it, whereas others (losers) are at the origin of its instability. These are the members to whom non-deviating agents need to grant ex-post transfers in order to obtain a stable coalition.

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