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Learning strategies in modelling economic growth

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Abstract

Cornerstone economic growth models as the Solow-Swan model and their modern extensions normally assume the rate of population growth as exogenous without any explanation of the links between economic growth and most important demographic variables. Recently, some articles have presented models to explain many phenomena of population dynamics, including evolution and ageing. This paper is a first exercise to include endogenous population dynamics and learning strategies as ingredients of an economic growth model. The model includes two ways of learning that determinate economic growth: individual and social learning. We study the dynamics through computer simulations and we show that the model reflects some features of real economies.

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1 Introduction

Economic growth has been center of attention for most of the economists around the world. Even if world is much richer today than it was hundred years ago, standard of living differs widely across countries. Finding determinants of long run economic growth is essential in order to improve the standard of living of population. To quote the Nobel Prize Robert Lucas (1988), "Once one starts to think about (economic growth), it is hard to think about anything else".

The starting point of modern economic growth analysis is the model proposed by Solow (1956) and Swan (1956). In this model, economic growth is explained by two exogenous variables: population growth (which is constant) and the growth of technological change (which is not explained). Modern extensions of this model are focused on the explanation of technological change. In this sense, there are two important lines of research. On the one hand, some models take a broad view of capital, including human capital, with education as an important part (Lucas, 1988). On the other hand, there are models interpreting technology accumulation as knowledge. These models differ in how knowledge is produced and what determines the allocation of resources to knowledge production. In this sense, Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) considered in their models a sector producing research and development (R&D). The explanation is as follow: some people working in the R&D sector produce knowledge which is used in the production sector.

Nonetheless, as Arrow (1962) asserts, the production of R&D sector cannot be considered the unique source of knowledge. Instead, Arrow proposes another source: the learning-by-doing process. This author considers this process as the final determinant of knowledge accumulation and observes that workers improve the production process as they work in it. Hence, knowledge accumulation depends not only on the resources engaged in R&D, but also on new knowledge generated by conventional economic activities.

Therefore, long run economic growth depends mainly on two variables: population growth and technological change, the last explained by accumulation of abstract knowledge or human capital. With respect to the first one, cornerstone economic growth models as the Solow-Swan model and their modern extensions cited above, make the simplistic assumption that population grows at an exogenous constant rate. Although important contributions in the economic growth literature with endogenous fertility of, for example, Becker and Barro (1988), Barro and Becker (1989) and Galor and Weil (1996), there are few macroeconomic models that study the relations between economic growth and population dynamics, with population as an endogenous variable. In contrast there is an extensive empirical literature about this issue (Furuoka (2009), Wan-Jun et al (2007) and references therein). Recently, some articles in Econophysics area have presented models to explain many phenomena of population dynamic, including evolution and ageing (see Bustillos and Oliveira (2004), Dabkowski et al (2000), Huang and Stauffer (2001), Magdon-Makymowicz et al. (2000), Maksymowicz et al. (2008), Mingfeng et al. (2006), Oliveira (1998), Pan et al. (2005), Penna (1995) and Stauffer (2007)). In particular, Penna (1995) presents a model for aging using "bit string strategy", which is also suitable for implementation in computers and is able to reflect some features of real populations. Bustillos and Oliveira (2004) combined the model presented in Penna (1995) and learning strategies to study the interaction of two learning strategies in population growth. Firstly, they analyze a person interacting with the environment producing a type of knowledge by "trial-and-error" called "individual learning". This kind of knowledge in a working environment is similar with the concept of "learning-by-doing" proposed by Arrow (1962). Secondly, people also accumulate knowledge when they interact with other members of the society, "social learning".

Although, as we said above, in recent years some articles have studied the interaction between population and economic growth in a demo-economic model which incorporates the age structure of the population (Manfredi and Fanti (2006), Lia et al (2007) and Shimasawa (2007)), no attempt has been made to integrate populations models mentioned above with an economic growth model. The main contribution of this paper is to combine the classic theories of economic growth with models of aging and learning strategies. In particular, we introduce the main elements of Bustillos and Oliveira (2004) model in a classic economic growth model. In the economic growth model that we present, population dynamics and knowledge evolution interact. We introduce a mechanism to study the interaction of two learning strategies in population growth and how these two factors (population and knowledge) affect economic growth. Then, we conduct some computer simulations to study the dynamics of the model.

The paper is organized as follows. In section 2 we develop the model. In section 3 some simulations are conducted to study the effects of changing some parameters. Finally, section 4 draws some conclusions.

2 An economic growth model with learning strategies

2.1 Population growth

As we claim in the introduction, cornerstone economic growth models as the Solow-Swan model and their modern extensions, make the simplistic assumption that population grows at an exogenous constant rate n. Based on data of human demography and experiments of other living organism, many important phenomena of longevity have been found. To reproduce and explain these phenomena, various models of senescence have been proposed. The one widely used by physicists is the Penna model (Penna (1995)) which presents a simple framework for biological aging, working under the effect of the Verhulst factor, mutation, death by genetic diseases or age and a minimum reproduction age. The standard Penna model assumes that each individual is characterized by a string (computer word) of 32 bits, and each bit is expressed as a particular age in individual life. A bit i is set to 1 if the individual presents a disease, and from this age i on this bit will continuously affect the survival probability of the individual. As it is remarked by Huang and Stauffer (2001), the individual's survival probability G up to age a is the product of contributions from all bits before a:

$$G(1,2,...,a) = (1-p_1)(1-p_2)...(1-p_a) = 1 - DP(a) \tag{1}$$

We take this survival probability as $1 - DP(a)$, that is, as one minus the death probability by accumulation of diseases DP, which can be estimated by using actuarial tables.

The individual surviving over the whole reproduction period is assumed to produce B (total fertility rate) offspring. At each step t, a Verhulst factor $V = 1 - N(t)/N_{max}$, denoting the survival probability of the individual due to space and food restrictions, is introduced; where $N(t)$ is the current population size and $N_{\max}$ is the carrying capacity of the environment. According to Livi-Bacci (1989), we assume the maximum capacity of the planet as 150 billions of persons. As will be explained later, Bustillos and Oliveira (2004) assume that an individual with age i accumulates a quantity $C(i)$, the sum of knowledge bits, which can be used to improve its survival capacity. In the Penna model an individual dies due to two reasons, accumulation of diseases or the Verhulst factor. Therefore, the survival probability of a person of age i taking into account the two factors is summarized as follows:

$$\text{Prob. survival up to age } i \text{ at time } t = (1 - DP(i))\left(1 - \frac{N(t)}{N_{\max}}\left(1 - \frac{C_t(i)}{N_{\max}}\right)\right) \tag{2}$$

On the one hand, note that if $C_t(i)$ is zero (no acquired knowledge), the old Verhulst factor is kept and the individual dies with the same probability as in the model without knowledge. On the other hand, $C(i)$ is a normalized variable: if the individual arrives to the maximum level of knowledge equal to the maximum population (N_{max}) its probability of die via Verhulst factor is zero.

2.2 Ways of learning

Learning strategies influence economic growth because they increase the survival probability. Bustillos and Oliveira (2004) proposed two ways of learning. The first way is due to the interaction of individuals with its natural environment; in a process of trial-and-error knowledge is acquired from an individual's own experiences, such as avoiding dangers in nature or determining some new food supplies. This concept is similar to the notion of "learning-by-doing": we propose that a worker in the production process sometimes finds, by trial-and-error, new ways of improving the production process. In particular, we assume that at some age an individual has a certain probability of having this kind of experience and to learn from it; this is represented by a switching the 0 bit to 1 in that position of its knowledge string. Therefore, accumulated knowledge by "individual learning" of people of age i at time t is given by the following equation:

$$C_i^1(t) = C_{i-1}^1(t-1) + pil.(n_{i-1}(t-1)) \tag{3}$$

The equation shows that cumulated knowledge, due to the first way of acquiring knowledge, of people aged i at time t ($C_i^1(t)$) is equal to the knowledge they had the year before ($C_{i-1}^1(t-1)$) plus the product of the probability of individual learning (pil) by the population in working age ($n_{i-1}(t-1)$) Thus, a fraction of the population $n_{i-1}(t-1)$ will have an experience and will learn from it, accumulating knowledge the following age. Bustillos and Oliveira (2004) assert that probability pil represents the cognitive capacity of individuals and it is different for each species because it depends on some physiological characteristics.

The second way, "social learning", is due to the interaction between individuals: a naive individual (observer) who spends some time near another one (teacher), can learn just by imitating. We assume that an individual of age i can learn with a probability psl from the elders. Therefore, its accumulated knowledge is given by the following equation:

$$C_i^2(t) = C_{i-1}^2(t-1) + psl.(C_{i+1}(t-1) + C_{i+2}(t-1) + ... + C_{A_{\max}}(t-1)) \tag{4}$$

Finally, the total accumulated knowledge of people of age i at time t ($C_i(t)$) is given by the sum of the two factors:

$$C_i(t) = C_i^1(t) + C_i^2(t) \tag{5}$$

and the total accumulated knowledge by population at moment t is the sum of all cumulated knowledge for all ages:

$$C(t) = \sum_{i=1}^{i=A_{max}} C_i(t) \tag{6}$$

Let's examine the relationships between fertility, population and knowledge more closely. When total fertility rate increases, the first consequence is an increment in population. Then we have two effects in opposite directions. On the one hand, accumulated knowledge by "individual learning" increases because it depends, by Equation (3), on $n_{i-1}(t-1)$. Then as $\frac{C_t(i)}{N_{\max}}$ is higher, survival probability increases and this reinforces the initial change. In the next periods the increment in the population also allows more "social learning" and then, more knowledge is generated. On the other hand, since population increases, it approaches to the carrying capacity of the environment and then, survival probability decreases with a negative effect on population. This relation drives the dynamic of population and accumulated knowledge in the period of analysis.

2.3 Economic growth

We assume that only a fraction of the population enters in the production process. In fact, only population in working age and their knowledge will enter into the production process. Therefore, labor at time t is defined as the sum of population in working age as follows:

$$L(t) = \sum_{i=W_{\min}}^{i=W_{max}} n_i(t) \tag{7}$$

where W_{min} is the minimum working age and W_{max} is the maximum working age before becoming into a retired worker. This distinction between population (N) and workers (L) will be useful later when we analyze some social security issues.

Economic growth is determined by knowledge (human capital or abstract knowledge) and population growth. For sake of simplicity we assume a production function where production is determined by the quantity of work adjusted by the knowledge as in the following equation:

$$Y(t) = H(t)L(t) \tag{8}$$

where $Y(t)$ is the economy production level, $L(t)$ is the number of workers and $H(t) = \sum_{i=W_{\min}}^{i=W_{\max}} C_i(t)$ is the accumulated knowledge by workers.

2.4 Dynamics of the model

To simulate the model we introduce the system of equations which defines our economic growth model. We set the initial conditions and parameters as:

Length of simulation horizon = 1500 years

$N_0 = 10$, initial level of population

$N_{max} = 150$ billions, the maximum level of population

$A_{max} = 120$, maximum age a person can arrive

$R_{min} = 18$, minimum reproduction age

$R_{max} = 45$, maximum reproduction age

$B = 2$ number of offspring or total fertility rate

$W_{min} = 15$, minimum working age

$W_{max} = 70$, maximum working age

$pil = 0.0003$, probability of "individual learning"

$psl = 0.00004$, probability of "social learning"

The survival probability $1 - DP(a)$ is estimated by using actuarial tables. In particular, the Social Security of USA (http://www.ssa.gov/OACT/STATS/table4c6.html) gives these probabilities for 120 ages.

Population initial conditions are given by:

$$n_1(1) = N_0, n_2(1) = 0, ..., n_{120}(1) = 0 \tag{9}$$

and

$$c_1(1) = 0, c_2(1) = 0, ..., c_{120}(1) = 0. \tag{10}$$

Then at time 1 we have a population of N_0 newborns with an accumulated knowledge equal to zero.

Population growth and knowledge is determined by the following system of 240 equations:

$$
\begin{cases}
n_1(t) = \frac{B}{R_{\max}+1-R_{\min}} \left(n_{R_{\min}}(t-1) + n_{R_{\min}+1}(t-1) + ... + n_{R_{\max}}(t-1)\right) \\
n_2(t) = n_1(t-1) \left(1 - \frac{N(t-1)}{N_{\max}} \left(1 - \frac{C_2(t-1)}{N_{\max}}\right)\right) (1 - DP(1)) \\
n_3(t) = n_2(t-1) \left(1 - \frac{N(t-1)}{N_{\max}} \left(1 - \frac{C_3(t-1)}{N_{\max}}\right)\right) (1 - DP(2)) \\
\qquad ... \\
n_{120}(t) = n_{119}(t-1) \left(1 - \frac{N(t-1)}{N_{\max}} \left(1 - \frac{C_{120}(t-1)}{N_{\max}}\right)\right) (1 - DP(119)) \\
C_1(t) = 0 \\
C_2(t) = pil.n_1(t-1) + psl.(C_3(t-1) + C_4(t-1) + ... + C_{120}(t-1)) \\
C_3(t) = C_2(t-1) + pil.n_2(t-1) + psl.(C_4(t-1) + C_5(t-1) + ... + C_{120}(t-1)) \\
\qquad ... \\
C_{120}(t) = C_{119}(t-1) + pil.n_{119}(t-1)
\end{cases}
\tag{11}
$$

The interaction between population and knowledge is clear. The first equation says that newborns are generated by population in reproduction age, that is, population between the minimum and maximum reproduction aged (18 and 45 years, respectively) at the period before and the total fertility rate. From the second equation until equation 120, population grows according to the survival probability proposed in Subsection 2.1. Specifically, total population aged 2 at time t is defined by the population aged 1 at the period before, corrected by the survival probability defined by equation (2). The same definition applies to the number of population until the maximum age that a person can arrive (equation 120). Equation 121 states that newborns have zero knowledge. From equation 122 to equation 240, people accumulate knowledge due to "individual learning" and "social learning" as it was explained in Subsection 2.2. For example, total cumulated knowledge by people of age 2 at time t is defined by the product of the probability of individual learning by the number of people of age 1 the period before, plus the product of the probability of social learning and the accumulated knowledge by older people. The cumulated knowledge by the rest of the population follows the same argument. Finally, note that the source of original knowledge is the trial-and-error process since at the initial period there is no knowledge and then people cannot learn by “social learning”.

3 Computer simulations

In this section we simulate the model. Figure (1) shows the evolution of total production for different values of the parameters. The continuous line indicates the production when total fertility rate is 1.8 and probability of social learning is 0.0002. Note that as we increase the value of the parameters, total production also increases. However, the effect of the increment in total fertility rate is stronger than the effect of knowledge.

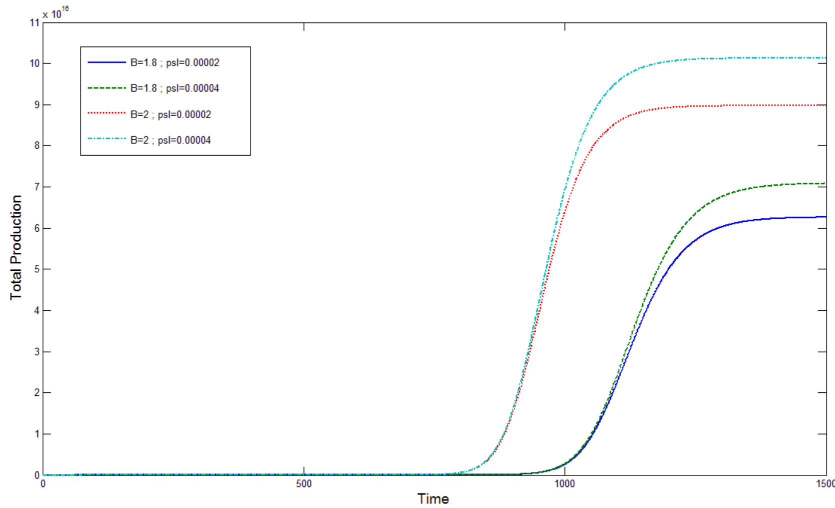

Figure 1: Total Production considering different parameters. B=2, B=1.8, psl=0.00002 and psl=0.00004.

Population size $N(t)$, labor $L(t)$, accumulated knowledge of population $C(t)$ and workers $H(t)$ follow the same shape. First, there is an exponential increase, later there is an inflexion point when variables are affected by the Verhulst factor. Notice that at the end of the time variables arrive to a kind of stationary state. Note also that total product evolution predicted by the model is in line with typical findings about this issue (see, for example, Fiaschi and Lavezzi (2007)).

Nowadays, an important question is what happens with social security of retired workers if total fertility rate decreases. As it is well-known, many social security systems use a percentage of the production generated by workers ($\%SS$) to the payment of retired workers (RET) (more details can be found in Gronchi and Nisticò (2008)). The following equation shows the quantity of production per retired work.

$$Social\ Security = \frac{\%SS(Y)}{RET} \tag{12}$$

Figure (2) shows the evolution of this quantity considering different values of the parameters.

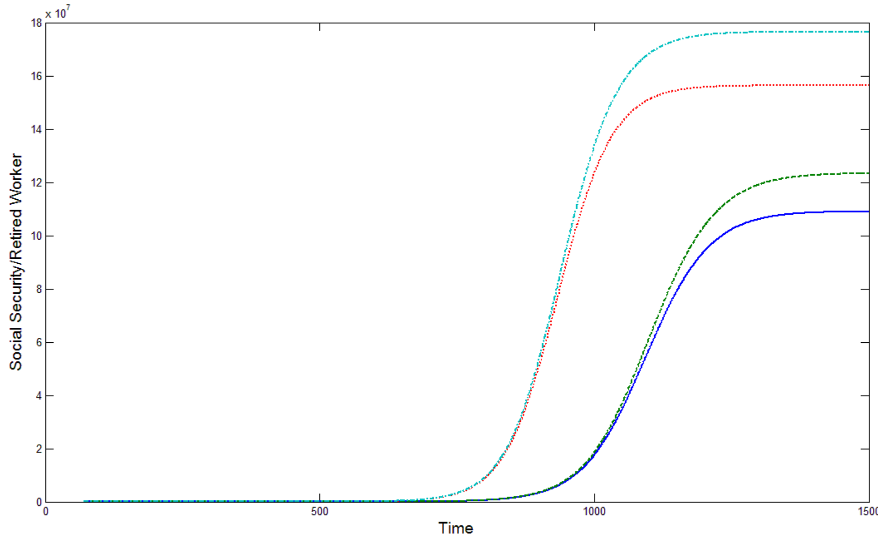

Figure 2: Evolution of Social Security per Retired Worker considering parameters B=2, B=1.8, psl=0.00002, psl=0.00004

As in the preceding case, the reduction of total fertility rate and accumulated knowledge has a negative effect in the quantity perceived by retired workers, but the effect of the reduction in the total fertility rate is stronger.

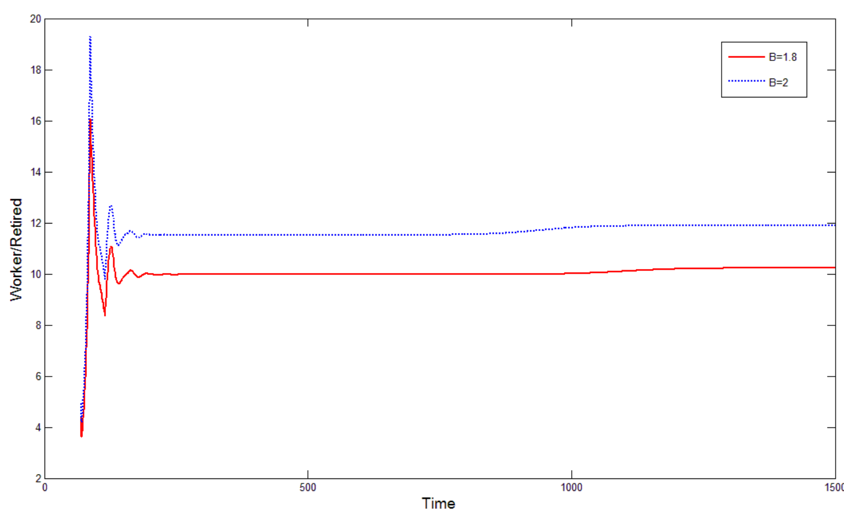

Figure 3: Evolution of ratio worker/retired

Once again, Figure (3) shows that when total fertility rate decreases, the number of worker supporting the retired workers also decreases. This is a common problem in many developed countries where total fertility rate has reduced.

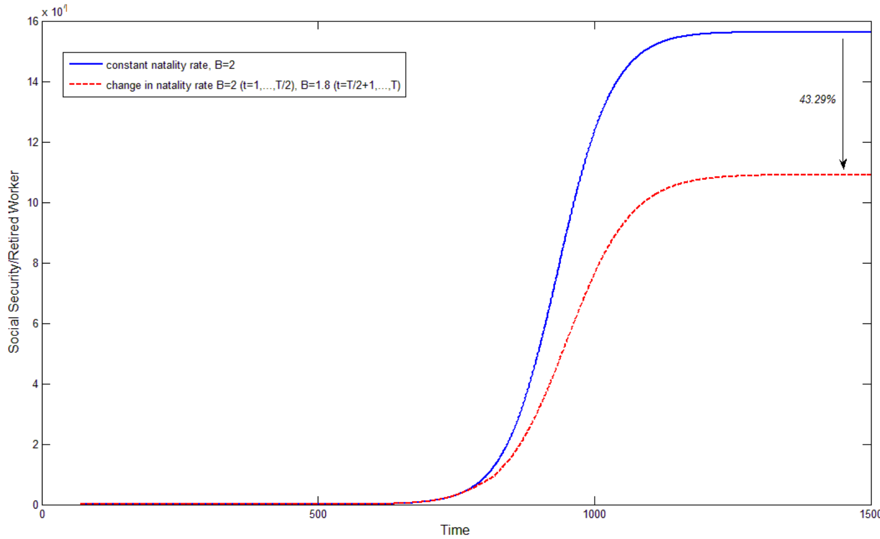

Figure 4: Evolution of Social Security per Retired Work considering constant and change in parameter B.

Figure (4) compares the evolution of two economies. Consider an economy starting with a total fertility rate of 2 (B=2) which is reduced at the middle of period to B=1.8. Figure (4) shows the evolution of social security comparing both cases, the hypothetical case of B=2 in all periods and the change to B=1.8. Note that the decrease in the total fertility rate implies a decrease of 43.29% in the Social Security perceived by a retired worker at the end of the period.

4 Conclusions

Economic growth is one of the most important topics in economics. Economic models explain economic growth basically due to two factors, population growth and technological change. However, cornerstone economic growth models as the Solow-Swan model and their modern extensions have focused on explaining technological change, assuming that population grows at an exogenous constant rate. Recently, new models of population dynamics where proposed, some of them to study the interaction of learning strategies and population growth.

We present an economic growth model where population dynamic and the evolution of knowledge interact and in which knowledge is generated by individual and social learning. We have simulated the model to study the effects of different probabilities of learning and total fertility rates. First, we show that a reduction of both total fertility rate and probability of learning, produce a reduction in total production. However, the effect of the decrease in total fertility rate is stronger than the effect of knowledge. In this sense, the results predicted by our model are in line with typical findings in empirical growth studies.

Second, we apply the model to study the effects of changes in the main variables on the evolution of social security system. In particular, we analyze a pay-as-you-go pension system and in that scheme we observe that the reduction of both total fertility rate and accumulated knowledge have a negative effect in the quantity perceived by retired workers. This is a common problem in many developed countries where total fertility rate has reduced. Hence, these results stress the importance of carrying out family policies by governments (a discussion about this issue can be found in Fanti and Gori (2010)).

Finally, future lines of research can include the generalization of our framework using another production function and/or the combination of the presented model of population growth with other economic growth models. With respect to the Penna model, it works under the assumption that fertility is constant. In this first exercise we hold that assumption. However, an interesting extension of our model would be to allow a varying fertility in time. Additionally, it is interesting to study the dynamics of other learning mechanism and their effects on economic growth. New simulations of the model with other parameters values could improve the robustness of our results.

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