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No Trade, Informed Trading, and Accuracy of Information

Shino Takayama
University of Queensland

Jayanaka Wijeratne
University of Queensland Alumnus

Abstract

We present a model in which there is uncertainty about realization of a risky asset value for an informed trader. We introduce two states such that in the "narrow" state the informed trader has better information than in the "wide" state. Then, we show that the informed trader in the wide state does not trade in equilibrium if the information that the informed trader with better information has is sufficiently accurate and the probability of the narrow state is sufficiently high. We use the framework presented by Glosten and Milgrom (1985) and extend the assumption that the informed trader knows the terminal value of the risky asset. Finally, we obtain the conditions under which the informed trader would not trade in equilibrium.

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1. Introduction

Jones et al. (1994) suggested that future theoretical research needs to develop scenarios in which (i) both the frequency and size of trades are endogenously determined, and yet (ii) the size of trades has no information content beyond that contained in the number of transactions. Ozsoylev and Takayama (2010) presented a model in which the size of trades is endogenously determined. In this article, we present a model in which the frequency of trades is endogenously determined. The contribution of this article is to propose a model of "no trade," in which an informed trader does not trade as an equilibrium outcome.

Milgrom and Stokey (1982) show that if there are no noise traders, and if the structure by which traders acquire information is itself common knowledge, then even though some traders may possess private information, the no-trade situation arises in equilibrium. In this article, we propose a model such that even if there are noise traders and common knowledge is assumed, an informed trader would not trade in equilibrium if the inside information that the informed trader has is not sufficiently accurate. We develop the model within the framework presented by Glosten and Milgrom (1985).

There are two changes that we will make to the Glosten–Milgrom model. First, we allow the informed traders to not trade. Second, we assume that the informed trader has some uncertainty about the terminal value of the risky asset. This creates a situation where the market maker's ask price is higher or the bid price is lower than the informed traders' expected asset value. Moreover, we consider the static version of the sequential trade model. In this article, we focus on the situation where the informed trader does not trade in equilibrium. When we assume that all the random variables such as the value of the asset, the probability of informed trading, or the liquidity of the traders' demand realizations are independent and identically distributed (i.i.d.) across all the periods, then each period is simply a replicate of other periods. Thus, we focus on one period as a representative of many identical periods and study the conditions under which there is a no-trade situation for the informed trader.

The model considers a market where a risky asset is traded between a market maker, strategic traders, and liquidity traders. First, the market maker, who is not informed of the risky asset payoff, quotes the bid and ask price. Then, either a strategic trader or a liquidity trader arrives in the market in a random manner. The liquidity trader's trading motive is not related to the risky asset payoff at all, whereas the strategic trader has information on the risky asset payoff. In the model, there are two states. In one state, called "the wide state", the informed trader has more coarse information about the terminal value of the asset, whereas in the other state, "the narrow state", the informed trader has better information. Then, we show that if the information that the informed trader with better information has is sufficiently accurate and the probability of the narrow state is sufficiently high, the informed trader with less accurate information does not trade in equilibrium.

In our model, the conditional variance of the asset with respect to the informed trader's information is higher in the wide state than in the narrow state. Therefore, we can interpret our result in the following way. When the difference in information accuracy between the two states is sufficiently large, the difference between the conditional variance in the two states is also large. In this situation, when the probability of the wide state is very low, the prices are set close to a true value. Then, the informed trader with more coarse information does not trade in equilibrium.

The organization of our paper is as follows. The next section presents the model and the equilibrium concept. In Section 3, we study the conditions under which the informed trader does not trade, and present our results.

2. The Model of Trading

There are three classes of risk-neutral market participants: a competitive market maker, an informed trader, and a liquidity trader. The game structure and the parameters of the joint distribution of the investor's state variables are common knowledge to all market participants. In the beginning, the market maker posts bid and ask prices equal to the expected value of the asset, conditional on the observed history of trades in equilibrium. The trader trades at those prices, or possibly chooses not to trade.

As noted above, there are two states of the world. These states, "wide" and "narrow", are denoted by W and N, respectively. The wide state occurs with probability ν and the narrow state occurs with probability $1-\nu$. The terminal value of the asset is represented by $V \in \{0,1\}$. Informed traders receive two signals, θ, about the asset's value, and τ, about the state of the world. The signal θ has two values, low (L) and high (H), where the probability that the signal is low is $1/2$. Let $p_\tau \in (0, 1/2)$ for each $\tau \in \{N, W\}$ such that $Pr(V = 1|H, \tau) = 1/2 + p_\tau$ in state τ and $Pr(V = 0|H, \tau) = 1/2 - p_\tau$ in state τ. Furthermore, $Pr(V = 1|L, \tau) = 1/2 - p_\tau$ in state τ and $Pr(V = 0|L, \tau) = 1/2 + p_\tau$ in state τ. We suppose that $p_W < p_N$. For the high signal, the conditional expected value of the asset is $\frac{1}{2} + p_\tau$, and for the low signal it is $\frac{1}{2} - p_\tau$. Thus, the conditional variance for the high signal is $(1/2+p_\tau)(1-1/2-p_\tau)^2 + (1/2-p_\tau)(0-1/2-p_\tau)^2 = \frac{1}{4} - p_\tau^2$ for each $\tau \in \{N, W\}$ and the conditional variance for the low signal is $(1/2-p_\tau)(1-1/2+p_\tau)^2 + (1/2+p_\tau)(0-1/2+p_\tau)^2 = \frac{1}{4} - p_\tau^2$ for each $\tau \in \{N, W\}$. Notice that in the wide state, the variance is larger than in the narrow state. In other words, in the narrow state, the probability of each signal is closer to certainty than is the case in the wide state. In this sense, we can think of p_τ as the accuracy of the informed trader's information. The market maker and the liquidity trader share common prior beliefs about the assets' values with a mean $V^* = 1/2$.

The probabilistic assumption about the states of the world can be interpreted as follows. The informed trader's knowledge of the terminal value of the asset is slightly better than that of the market maker because that trader has "more accurate" information prior to the market maker. However, the informed trader does not know the exact terminal value of that asset. In the wide state, given a signal, the probability difference between the high and low values is wider than would be the case for the narrow state. Therefore, we can say that in the wide state, the informed trader has less accurate information than in the narrow state.

Let $E = \{-1, 0, +1\}$[1] denote the set of possible trades available to the trader in each period, with e its generic element. That is, $e = +1$ denotes a buy order, $e = -1$ denotes a sell order, and $e = 0$ denotes no trade. Let $\Delta(E)$ denote the set of probability distributions on E. The market maker posts an ask price and a bid price. The trader can choose to buy the asset at the ask price, or sell the asset at the bid price, or choose not to trade. Let α be the ask price posted and let β be the bid price. Let $p \equiv (\alpha, \beta) \in [0, 1]^2$.

We consider the following game: with probability μ, the informed trader will be chosen, and with probability $1 - \mu$, the liquidity trader will be chosen. If a trader is uninformed, his or her demand is determined by the random variable $\tilde{Q}$, which takes a value from E. We assume that $Pr(Q = e) = \gamma_e > 0$ for every $e \in E$. We suppose that all the random variables are mutually independent. The probability distribution is common knowledge to everybody in the model. In this paper, we simply consider a static model.

The timing structure of the trading game is as follows.

1. At the beginning of the game, nature chooses the realization $V \in \{0, 1\}$ of the risky-asset payoff $\tilde{V}$ and the type of trader θ. The informed trader observes θ.

2. At the end of the game, the realization $\tilde{V}$ is publicly disclosed, and consumption takes place.

For each type of trader, a trading strategy specifies a probability distribution over trades with respect to the ask and bid prices p. A strategy for the trader is defined as a function $\sigma : P \rightarrow$

[1] It might be interesting to consider the multiple-size setting. Although the main intuition of our result still holds even in the multiple-size setting, it could complicate the analysis, and so we focus on the single-unit setting to point out a simple intuition.

$\Delta(E)$. Let $\sigma(e|p)$ be the probability that σ assigns to action e conditional on p. For simplicity of notation we denote each type's strategy by $\{\sigma^{\tau}_{\theta,e}(p)\}$ for $\theta \in \{H, L\}$, $\tau \in \{N, W\}$, and $e \in E$. To avoid complication, we assume that when trading yields zero profit, the informed trader would trade.

Now, we consider the market maker's belief. Let $\delta \in \Delta(\Theta)$ denote the market maker's prior belief; that is, $\delta(e)$ denotes the market maker's belief that the trader is the high type after observing trade e. Then, the (Bayesian) market maker's belief is updated through Bayes' rule; that is, for all $e \in E$:

$$\begin{aligned} \delta(e) &:= \Pr(\tilde{\theta} = H|e) \\ &= \Pr(\tilde{\theta} = H) \cdot \frac{\mu \sum_{\tau=N,W} \sigma^{\tau}_{He}(p) \cdot Pr(\tau) + (1-\mu)\gamma_e}{\mu \sum_{\tau=N,W} \sum_{\theta\in\Theta} Pr(\tau)\Pr(\theta)\sigma^{\tau}_{\theta e}(p) + (1-\mu)\gamma_e}. \end{aligned}$$

Definition 1 *An informed trader's strategy profile σ^* is optimal for price rule p if it prescribes a probability distribution $\sigma^{\tau *}_{\theta}$ over E for each $\theta \in \{L, H\}$ and $\tau \in \{N, W\}$ such that:*

$$\sigma^{\tau *}_{\theta}(p) \in \arg \max_{\sigma \in \Delta(E)} \left[\sum_{e\in E} \sum_{v\in\{0,1\}} Pr(V = v|\theta, \tau)\sigma^{\tau}_{\theta e}(p)(v - p)e \right].$$

Definition 2 *An equilibrium consists of the market maker's prices, the informed traders' trading strategies, and posterior belief such that:*

(P1) *the bid and ask prices satisfy the zero-profit condition, given the posterior belief;*

(P2) *σ^* is the informed traders' optimal trading strategy for the price;*

(B) *the market maker's belief satisfies Bayes' rule.*

3. No Trade in Equilibrium

Observe that $\Pr(\tilde{\theta} = L| + 1) = 1 - \delta(+1)$ and $\Pr(\tilde{\theta} = L| - 1) = 1 - \delta(-1)$. Therefore, we obtain:

$$\begin{aligned} \alpha &= \textstyle\sum_{\tau=N,W} \Pr(\tau) \left(\Pr(V = 1|H, \tau) \cdot \Pr(H|e = +1) + \Pr(V = 1|L, \tau) \cdot \Pr(L|e = +1)\right) \\ &= \textstyle\sum_{\tau=N,W} \Pr(\tau) \left((\frac{1}{2} + p_\tau) \cdot \delta(+1) + (\frac{1}{2} - p_\tau) \cdot (1 - \delta(+1))\right) \\ &= \textstyle\sum_{\tau=N,W} \Pr(\tau) \left(\frac{1}{2} - p_\tau + 2p_\tau \cdot \delta(+1)\right); \\ \beta &= \textstyle\sum_{\tau=N,W} \Pr(\tau) \left(\Pr(V = 1|H, \tau) \cdot \Pr(H|e = -1) + \Pr(V = 1|L, \tau) \cdot \Pr(L|e = -1)\right) \\ &= \textstyle\sum_{\tau=N,W} \Pr(\tau) \left((\frac{1}{2} + p_\tau) \cdot \delta(-1) + (\frac{1}{2} - p_\tau) \cdot (1 - \delta(-1))\right) \\ &= \textstyle\sum_{\tau=N,W} \Pr(\tau) \left(\frac{1}{2} - p_\tau + 2p_\tau \cdot \delta(-1)\right). \end{aligned} \tag{1}$$

The expected value of the asset is: $\frac{1}{2} + p_\tau$ for the high type and $\frac{1}{2} - p_\tau$ for the low type for each $\tau \in \{N, W\}$. The informed trader buys if his expected value is greater than α and sells if it is smaller than β. We can see that the high type in the narrow state always buys and the low type in the narrow state always sells, because from (1) ask and bid prices are convex combinations of those four expected values of the asset. The following proposition provides a formal statement.

Proposition 1 *In equilibrium, the high type informed trader always buys and the low type always sells in the narrow state.*

Proof of Proposition 1: As the argument is symmetric, we will only prove the statement for the high-type informed trader. The high type informed trader buys if $\alpha \leq \frac{1}{2} + p_N$, and, similarly to the above, we obtain the following condition:

$$p_N > (\nu p_W + (1-\nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_{+1} + 1/2 \cdot \mu(1-\nu)}. \tag{2}$$

Note that $p_N > (\nu p_W + (1-\nu)p_N)$ as $p_N > p_W$ by assumption, and also $\frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)} < 1$. Therefore, (2) always holds. Therefore, we can conclude that the high type informed trader in the narrow state buys. This completes our proof. ■

Thus if there is any possibility of no trade, it must be in the wide state. Now focus on the wide state and notice that if the high type in the wide state sells, then the low type must also sell, and if the low type in the wide state buys, then the high type must also buy. Therefore, if there is no trade from the informed trader, it must be one of the following three cases: in the wide state,

Case 1. the high type and the low type do not trade – this situation arises if and only if $\alpha > \frac{1}{2} + p_W$ and $\beta < \frac{1}{2} - p_W$;

Case 2. the high type does not trade and the low type sells – this situation arises if and only if $\alpha > \frac{1}{2} + p_W$ and $\beta \geq \frac{1}{2} - p_W$;

Case 3. the high type buys and the low type does not trade – this situation arises if and only if $\alpha \leq \frac{1}{2} + p_W$ and $\beta < \frac{1}{2} - p_W$.

We start with **Case 1**. If $\sigma^N_{H,+1} = 1$, $\sigma^W_{H,0} = 1$, $\sigma^N_{L,-1} = 1$, and $\sigma^W_{L,0} = 1$, then we obtain:

$$\begin{aligned} \delta(+1) &= \Pr(\tilde{\theta} = H | +1) = \tfrac{1}{2} \cdot \tfrac{(1-\mu)\gamma_{+1} + \mu(1-\nu)}{(1-\mu)\gamma_{+1} + 1/2 \cdot \mu(1-\nu)}; \\ \delta(-1) &= \Pr(\tilde{\theta} = H | -1) = \tfrac{1}{2} \cdot \tfrac{(1-\mu)\gamma_{-1}}{(1-\mu)\gamma_{-1} + 1/2 \cdot \mu(1-\nu)}. \end{aligned} \tag{3}$$

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By substituting (3) into (1) and then simplifying the two conditions $\alpha > \frac{1}{2} + p_W$ and $\beta < \frac{1}{2} - p_W$, we obtain:

$$\begin{aligned} \sum_{\tau=N,W} Pr(\tau) \cdot \left(-p_\tau + p_\tau \cdot \frac{(1-\mu)\gamma_{+1}+\mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)}\right) &> p_W; \\ \sum_{\tau=N,W} Pr(\tau) \cdot \left(-p_\tau + p_\tau \cdot \frac{(1-\mu)\gamma_{-1}}{(1-\mu)\gamma_{-1}+1/2\cdot\mu(1-\nu)}\right) &< -p_W. \end{aligned} \tag{4}$$

In the end, we obtain:

$$p_W < (\nu p_W + (1-\nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_{+1} + 1/2 \cdot \mu(1-\nu)} \tag{5}$$

$$-p_W > -(\nu p_W + (1-\nu)p_N) \cdot \frac{1/2 \cdot \mu(1-\nu)}{(1-\mu)\gamma_{-1} + 1/2 \cdot \mu(1-\nu)}. \tag{6}$$

Therefore we can say: **Case 1** arises if (5) and (6) hold. Note that $p_W < (\nu p_W + (1-\nu)p_N)$, and as $\frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)} < 1$, the condition (5) does not always hold. Whether it holds depends on p_N, p_W, γ_{+1}, or ν. If p_N is close to $\frac{1}{2}$, p_W is close to 0, and ν is low, then the informed trader's information is accurate with very high probability. Therefore, the prices are set very close to 0 or 1. Thus, the informed trader who has less accurate information would not trade. In other words, the informed trader does not trade in the wide state. This is true for both the buy and the sell sides. The symmetry of the conditions (5) and (6) gives us the following result.

Proposition 2 *Neither type of informed trader trades in the wide state if and only if* $p_W < (\nu p_W + (1-\nu)p_N) \cdot \frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\max\{\gamma_{+1},\gamma_{-1}\}+1/2\cdot\mu(1-\nu)}$.

Proof of Proposition 2: By Proposition 2, the informed trader in the wide state does not trade if (5) and (6) both hold. The condition (6) can be rewritten as: $p_W < (\nu p_W + (1-\nu)p_N) \cdot \frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{-1}+1/2\cdot\mu(1-\nu)}$. Therefore, we can say that the informed trader in the wide state does not trade if $p_W < (\nu p_W + (1-\nu)p_N) \cdot \min\{\frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)}, \frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{-1}+1/2\cdot\mu(1-\nu)}\}$. This gives us the desired result. ■

Notice that the conditional variance is higher in the wide state than in the narrow state because $\frac{1}{4} - p_W^2 > \frac{1}{4} - p_N^2$. The difference between the two is $p_N^2 - p_W^2$. When p_N is close to $\frac{1}{2}$ and p_W is close to 0, $p_N^2 - p_W^2$ is close to its supremum. This allows us to say that when the difference between the conditional variances is large with respect to the informed trader's information and the probability that the narrow state is sufficiently high, then the informed trader in the wide state does not trade.

It is worth mentioning the relationship between the no-trade situation and the probability of informed trading (μ). If μ is sufficiently high, then $\frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\max\{\gamma_{+1},\gamma_{-1}\}+1/2\cdot\mu(1-\nu)}$ is also close to 1. Therefore, the possibility of no trade increases because $p_W < (\nu p_W + (1-\nu)p_N)\cdot\frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\max\{\gamma_{+1},\gamma_{-1}\}+1/2\times\mu(1-\nu)}$ would hold for a wider range of p_W. This is because if the probability of informed trading is higher, the prices more accurately reflect true values. Therefore, the ask price is set higher and the bid price is set lower. Hence, the informed trader who has more coarse information would not trade.

Finally, we consider the other two cases. Notice that when $\gamma_{+1} = \gamma_{-1}$, (5) holds if and only if (6) holds. Thus if the informed trader does not trade in equilibrium when $\gamma_{+1} = \gamma_{-1}$, only **Case 1** could arise. It might be interesting to see how asymmetry of liquidity distribution relates to the no-trade situation. Since the argument is symmetric, we only consider **Case 2**. If $\sigma^N_{H,+1} = 1$, $\sigma^W_{H,0} = 1$, $\sigma^N_{L,-1} = \sigma^W_{L,-1} = 1$, then we obtain:

$$\begin{aligned} \delta(+1) &= \Pr(\tilde{\theta} = H| + 1) = \tfrac{1}{2}\cdot\tfrac{(1-\mu)\gamma_{+1}+\mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)}; \\ \delta(-1) &= \Pr(\tilde{\theta} = H| - 1) = \tfrac{1}{2}\cdot\tfrac{(1-\mu)\gamma_{-1}}{(1-\mu)\gamma_{-1}+1/2\cdot\mu}. \end{aligned} \tag{7}$$

Similarly with **Case 1** we obtain:

$$\begin{aligned} p_W &< (\nu p_W + (1-\nu)p_N)\cdot\tfrac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)}; \\ -p_W &\leq -(\nu p_W + (1-\nu)p_N)\cdot\tfrac{1/2\cdot\mu}{(1-\mu)\gamma_{-1}+1/2\cdot\mu}. \end{aligned} \tag{8}$$

By using a similar argument for **Case 3** we obtain the following result:

Proposition 3 *In the wide state, the high type does not trade and the low type sells if and only if the following holds:*

$$\frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{+1}+1/2\cdot\mu(1-\nu)} > \frac{p_W}{\nu p_W + (1-\nu)p_N} \geq \frac{1/2\cdot\mu}{(1-\mu)\gamma_{-1}+1/2\cdot\mu}.$$

On the other hand, the high type buys and the low type does not trade if and only if the following holds:

$$\frac{1/2\cdot\mu(1-\nu)}{(1-\mu)\gamma_{-1}+1/2\cdot\mu(1-\nu)} > \frac{p_W}{\nu p_W + (1-\nu)p_N} \geq \frac{1/2\cdot\mu}{(1-\mu)\gamma_{+1}+1/2\cdot\mu}.$$

Note that all the terms in Proposition 3 are strictly smaller than 1. A similar intuition to Proposition 2 holds here. In addition, if p_N is close to $\frac{1}{2}$, p_W is close to 0, ν is low, μ is close to 1, and $(1-\nu)\gamma_{-1} > \gamma_{+1}$, then the informed trader's information is accurate with very high probability and the buy order would be coming from the high type informed trader in the narrow state. Therefore, the ask price is set very close to 1. Thus, the high type informed trader in the wide state would not trade, whereas asymmetric distribution of liquidity trade makes space for the low type to sell.

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