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An econometric analysis of the output gap fluctuations: The case of Lebanon

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Abstract

This paper shows the reasons for output gap fluctuations in Lebanon during the period 1970-2009 and causal relationships between macroeconomic variables. It indicates that the output gap fluctuations, that measures observed GDP fluctuations around its long-run trend, can be explained by macroeconomic variables and war periods. By means of econometric methods, this research proposes to estimate the elasticity of the output gap with regard to others macroeconomic variables such as household consumption, expenditure government, gross fixed capital formation, and rate of economic dependence. Furthermore, it shows the causality direction between macroeconomic variables and the output gap in the short-run and in the long-run. The output gap is explained by all the macroeconomic variables in the long-run. But in the short-run, the output gap is explained by the household consumption expenditure only. In the short-run, the household consumption expenditure entails a decrease of output gap whereas it increases it in the long-run. In addition, the war does increase the output gap fluctuations.

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1. Introduction

In Lebanon political instabilities and variations of the economic activity are seen to be significant. During the period of 1975-1990, the civil war was certainly an important explicative factor of these instabilities. In fact, before the civil war, the Gross Domestic Product (GDP) showed a growth rate of 12% in 1972. Then this rate fell drastically to – 58% in 1976 and increased to 83% the next year (United Nations, 2009). This instability continued until the end of the civil war in 1990. These irregularities of growth rates indicate that in Lebanon (as in others countries) the economic activity fluctuates. Consequently, we see movements of GDP around a long-run trend. This trend is also called potential output, so we understand the potential output as the potential GDP or as the long-run trend of GDP. The potential GDP measures the optimal level of product obtained with the complete use of production factors. But statistical data regarding employment and unemployment rates are missing in Lebanon as a lot of developing countries. So we estimate the long-run trend by means of statistical methods.
Several studies regarding the output gap, i.e., concerning the measures of the GDP magnitude with regard to its long-run trend, have been conducted in developed OECD countries (Cette, 1997, OCDE, 2002, Ferrara, 2008 and 2009) and in developing countries like Tunisia (Fathi, 2009), Cameroun (Odia Ndongo, 2007), Brazil and Korea (Holfmaister and Roldôs, 2001). However, no study analyzes the causes of the output gap evolution in Lebanon actually. We can say that this study permits to improve the comprehension of factors influencing the economic functioning in a country like Lebanon where politics instabilities are becoming the rule.
During the war, the GDP vary strongly around its long-run trend but after 1990 the gap between GDP and its long-run trend shows an important reduction. So we raise the following questions: Which macroeconomic variables are influencing output gap fluctuations in Lebanon? What is the causality direction between macroeconomic variables and output gap? Are there relationships between war and output gap fluctuations?
To answer these questions, we shall present characteristics of the GDP evolution over the period of 1970-2009 and decomposition methods concerning the economic cycle extraction to estimate output gap evolution in Section 2. Section 3 presents an econometric model which takes into account the output gap and others macroeconomic variables such as household consumption expenditure, expenditure government, gross fixed capital formation and rate of economic dependence but also the war periods. In this section, we shall demonstrate the sources of the output gap fluctuations in the long and short-run and the causality direction between the output gap and others variables. Section 4 analyzes the results and concludes.

2. GDP and output gap evolution : an approach by the cycle extraction method

In Lebanon, GDP fluctuations are marked substantially over the period of 1975-1990 and they diminish after the end of the civil war in 1990. The following chart shows the logarithm of real GDP (at constant 2005 prices in US Dollars) evolution over the period of 1970-2009 by using the data of The United Nations (2010).

Chart 1: GDP evolution at constant 2005 prices of US Dollars 1970-2009

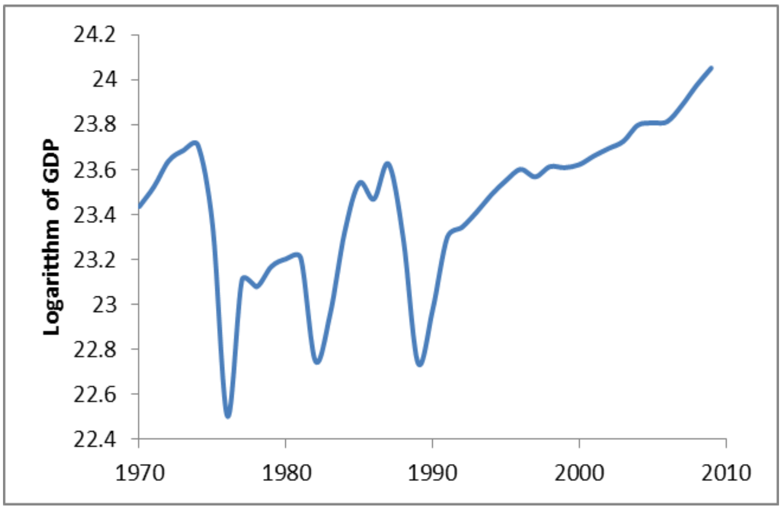


Chart 1 indicates that during the period 1970-1990, the Lebanese GDP follows a cyclical evolution in the sense of Burns and Mitchell's definition. These authors affirm: "*a cycle consists of expansions occurring at about the same time in many economic activities, followed by similar general recessions, contractions and revivals which merge into expansion of the next cycle…*" (Burns and Mitchell, 1946). In Lebanon, we see that this definition is obvious during the period 1970-1990. But after 1990, we notice that GDP fluctuations are weaker but do exist again. On the other hand, the GDP growth seems more regular from 1990 to 2009. However, this chart does not show clearly GDP fluctuations around a long-run trend of GDP. In order to see output gap fluctuations, we must separate cycles and trend. To make this operation, we can use the Hodrick-Prescott Filter's decomposition trend-cycle method (1991). This method consists to separate the trend (or long-term) movements in output cyclical (or short-term) movements with deviation of the output from trend representing a statistical measure of the output gap. This method allows the calculation of economic cycle magnitudes. It uses the minimization technique of a quadratic function as follow:

$$\text{Min } \Sigma(Y_t - Y^*_t)^2 + \lambda\, \Sigma[(Y^*_{t+1} - Y^*_t) - (Y^*_t - Y^*_{t-1})]^2 \qquad (1)$$

Relation [1] shows the GDP Y_t, that represents the logarithm of real GDP Lebanese over the period of 1970-2009. This one is decomposed in trend Y^*_t and cyclical factor Y_t^c. As the result we have:

$$Y_t^c = Y_t - Y^*_t \qquad (2)$$

Smoothing parameter λ indicates the regularity of the trend: if $\lambda \to \infty$ then the trend takes a linear form. On the contrary, if $\lambda = 0$, the trend is like the gross series. Generally, in the literature, we can use a value of 100 for the smoothing parameter λ if we have annual data, as Backus and Kehoe (1992) do it by studying cycle properties of cross section countries. But on the other hand, Ravn and Uhlig (2002) recommend adjusting this filter parameter by multiplying it with the fourth power of the observation frequency ratio. As the result, for annual data, this implies a value of 6.25 for the smoothing parameter. If we apply the Ravn and Uhlig's suggestion, regarding the Lebanese GDP, we choose $\lambda = 6.25$ and obtain the following chart.

Chart 2: Trend and cycle of Lebanese real GDP (1970-2009)

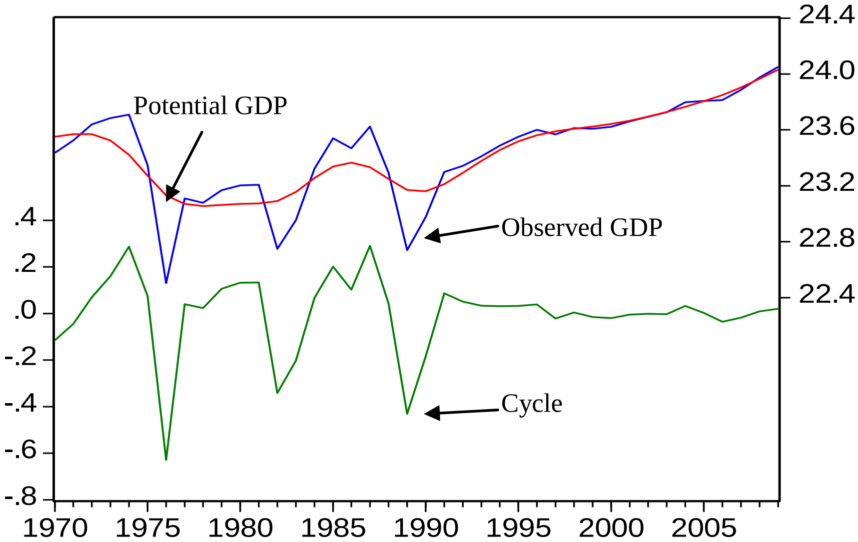
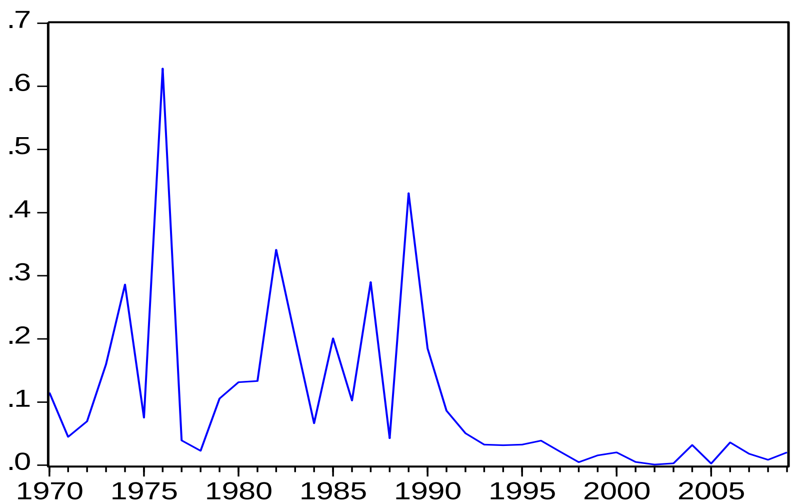


Chart 2 shows the observed and potential GDP variation and consequently the output gap evolution during the period 1970-2009. The right scale indicates the GDP written in logarithm and the left scale shows the gap between observed GDP and trend (potential GDP) in percentage.

We notice that during the period 1970-1990, observed GDP fluctuations with regard to potential GDP are significant and as a result, the output gap is seen to be important. After 1990, we see a reduction of output gap because the observed GDP is close to the level of potential GDP. Observed GDP and trend are increasing weakly but smoothly since the end of the war in 1990. There is a decline of GDP around its long-run trend. To show this, we take the absolute value of cyclic component that results from HP's filter (called Y_t^c in the relation (2)), and obtain the following chart.

Chart 3: GDP standard deviation with regard to the long run trend



Between 1970 and 1990, the GDP fluctuated strongly around its long-run trend. After 1990, at the end of the war, we see a drastic diminution of observed GDP with regard to its long-run trend. Regarding this last period, the observed GDP is very close to the long-run trend.

Finally, the evolution of the output gap magnitude is not regular and we can search the sources of these output gap fluctuations in Lebanon by means of econometric models that encompass macroeconomics variables and war periods also.

3. Short and long-run analysis of output gap

Before analyzing the relation between output gap in absolute value (Y_t^c) and others macroeconomic variables, such as household consumption expenditure (C_t), expenditure government (G_t), gross fixed capital formation (I_t), and economic dependence (TO_t)[1], we have to study the stochastic characteristics of these variables. To do this, we use unit root tests of Phillips-Perron (1988) based on the test strategies initiated by Dickey-Fuller (1981). We choose the Phillips-Perron's method because it includes residuals autocorrelation in regressions models. If we want to know if a series comports a unit root, we must estimate by the OLS (Ordinary Least Square) method the following models:

$$\text{Model [1]: } \Delta Y_t^c = \Phi Y_{t-1}^c + \varepsilon_t$$
$$\text{Model [2]: } \Delta Y_t^c = \Phi Y_{t-1}^c + c + \varepsilon_t$$
$$\text{Model [3]: } \Delta Y_t^c = \Phi Y_{t-1}^c + c + bt + \varepsilon_t$$

ΔY_t^c represents the absolute value of output gap series set in the first differences and ε_t indicates the error term.

After conducting these test strategies, we find that only the output gap series is stationary. Others macroeconomic variables have a unit root and follow a DS process (appendix 1). They need to be differenced once to become individually stationary. Such time series are called integrated of order one and denoted I(1). As the result, they can be cointegrated. To test whether a cointegration relationship does exist, we have to proceed with the Johansen-Juselius's method (1990). So, we estimate the VAR model with macroeconomic variables written in level. We choose one lag determined by the Schwarz information criterion. We use Johansen and Juselius's test by choosing, in Eviews command, "Intercept (no trend in CE) – no intercept in VAR". The result of this test indicates one relation of cointegration (appendix 2). Consequently, we can estimate the following VECM model to analyze the influence of macroeconomic variables on short-run and long-run output gap fluctuations.

$$DY_t^c = b_1 DY_{t-1}^c + b_2 DC_{t-1} + b_3 DI_{t-1} + b_4 DG_{t-1} + b_5 DTO_{t-1} + b_6 WAR + \alpha^1(Y_{t-1}^c - \beta_1 C_{t-1} - \beta_2 I_{t-1} - \beta_3 G_{t-1} - \beta_4 TO_{t-1} - \beta_5) + \varepsilon_t^1 \quad (3)$$

$$DC_t = c_1 DY_{t-1}^c + c_2 DC_{t-1} + c_3 DI_{t-1} + c_4 DG_{t-1} + c_5 DTO_{t-1} + c_6 WAR + \alpha^2(Y_{t-1}^c - \beta_1 C_{t-1} - \beta_2 I_{t-1} - \beta_3 G_{t-1} - \beta_4 TO_{t-1} - \beta_5) + \varepsilon_t^2 \quad (4)$$

$$DI_t = d_1 DY_{t-1}^c + d_2 DC_{t-1} + d_3 DI_{t-1} + d_4 DG_{t-1} + d_5 DTO_{t-1} + d_6 WAR + \alpha^3(Y_{t-1}^c - \beta_1 C_{t-1} - \beta_2 I_{t-1} - \beta_3 G_{t-1} - \beta_4 TO_{t-1} - \beta_5) + \varepsilon_t^3 \quad (5)$$

$$DG_t = e_1 DY_{t-1}^c + e_2 DC_{t-1} + e_3 DI_{t-1} + e_4 DG_{t-1} + e_5 DTO_{t-1} + e_6 WAR + \alpha^4(Y_{t-1}^c - \beta_1 C_{t-1} - \beta_2 I_{t-1} - \beta_3 G_{t-1} - \beta_4 TO_{t-1} - \beta_5) + \varepsilon_t^4 \quad (6)$$

[1] TO_t variable indicates the rate of economic dependence measured by the following formula: (exports of goods and services + imports of goods and services) / 2*GDP. It shows the open rate of economic.

$$DTO_t = f_1DY_t^c{}_{-1} + f_2DC_{t-1} + f_3\,DI_{t-1} + f_4DG_{t-1} + f_5DTO_{t-1} + f_6WAR + \alpha^5(Y_t^c{}_{-1} - \beta_1C_{t-1} - \beta_2I_{t-1} - \beta_3G_{t-1} - \beta_4TO_{t-1} - \beta_5) + \varepsilon_t^5 \qquad (7)$$

Variables in the parenthesis indicate the long-run relationship and α coefficient is seen to be the force of return towards the equilibrium. This coefficient has to be negative and statistically significant for the validation of VECM model.

Results concerning relations ranging (3) to (7) are put in the appendix 3. We see that the long-run relation is confirmed because the force of return (coefficient α) regarding the output gap variable (Y_t^c) is negative and significant at the 5% threshold value.
However, others α coefficients are not significant at the threshold of 5%. The VECM model contains restrictions. We estimate a partial VECM concerning relations ranging (3) to (7) (see appendix 4) and obtain the following results:

$$DY_t^c = -\,0.21DY_t^c{}_{-1} - \;0.2DC_{t-1} + 0.06DI_{t-1} - 0.00DG_{t-1} - 0.00\,DTO_{t-1} - 0.00WAR$$
$$(-\,1.31) \qquad (-\,2.41) \qquad (1.16) \quad (-\,0.06) \qquad (-\,1.47) \qquad (-\,0.38)$$

$$-\,1.48(Y_t^c{}_{-1} - 0.29C_{t-1} + 0.1I_{t-1} + 0.12G_{t-1} + 0.008TO_{t-1} - 2.02) + \varepsilon_t \qquad (8)$$
$$(-\,7.41) \qquad (-4.87) \qquad (3.62) \qquad (5.08) \qquad (-\,9.92) \qquad (2.1)$$

$N = 38$; $R^2adj = 0.8$; $F = 26.9$

(.) indicates the Student statistic; N, the observation number, F, the global significance and R^2adj, the determination adjusted coefficient.
Furthermore, we verify if residuals of each equation are seen to be a white noise by using the Q Statistic of Ljung-Box (with 15 periods lag).

Equation (3): Q(15) = 24.08 (Prob = 0.06)
Equation (4): Q(15) = 13.31 (Prob = 0.58)
Equation (5): Q(15) = 4.31 (Prob = 0.99)
Equation (6): Q(15) = 14.25 (Prob = 0.51)
Equation (7): Q(15) = 12.89 (Prob = 0.61)

All residuals of equation ranging (3) to (7) follow a white noise process because they show probabilities larger than 5% threshold value. The statistic model is valid.
Regression (8) displays a correct linear adjustment ($R^2adj = 0.8$) and a good global significance (F Statistic is high). Student statistics below coefficients of the long-run relation (in parenthesis) are all significant at the 5% threshold value. This equation represents a partial VECM and shows the output gap evolution in the long-run and short-run after a rising of the 1% macroeconomic variables (all variables are written in logarithm). The coefficient about the force of return towards the equilibrium is negative and significant statistically. Regarding the short-run relationship, only the coefficients of household consumption expenditure is significant at the 5% threshold value. Consequently, we can analyze the long-run elasticity of the output gap with regard to others macroeconomic variables. But if we want to search the causal link between variables and estimate the long-run elasticity, we have to implement Granger causality tests (1969). The results are put in the table 1.

Table 1: Granger Causality test

Null Hypothesis:	Obs	F-Statistic	Probability
DC does not Granger Cause Y_t^c	37	8.81727	0.00089
Y_t^c does not Granger Cause *DC*		1.21065	0.31129
DG does not Granger Cause Y_t^c	37	4.73270	0.01583
Y_t^c does not Granger Cause *DG*		2.55845	0.09316
DI does not Granger Cause Y_t^c	37	5.96494	0.00628
Y_t^c does not Granger Cause *DI*		0.55434	0.57987
DTO does not Granger Cause Y_t^c	37	9.29814	0.00066
Y_t^c does not Granger Cause *DTO*		0.31189	0.73426

Table 1 shows causal links between output gap and macroeconomic variables. The output gap is explained by all variables such as household consumption expenditure (C_t), expenditure government (G_t), gross fixed capital formation (I_t), and economic dependence (TO_t) in the long-run. However, in the short-run, the relation (8) shows the causality existence between output gap and household consumption expenditure only.

Indeed, in the short-run, the 1% increase in household consumption expenditure does cause a reduction of 0.2% output gap. In contrast, the 1% increase in the household consumption expenditure leads to a 0.43% output gap increase (– 1.48*(– 0.29) = 0.43) in the long-run.
Regarding the gross capital formation (the investment I_t), there is a long-run causality only: An increasing of the 1% gross capital formation enhances a decrease of the 0.148% (-1.48*0.1) output gap. We find also in the long-run the causality existence between expenditure government and output gap: An increasing of the 1% expenditure government entails a decrease of the 0.177% (-1.48*0.12) output gap. In addition, we see a causality existence between economic dependence and output gap in the long-run but the impact of the 1% increase in economic dependence rate on the output gap is very weak.
The short-run relationship between output gap and household consumption expenditure differs with regard to the long-run relationship. In the short-run, the household consumption expenditure has a negative effect on the output gap: If the household consumption expenditure is increasing, then the output gap is falling but we see the contrary phenomenon in the long-run. However, the war seems to have no effect on the output gap fluctuations if we analyze the significance of its coefficient in the relation (8). But can we be persuaded that the civil war was just a coincidence and did not influence the output gap evolution? Certainly no, if we test the following VAR model taking into account the output gap Y_t^c , household consumption expenditure (*Ct*) (the only variable that explains the output gap fluctuations in the short-run), *WAR* (as dummy variable) and a trend called *t*.

By means of the OLS method, we obtain the following equation (see the complete results in appendix 5):

$$Y_t^c = 0.04 - 0.48\, Y_{t-1}^c - 0.22 DC_{t-1} + 0.08 WAR - 0.07t + \varepsilon_t^1 \qquad (9)$$
$$(1.7) \quad (-2.77) \quad (-2.9) \quad (2.07) \quad (-4.09)$$

$N = 38$; $R^2 adj = 0.42$; $F = 7.73$

$$DC_t = 0.04 + 0.82Y_{t-1}^c - 0.06DC_{t-1} - 0.14WAR + 0.004t + \varepsilon_t^2 \qquad (10)$$
$$(0.6) \quad (1.96) \quad (-0.31) \quad (-1.52) \quad (1)$$

$N = 38$; $R^2adj = 0.04$; $F = 1.4$

(.) indicates the Student statistic; N, the observation number, F, the global significance and R^2adj, the determination adjusted coefficient.

As we can see, the equation (9) presents a good global significance (F value as well as R^2adj are high) with regard to equation (10) where the statistics values are weak. On the other hand, concerning the relation (9), the war variable does increase the output gap whereas the household consumption expenditure has a negative impact on it. Except the intercept, all coefficients regarding the relation (9) are significant at the 5% threshold value. With the time, the output gap fluctuations are decreasing as the negative sign of the coefficient concerning the t variable show it. In addition, the war entails a rising of the 0.08% output gap fluctuations.

Section 4: Results and conclusion

In Lebanon, the sources of output gap fluctuations do result from macroeconomic variables and the civil war over the period of 1975-1990. Indeed, we notice a drastic reduction of the output gap fluctuations after 1990. In war situation, the observed GDP increases of 0.08% with regard to its long-run trend. This means that the peace periods constitute an important factor for the economic stability. Furthermore, others macroeconomic variables have an impact in the long-run only except the household consumption expenditure that influences the output gap in the short-run also.

An increasing of the household consumption expenditure entails a reduction of the output gap in the short-run and an increase in the long-run. Indeed, the short-run elasticity of the output gap with regard to the household consumption expenditure is negative: a 1% increase in the household consumption expenditure variable leads to 0.2% output gap reduction. However, in the long-run, a 1% increase in the household consumption expenditure enhances a 0.43% output gap augmentation. Finally, the household consumption expenditure variable is seen to be a factor of economic stabilization in the short-run whereas it provokes the enlargement of economic fluctuations in the long-run. This means that the increase of the household consumption expenditure during a long time entails economic instability and inflation consequently. When the observed GDP is larger than the potential GDP, during a long period, inflationary situations are appearing. On the other hand, the reduction of the output gap fluctuations does result from the growing of macroeconomic variables in the long-run too: The GDP observed becomes close to the potential GDP in the long-run and this means an improvement of the economic stability. When the expenditure government and investment are growing, the economic growth is more regular and less instable. Furthermore, when the rate of economic dependence is seen to be more important, the output gap fluctuations are decreasing too. But in Lebanon, except the political instability, two major problems persist: First, the national debt represents, in 2010, 164% of GDP (Central Bank of Lebanon), and the interests of the debt are financed by the expenditure government; Second, the trade balance is overdrawn since 1970 and this country is culturally dependent toward the rest of others countries. In fact, it is also necessary to reduce the public debt and improve the trade balance to obtain the steady growth and economic stability.

REFERENCES

Backus, David K and Patrick, J. Kehoe (1992) "International Evidence on the Historical Properties of Business Cycle", *American Economic Review*, Vol.82, Iss.4, pp. 864-888.

Bernanke, Ben S. (2004) "The Great Moderation," Remarks at the meetings of the *Eastern Economic Association*, Washington, DC, February 20, 2004.

Beveridge, Stephen and Charles R. Nelson (1981) "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components", *Journal of Monetary Economics*, Vol. 7, Iss.2, pp. 151-174.

Bourbonnais, Régis (2009) *Econométrie*, Dunod, 7ème édition.

Burns, Arthur F and Wesley C. Mitchell (1946) *Measuring Business Cycle*, New-York, NBER.

Central Bank of Lebanon, **www.bdl.gov.lb**

Cette, Gilbert (1997) "Ecart de positionnement dans le cycle économique : quelques évaluations pour l'économie française ", *Bulletin de la Banque de France*, No 38, Février, pp. 91-104.

Engle, Robert F and Clive W.J. Granger (1987) "Co-integration and Error Correction: Representation, Estimation and Testing", *Econometrica*, Vol. 55, Iss.2, pp. 251-276.

Ferrara, Laurent (2009) "Caractérisation et datation des cycles économiques en zone euro ", *Revue Economique*, Vol. 60, No 3, pp. 703-712.

Ferrara, Laurent (2008) " L'apport de indicateurs de retournement cyclique à l'analyse conjoncturelle", *Bulletin de la Banque de France*, No 171, mars.

Granger, Clive W.J. (1969) "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods", *Econometrica*, Vol. 37, Iss.3, pp. 424-438.

Granger, Clive W.J. (2003) "Some Aspects of Causal Relationships", *Journal of Econometrics*, 112, pp. 69-71.

Hodrick, Robert and Edward C. Prescott (1997) "Postwar U.S. Business Cycles: an Empirical Investigation'', *Journal of Money, Credit, and Banking*, Vol. 26, pp. 1-16.

Hoffmaister, Alexander W and Jorge E. Roldôs (2001) "The Source of Macroeconomics Fluctuations in Developing Countries: Brazil and Korea", *Journal of Macroeconomics*, Vol. 23, Spring, pp. 213-239.

Johansen, Soren and Katarina, Juselius (1990) "Maximum Likelihood Estimation and Inference on Cointegration - With Applications to the Demand for Money", *Oxford Bulletin of Economics and Statistics* Vol. 52, Iss.2 (May), pp. 169-210.

OCDE, (2002) "Les changements en cours du cycle économique", in *Perspectives économiques de l'OCDE*, No 71, pp. 173-190.

Odia Ndongo, Yves-Francis (2007) "Les sources des fluctuations macroéconomiques au Cameroun", MPRA, Munich Personal RePc Archive, Paper No 1308, Online at **http://mpra.ub.uni-muenchen.de/1308/**

Okun, Arthur M. (1962) "Potential GNP: Its Measurement and Significance", In Proceedings *Business and Economic Statistics Section of the American Statistical Association*, pp. 89–104.

Ravn, Morten O and Harald Uhlig (2002) "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations", *The Review of Economics and Statistics*, Vol. 84, Iss.2, pp. 371-375.

Sims Christopher A. (1980) "Macroeconomics and Reality", *Econometrica*, Vol. 48, Iss.1, January, pp. 1-48.

United Nations, Statistic Division, National Account Main Aggregates Database; **http://unstats.un.org/unsd/snaama/dnlList.asp**

Appendix 1: Unit root tests

Table 1: Phillip-Perron's unit root test on output gap Y_t^c series

Null Hypothesis : Y_t^c has a unit root Troncature l = 3		
Phillips-Perron test	Adjusted t Stat	Prob.
Model [3] : constant and trend	– 6.19	0.0000
Model [2]: without trend	– 5.39	0.0001
Model [1]: without constant and trend	– 3.78	0.0004

The output gap Y_t^c series is stationary because whatever the model, the probability regarding the adjusted t stat is inferior to 0.05.

Table 2: Phillip-Perron's unit root test on household expenditure C_t series

Null Hypothesis : C_t has a unit root Troncature l = 3		
Phillips-Perron test	Adjusted t Stat	Prob.
Model [3] : constant and trend	– 3.32	0.0774
Model [2]: without trend	– 2.49	0.125
Model [1]: without constant and trend	0.40	0.7955

The household expenditure series C_t has a unit root because whatever the model, the probability regarding the adjusted t stat is larger than 0.05.

Table 3: Phillip-Perron's unit root test on gross capital formation I_t series

Null Hypothesis : I_t has a unit root Troncature l = 3		
Phillips-Perron test	Adjusted t Stat	Prob.
Model [3] : constant and trend	– 3.22	0.0952
Model [2]: without trend	– 2.0	0.2844
Model [1]: without constant and trend	0.63	0.8479

The gross capital formation series I_t has a unit root because whatever the model, the probability regarding the adjusted t stat is larger than 0.05.

Table 4: Phillip-Perron's unit root test on expenditure government G_t series

Null Hypothesis : G_t has a unit root Troncature l = 3		
Phillips-Perron test	Adjusted t Stat	Prob.
Model [3] : constant and trend	– 2.55	0.3017
Model [2]: without trend	– 2.42	0.1429
Model [1]: without constant and trend	0.64	0.8502

The expenditure government series G_t has a unit root because whatever the model, the probability regarding the adjusted t stat is larger than 0.05.

Table 5: Phillip-Perron's unit root test on economic dependence rate TO_t series

Null Hypothesis : G_t has a unit root Troncature l = 3		
Phillips-Perron test	Adjusted t Stat	Prob.
Model [3] : constant and trend	– 2.90	0.172
Model [2]: without trend	– 2.57	0.1075
Model [1]: without constant and trend	– 0.57	0.4639

The economic dependence rate series TO_t has a unit root because whatever the model, the probability regarding the adjusted t stat is larger than 0.05.

Appendix 2: Results of cointegration test

Sample (adjusted): 1972 2009
Included observations: 38 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: Y_t^c LNC LNI LNG LNTO
Exogenous series: *WAR*
Warning: Critical values assume no exogenous series
Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.663787	95.07183	76.97277	0.0011
At most 1	0.551196	53.65147	54.07904	0.0546
At most 2	0.244555	23.20708	35.19275	0.5146
At most 3	0.235512	12.55006	20.26184	0.4008
At most 4	0.059850	2.345201	9.164546	0.7085

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

This table indicates that a relationship of cointegration does exist. The statistic trace value is larger than the critical value (line 1). The matrix range is not equal to 0 but we can’t reject the H0 hypothesis indicating that the matrix range is not larger than 1 (line 2). As the result, the model contains one equation of cointegration and we must estimate a VECM model by using maximum likelihood method.

Appendix 3: Results of VECM model

Vector Error Correction Estimates
Sample (adjusted): 1972 2009
Included observations: 38 after adjustments
t-statistics in [ ]

Cointegrating Eq:	CointEq1				
Y_t^c (-1)	1.000000				
LNC(-1)	-0.561598 [-7.05939]				
LNI(-1)	0.184368 [4.77780]				
LNG(-1)	0.168007 [5.08743]				
LNTO(-1)	-0.010698 [-9.20332]				
C	5.754828 [4.54307]				
Error Correction:	**D(Y_t^c)**	**D(LNC)**	**D(LNI)**	**D(LNG)**	**D(LNTO)**
CointEq1	-0.730104 [-3.37333]	0.650309 [1.73028]	-0.143013 [-0.16090]	0.059418 [0.08783]	-8.375809 [-0.27148]
D(Y_t^c (-1))	-0.489668 [-3.33083]	0.239866 [0.93960]	0.650979 [1.07824]	0.392841 [0.85494]	10.57661 [0.50470]
D(LNC(-1))	-0.258119 [-2.70356]	0.317989 [1.91800]	0.513126 [1.30870]	-0.361657 [-1.21194]	-0.123722 [-0.00909]
D(LNI(-1))	0.087451 [1.43301]	-0.101788 [-0.96051]	-0.553223 [-2.20742]	-0.320260 [-1.67902]	6.030376 [0.69321]
D(LNG(-1))	-0.072997 [-0.88845]	0.164496 [1.15294]	0.566930 [1.68018]	0.533865 [2.07887]	9.719710 [0.82989]
D(LNTO(-1))	-0.001298 [-0.41040]	-0.010042 [-1.82816]	-0.020792 [-1.60048]	-0.011405 [-1.15351]	-0.422627 [-0.93724]
WAR	0.000960 (0.02075) [0.04627]	-0.008560 (0.03603) [-0.23761]	0.047008 (0.08520) [0.55172]	0.041266 (0.06485) [0.63638]	0.714538 (2.95738) [0.24161]

R-squared	0.793419	0.701602	0.320999	0.471948	0.165684
Adj. R-squared	0.753435	0.643847	0.189580	0.369745	0.004203
Sum sq. resids	0.239853	0.723275	4.045278	2.343211	4873.836
S.E. equation	0.087961	0.152746	0.361238	0.274932	12.53876
F-statistic	19.84368	12.14801	2.442556	4.617727	1.026028
Log likelihood	42.32130	21.34983	-11.35898	-0.984447	-146.1466
Akaike AIC	-1.859016	-0.755254	0.966262	0.420234	8.060349
Schwarz SC	-1.557355	-0.453594	1.267923	0.721895	8.362009
Mean dependent	-0.000653	0.011201	0.030360	0.028124	0.188693
S.D. dependent	0.177144	0.255949	0.401271	0.346311	12.56520

Determinant resid covariance (dof adj.)	3.45E-05
Determinant resid covariance	1.25E-05
Log likelihood	-55.03160
Akaike information criterion	5.054295
Schwarz criterion	6.821164

Appendix 4: Table of the partial VECM estimation

Vector Error Correction Estimates
Sample (adjusted): 1972 2009
Included observations: 38 after adjustments
t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1, A(2,1)=0, A(3,1)=0, A(4,1)=0, A(5,1)=0
Convergence achieved after 23 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(4)	3.862666
Probability	0.424911

Cointegrating Eq:	CointEq1
Y_t^c (-1)	1.000000
LNC(-1)	-0.293859
	[-4.87361]
LNI(-1)	0.105914
	[3.62133]
LNG(-1)	0.127245
	[5.08374]
LNTO(-1)	-0.008741
	[-9.92132]
C	2.022426
	[2.10649]

Error Correction:	D(Y_t^c)	D(LNC)	D(LNI)	D(LNG)	D(LNTO)
CointEq1	-1.480196	0.000000	0.000000	0.000000	0.000000
	[-7.41280]	[NA]	[NA]	[NA]	[NA]
D(Y_t^c (-1))	-0.209586	0.328824	0.668915	0.316481	-2.824215
	[-1.31134]	[1.01117]	[0.89989]	[0.55985]	[-0.10966]
D(LNC(-1))	-0.206262	0.309654	0.519153	-0.371869	-1.387152
	[-2.41365]	[1.78090]	[1.30622]	[-1.23031]	[-0.10074]
D(LNI(-1))	0.062974	-0.108345	-0.554923	-0.313780	7.141738
	[1.15565]	[-0.97719]	[-2.18959]	[-1.62802]	[0.81336]
D(LNG(-1))	-0.004924	0.241441	0.565253	0.506549	3.746718
	[-0.06562]	[1.58165]	[1.61995]	[1.90890]	[0.30993]

D(LNTO(-1))	-0.004213	-0.013990	-0.020649	-0.010132	-0.134806
	[-1.47012]	[-2.39928]	[-1.54922]	[-0.99958]	[-0.29193]
WAR	-0.006926	0.001534	0.045128	0.041422	0.473339
	[-0.38467]	[0.04188]	[0.53889]	[0.65042]	[0.16315]
R-squared	0.838899	0.680529	0.320886	0.472677	0.168653
Adj. R-squared	0.807718	0.618696	0.189445	0.370614	0.007747
Sum sq. resids	0.187049	0.774353	4.045953	2.339979	4856.493
S.E. equation	0.077678	0.158048	0.361268	0.274742	12.51643
F-statistic	26.90421	11.00589	2.441286	4.631241	1.048144
Log likelihood	47.04582	20.05329	-11.36215	-0.958222	-146.0789
Akaike AIC	-2.107675	-0.687015	0.966429	0.418854	8.056784
Schwarz SC	-1.806014	-0.385354	1.268090	0.720514	8.358444
Mean dependent	-0.000653	0.011201	0.030360	0.028124	0.188693
S.D. dependent	0.177144	0.255949	0.401271	0.346311	12.56520

Determinant resid covariance (dof adj.)	3.63E-05
Determinant resid covariance	1.31E-05
Log likelihood	-56.96293
Akaike information criterion	5.155944
Schwarz criterion	6.922813

We see that the probability concerning the LR test for biding restriction is inferior to 0.05. This means that others α coefficients are not significant and the partial VECM can be estimated with $\alpha^2 = \alpha^3 = \alpha^4 = \alpha^5 = 0$

Appendix 5: Table of the causal VAR estimation

Vector Autoregression Estimates
Sample (adjusted): 1972 2009
Included observations: 38 after adjustments
t-statistics in [ ]

	Y_t^c	DC
Y_t^c (-1)	-0.480573	0.826223
	[-2.76836]	[1.96277]
DC(-1)	-0.227656	0.060077
	[-2.90245]	[0.31587]
C	0.045765	0.039256
	[1.70011]	[0.60138]
WAR	0.078925	-0.140407
	[2.07489]	[-1.52223]
@TREND(0)	-0.007427	0.004405
	[-4.09656]	[1.00198]
R-squared	0.483890	0.145554
Adj. R-squared	0.421331	0.041985
Sum sq. resids	0.352217	2.071056
S.E. equation	0.103311	0.250518
F-statistic	7.734969	1.405381
Log likelihood	35.02113	1.361359
Akaike AIC	-1.580059	0.191507
Schwarz SC	-1.364588	0.406979
Mean dependent	0.104653	0.011201
S.D. dependent	0.135810	0.255949
Determinant resid covariance (dof adj.)		0.000498
Determinant resid covariance		0.000376
Log likelihood		41.99883
Akaike information criterion		-1.684149
Schwarz criterion		-1.253205