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Tactical transfers in a federal institutional setting

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Abstract

One of the main aim of political economy is to understand how income redistribution is determined. In the paper tactical redistribution, through which parties aim at maximizing the share of votes obtained in an election, is analyzed in a federal institutional setting, where different levels of government coexist. Dixit and Londregan (1996)'s model is taken as a starting point; their model is extended in order to allow the analysis of the interactions between the various government levels. Four institutional settings are considered, entailing different rules and a different degree of decentralization in the policy and transfer determination process: fully localized and fully centralized governments, federal government with transfers among regions and federal government with transfers among social groups.

1. Introduction

There are at least two forces driving redistribution. On one hand (ideological component) redistribution among citizens is desirable in order to reach some social objective (e.g.: total welfare maximization). On the other hand (tactical component) redistribution may serve parties' objective of maximizing their share of votes. The ideal recipients of tactical transfers may be either swing voters (see Dixit and Londregan, 1996)¹ or parties' constituencies (Cox and McCubbins, 1986).²

This paper analyzes tactical transfers in a federal setting, where parties compete to maximize the share of votes in the elections at a central and at a local level. Four settings will be considered, entailing a different degree of centralization. Fully localized and fully centralized governments, with both policy and transfers decided at a local and central level respectively, will be considered as benchmark cases. In federal governments, policies are the result of a bargaining between local and central governments. In a federal government with transfers among regions, the central government performs transfers among regions, and local governments perform transfers among different social groups within their regions. In a federal government with transfers among social groups, both the local and the central governments perform transfers among social groups, the central government at a national and the local government at a regional level.

Dixit and Londregan's (1996) model is chosen as a starting point, for two reasons. First, the evidence available in the literature tends to confirm the prediction of this model as compared to Cox and McCubbins's (1986) one (see, among others, Dahlberg and Johansson, 2002). Second, this model is more general and, under special circumstances, may lead to similar conclusions as Cox and McCubbins's (1986) one.

The paper proceeds as follows: Section 2 describes the elements of the model; Section 3 analyzes political equilibria under the different institutional settings; Section 4 concludes.

2. Elements of the model

A country formed by different regions is considered; within each region there are groups of citizens with different income per capita. Citizens have preferences over the implemented policy and over their private consumption. A local and a central government level coexist and parties compete at both levels, using monetary transfers in order to attract votes. In detail, the elements of the model are as follows.

Two parties, *L* and *R*, run an electoral competition and aim at maximizing their share of votes at a local and at a central level.³ Parties have a fixed political position and, for simplicity, *L* and *R* will denote both the parties and their political platforms. Parties pursue tactical redistribution in order to maximize the number of votes obtained.

Citizens care about private consumption and have ideological preferences over the different policies proposed by the parties. As in Dixit and Londregan (1996), voters are considered as a continuum distributed along the real line: a voter located at point *X* has an (ideological) preference

¹The model is extended by Dixit and Londregan (1998) where citizens and parties also care about income distribution.

²? analyze the incentives for candidates to creates inequalities among voters. His model, however, considers homogeneous voters and therefore does not provide useful predictions for the purpose of our analysis.

³The precise objective function of the parties will depend on the institutional setting and will be specified in detail in the following section.

of X for party R over party L . The utility over consumption is increasing and strictly concave (e.g.: $U'(c) > 0, U''(c) < 0$).

The country is formed by F regions denoted by i . Each region has a population N_i , and N is the total population of the country (i.e.: $N = \sum_{i=1}^F N_i$).

Citizens belong to G different social groups, denoted by j . Every citizen belonging to group j has a pre-transfers income of y_j . Citizens in each group exhibit heterogeneous ideological positions, but it is realistic to suppose some kind of “political orientation” (e.g.: the poor may be more left oriented and the rich more right oriented). This phenomenon is modeled by considering different distributions of X among different groups. $\Phi_j(\cdot)$, defined over the range of values of X for citizens of group j , denotes the cumulative frequency distribution (and $\phi_j(\cdot)$ the density function) of voters of group j over this range.⁴ Therefore $\Phi_j(X')$ denotes the proportion of voters of group j with $X \leq X'$. Two restrictions will be imposed on the distribution functions. Each density function $\phi_j(\cdot)$ must be single peaked and, for each group, there must be at least one citizen strictly preferring each of the two parties’ political platform (e.g.: $0 < \Phi_j(0) < 1$).

The policy outcome is identified by a vector $P = (P_1, P_2, \dots, P_F)$ representing the policies implemented in each region. Policy outcome and monetary transfers will be determined following different rules depending on the institutional setting adopted.

3. Political Equilibria

Four different political settings are considered, entailing a different degree of centralization: localized and centralized governments and federal governments with transfers among social groups or among regions.

3.1 Equilibrium in a fully localized government

The results obtained in this framework largely reflect the results obtained in Dixit and Londregan (1996). In a fully localized government the game develops as follows.

During the electoral campaign (stage 1) parties in region i propose a vector of transfers $t_i^k = (t_{i1}^k, \dots, t_{iG}^k)$, $k = L, R$ towards each social group. Transfers are subject to the budget constraint: $\sum_{j=1}^G t_{ij}^k \times N_{ij} \leq B_i$, $k = L, R$, where t_{ij}^k is the transfer proposed by party k to a citizen of region i and group j and B_i is an exogenous given monetary amount available to the government of region i .

B_i is identical for the two parties, since it represents an amount of resources available for redistribution in region i , irrespective of which party wins the elections. This assumption is crucial and will be maintained through all the different institutional settings, as well as it was present in Dixit and Londregan (1996). The results of the model are not robust to a modification of this assumption.⁵ Moreover, B_i is not dependent on any social group (j) and may vary only across

⁴ $\Phi_j(\cdot)$ is supposed to be concave. However, for a discussion on the necessity of this assumption, see the Mathematical Appendix in Dixit and Londregan (1996).

⁵In Dixit and Londregan (1996), the assumption that $B_L = B_R$ is crucial in order to obtain the result that the Lagrange multipliers related to the maximization problem faced by the two parties are the same ($\lambda_L = \lambda_R$). Since through the maximization problem the Lagrange multipliers are the only party-specific information, an equal Lagrange multiplier for both parties implies that the same strategy represents an optimal solution for L and R (so that the transfer

regions, since it is available to each party that will win the elections irrespective of the social groups that will benefit from the tactical transfers. Also this assumption is crucial in order to obtain the equilibrium results, and will be maintained through the various settings.⁶

The consumption enjoyed by a citizen of region i and group j is $c_{ij} = y_j + t_{ij}$.

Denoting V_i^k as the share of votes obtained in region i by party k , the policy implemented in region i is L iff $V_i^L \geq V_i^R$, otherwise the policy implemented is R . Similarly, the transfers implemented in region i are t_i^L iff $V_i^L \geq V_i^R$, (t_i^R otherwise).

Therefore, a political equilibrium for region i in a fully localized setting is characterized by two sets of transfers $t_i^k = (t_{ij}^k, \dots, t_{ij}^k)$, $k = L, R$ and a set of voting decisions by citizens such that, in stage 1, $t_i^L(t_i^R)$ maximizes party L (R) shares of votes in region i given $t_i^R(t_i^L)$ and, in stage 2, every citizen Z votes for party L iff $U_Z(L) \geq U_Z(R)$ (and for party R otherwise). Each party chooses a transfer proposal that maximizes its own share of votes (tactical redistribution) and each citizen casts his vote for the preferred party (considering both ideological position and transfer proposals).⁷

In order to find a closed form equilibrium the following specification of the utility function, as in Dixit and Londregan (1996), will be assumed: $U_{ij}(c_{ij}) = k_j \times \frac{c_{ij}^{1-\varepsilon}}{1-\varepsilon}$. k_j is a group specific parameter on the relative importance of consumption with respect to ideology. This specification will be maintained through the rest of the paper. We look for subgame perfect equilibria. We will limit the analysis to interior equilibria (for a discussion on the reasonableness of this approach and on the conditions for excluding corner solution, see the Mathematical Appendix in Dixit and Londregan, 1996).

Proposition 1. In a fully localized government there exists a unique SPE in which:

(1) Both parties propose, in each region i , the transfer schedule $t_{ij} = \frac{[k_j \times \phi_j(0)]^{1/\varepsilon}}{\sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon}} \times (Y_i + B_i) - y_j$, where $\phi_j(0)$ is the density of group j at $X = 0$ and $Y_i = \sum_j N_{ij} \times y_j$ represents the total income of region i citizens;

(2) All citizens positioned on the real line of ideological positions at a point $X \leq 0$ vote for party L , all citizens positioned at $X > 0$ vote for R .

Proof: see Dixit and Londregan (1996).

The equilibrium consumption of each citizen belonging to group j is therefore $c_{ij} = y_j + t_{ij} = \frac{[k_j \times \phi_j(0)]^{1/\varepsilon}}{\sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon}} \times (Y_i + B_i)$ and groups with higher k and higher $\phi_j(0)$ obtain better results in the

schedules proposed are the same in equilibrium). Clearly, this result does not hold if $\lambda_L \neq \lambda_R$ (e.g. if from the initial case in which $B_L = B_R$ we raise either B_L or B_R , then the two Lagrange multipliers will not be equal between them and parties' strategies will diverge). For further details concerning the derivation of the SPE equilibria see the Mathematical Appendix in Dixit and Londregan (1996).

⁶If B_i had been different depending on the social group receiving the monetary transfer, the maximization problem would have been more complicated, since parties would have faced a different budget constraint for each social group. The proof of equilibrium existence and uniqueness as it is presented in Dixit and Londregan (1996) would not hold in such a modified setting.

⁷The assumptions that, when indifferent, a citizen chooses L and that, in case of $V^L = V^R$, the policy and the transfer proposal of party L are implemented will not have any effect on equilibrium implications.

redistribution game.

3.2 Equilibrium in a fully centralized government

In a fully centralized government only the national electoral campaign and the national elections matter and the game develops as follows: in stage 1 the national electoral campaign takes place and each party proposes a transfer schedule among different social groups; in stage 2 the national elections take place; in stage 3 the policy and transfer schedule of the winning party are implemented.

In particular, during the national electoral campaign (stage 1) parties propose two transfer vectors $t^{N,k} = (t_1^{N,k}, \dots, t_G^{N,k})$, $k = L, R$ where $t_j^{N,k}$ represents the transfer proposal of party k towards each citizen belonging to social group j at a national level. Transfers are subject to the budget constraint $\sum_{i=1}^F \sum_{j=1}^G t_{ij}^N \times N_{ij} \leq B$, where B is an exogenous given amount available to the central government for transfers.⁸

Denoting V_N^k as the share of votes obtained at a national level by party k , the policy implemented in region i is L iff $V_N^L \geq V_N^R$, (R otherwise). The transfers implemented are $t^{N,L}$ iff $V_N^L \geq V_N^R$ ($t_i^{N,R}$ otherwise).

Similarly to the local government case, a political equilibrium for region i in a fully centralized setting is characterized by two sets of transfers $t_i^{N,k} = (t_1^{N,k}, \dots, t_i^{N,k})$, $k = L, R$ and a set of voting decisions by citizens such that, in stage 1, $t_i^{N,L}(t_i^{N,R})$ maximizes party L (R) shares of votes in region i given $t_i^{N,R}(t_i^{N,L})$ and, in stage 2, every citizen Z votes for party L iff $U_Z(L) \geq U_Z(R)$ (and for party R otherwise).

Proposition 2. In a fully centralized government there exists a unique SPE, in which:

$$(1) \text{ Both parties propose the transfer schedule: } t_{ij}^N = \frac{[k_j \times \phi_j(0)]^{1/\varepsilon}}{\left\{ \sum_{i=1}^N \sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon} \right\}} \times (Y + B) - y_j,$$

where $\phi_j(0)$ is the density of group j at $X = 0$ and $Y = \sum_i \sum_j N_{ij} \times y_j$ represents the total income of the country;

(2) All citizens positioned on the real line of ideological positions at a point $X \leq 0$ vote for party L , all citizens positioned at $X > 0$ vote for R .

Proof: the proof is similar to the case of fully localized government, it is sufficient to consider the whole nation as a unique region.

As in the case of completely centralized government groups with an higher k and $\phi_j(0)$ perform better in the redistribution game.

3.3 Equilibria in federal governments

In federal governments, policies are the result of a bargaining between local and central governments. In a federal government with transfers among regions, the central government performs transfers among regions, and local governments perform transfers among different social groups within their regions.⁹ In a federal government with transfers among social groups both the local

⁸In order to maintain consistency with the previous section assume $B = \sum_i B_i$.

⁹This model might stylize the E.U. context in which, at a federation level, there is a certain degree of redistribution

and the central governments perform transfers among groups of citizens, the central government at a national and the local government at a regional level.¹⁰ In order to maintain the analysis tractable, the discussion will be limited, from now on, to the case in which two regions and two income groups exist.

In particular, in a federal government with transfers among regions, the game develops as follows: in stage 1 the national electoral campaign takes place and parties propose an inter-regional transfer schedule; in stage 2 national elections are held; in stage 3, the local electoral campaign takes place and parties propose transfers among different social groups at a regional level; in stage 4 local elections are held; in stage 5 policies and transfers are implemented according with the rules that will be described below.

During the national campaign (stage 1) parties propose a transfer vector $M^k = (M_1^k, M_2^k)$, $k = L, R$ where M_i^k represents the transfers proposed by party k towards region i . Transfers are subject to the budget constraint $\sum_{i=1}^2 M_i \leq 0$. The inter-regional transfer vector implemented is M^L iff $V_N^L \geq V_N^R$ in the national elections (M^R otherwise).

During the local electoral campaign (stage 3) parties in region i propose a vector of transfers $t_i^k = (t_{i1}, t_{i2})$, $k = L, R$. Transfers are subject to the budget constraint $\sum_{j=1}^2 N_{ij} \cdot t_{ij}^k \leq B_i + M_i$, where t_{ij}^k is the transfer proposed by party k to a citizen of region i and group j and B_i is an exogenous given amount available to government of region i for transfers.¹¹ The intra-regional implemented transfers are represented by t_i^L iff $V_i^L \geq V_i^R$ (t_i^R otherwise).

The final policy implemented in each region will be a linear combination of the policy platforms of the party governing at a central and at a local level, $P_i = \gamma \cdot P_i^{Cen} + (1 - \gamma) \cdot P_i^{Loc}$.

Parties aim at maximizing a weighted average of the share of votes in the two levels of elections. Part k objective function is therefore $\Omega^k = \varphi(\gamma) \cdot V_N^k + \sum_{i=1}^2 \mu_i(\gamma) \cdot V_i^k$ with $\varphi(\gamma) + \sum_{i=1}^2 \mu_i = 1$. φ is the weight given to the national elections, μ_i is the weight given to the local election in region i , V_N^k and V_i^k are the shares of votes obtained by party k in the national and in region i local election, respectively.

A political equilibrium in a federal government with transfers among regions is characterized by two proposed sets (one for each party) of inter-regional transfers $M^k = (M_1^k, M_2^k)$, two sets (for each region) of intra-regional transfers $t_i^k = (t_{i1}^k, t_{i2}^k)$, $k = L, R$ and two sets of voting decision by citizens (both in the central both in the local elections) such that: in stage 3 (local electoral campaign), in each region, t_i^L (t_i^R) maximizes party L (R) objective function value given t^R (t^L), M^R and M^L ; in stage 1 (national electoral campaign), M^L (M^R) maximizes party L (R) objective function given M^R (M^L) and assuming agents' rational behavior in the following stages of the game; in stage 2 (national elections) and in stage 4 (local elections) every citizen Z votes for party L iff

among different member states while each country implements redistribution policies among the different groups of citizens.

¹⁰In the U.S. both at a federal and at a national level there are policy programs that entail a certain degree of redistribution among social groups.

¹¹Note that, in fact, the real amount of resources available to the regional government for transfers include M_i , the transfer operated by the central government towards region i .

$U_Z(L) \geq U_Z(R)$, vice versa votes for party R .

On the other side, in a federal government with transfers among social groups the game develops as follows: in stage 1 the national electoral campaign takes place and parties propose transfer schedules among different social groups at a central level; in stage 2 national elections are held; in stage 3, the local electoral campaign takes place and in each region parties propose transfer schedules among different social groups at a local level; in stage 4 local elections are held; in stage 5 policy and transfers are implemented in accordance with the rules that will be described below. Transfers at a national level will be denoted with t^N and transfers at a local level with t .

During the national electoral campaign parties propose a vector of transfers $t^{N,k} = (t_1^{N,k}, t_2^{N,k})$, $k = L, R$. Transfers are subject to the constraint $\sum_{i=1}^2 \sum_{j=1}^2 N_{ij} \cdot t_j^{N,k} \leq 0$; $k = L, R$, where $t_j^{N,k}$ is the transfer proposed by party k to all the citizens of the country belonging to group j . The transfers implemented by the central government are $t^{N,L}$ iff $V_N^L \geq V_N^R$ in the national elections ($t^{N,R}$ otherwise).

During the local electoral campaign parties in region i propose a vector of transfers $t_i^k = (t_{i1}^k, t_{i2}^k)$, $k = L, R$. Transfers are subject to the budget constraint $\sum_{j=1}^2 N_{ij} \cdot t_{ij}^k \leq B_i + \sum_{j=1}^2 N_{ij} \cdot t_j^{N,k}$, $k = L, R$, where t_{ij}^k is the transfer proposed by party k to a citizen of region i and group j and B_i is an exogenous given amount available to government of region i for transfers.¹²

The policy implemented in each region i is determined according to the same rules as in the case of a federal government with transfers among regions; parties' objective functions are, as well, the same as in the previous setting.

A political equilibrium in a federal government with transfers among social groups is characterized by two proposed sets (one for each party) of transfers $t^{N,k} = (t_1^{N,k}, t_2^{N,k})$, two sets (for each region) of intra regional transfers $t_i^k = (t_{i1}^k, t_{i2}^k)$, $k = L, R$ and two sets of voting decision by citizens (both in the central both in the local elections) such that: in stage 3 (local electoral campaign), in each region, t_i^L (t_i^R) maximizes party L (R) objective function value given t_i^R (t_i^L), $t^{N,L}$ and $t^{N,R}$; in stage 1 (national electoral campaign) $t^{N,L}$ ($t^{N,R}$) maximizes party L (R) objective function given $t^{N,R}$ ($t^{N,L}$) and assuming agents' rational behavior in the following stages of the game; in stage 2 (national elections) and in stage 4 (local elections) every citizen Z votes for party L iff $U_Z(L) \geq U_Z(R)$, vice versa votes for party R .

Proposition 3. In federal governments with transfers among regions and social groups there exists a SPE, in which:

(1) *Equivalence result - A citizen of region i and group j receives a transfer*

$$T_{ij} = \frac{[k_j \times \phi_j(0)]^{1/\varepsilon}}{\sum_{j=1}^G \left\{ \sum_{i=1}^M N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon} \right\}} \times (Y + B) - y_j \tag{1}$$

¹²Note that, in fact, the amount of resources available to the regional government for transfers include $\sum_{j=1}^2 N_{ij} \cdot T_j^{N,k}$, the sum of the transfers operated by the central government towards citizens of region i .

where T_{ij} is the final monetary transfer (including transfers from local and central government), $\phi_j(0)$ is the density of group j at $X = 0$, $Y = \sum_i \sum_j N_{ij} \times y_j$ represents the total income of the country and $B = \sum_i B_i$;

(2) All citizens positioned on the real line of ideological positions at a point $X \leq 0$ vote for party L , all citizens positioned at $X > 0$ vote for R both in the elections for the central government and in the elections for the local one.

Proof: see Appendix I.

4. Discussion of the results and final remarks

The paper analyzed a model in which, in a federal country, redistribution may occur at different levels and towards different social groups or regions. Local and central governments interact in order to determine the implemented policy and the transfer schedule, parties compete in order to maximize their share of votes in local and central elections and citizens vote so to maximize their utility, given by consumption possibilities and implemented policies. In this basic model an equivalence result was obtained: for what concerns the distributional effects, the equilibria obtained in a context of centralized government and in a context of federal government with transfers (among regions or among social groups) are equivalent. That is: within these equilibria, given the same initial global endowment of resources, the transfer schedules implemented are the same.

Three assumptions were basic in the derivation of this result. First, the timing of the game is crucial. Given that the national electoral campaign precedes local ones, candidates to the central government have a higher commitment ability and can correctly anticipate the behavior of parties and citizens in the local electoral cycle. Since it is impossible for the winner of the national elections to influence local electoral results, the optimal behavior for candidates to the central government is to choose transfers so to maximize the total share obtained in the national elections (e.g.: the same as in the centralized government). Second, parties have the same redistributive abilities and differ only in their political position so that, in equilibrium, both parties propose the same set of transfers. Third, given the structure of citizens' political preferences and given that, in equilibrium, candidates propose the same set of transfers in both electoral cycles, sincere voting will always be optimal. This is due to the fact that, if equilibrium transfers are the same, everyone will vote for the preferred party in term of proposed policy platform.¹³

Comparing the equilibria obtained in the case of localized government two settings (localized and centralized) we observe that a citizen of region i and group j obtain higher consumption opportunities in a context of localization, with respect to centralization and federal government

institutional settings, if $\frac{Y_i + B_i}{Y + B} > \frac{\sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon}}{\sum_{i=1}^F \left\{ \sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon} \right\}}$.

Citizens living in high income regions tend to prefer, *ceteris paribus*, localization over centralization. Citizens living in regions with an higher value of $A = \sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon}$ tend to

¹³If the preferences were specified also over linear combinations of the two policies, a balancing effect (similar to divided government effect pointed out in ?) between central and local elections would have emerged, with someone voting strategically in the local election in order to promote a policy platform intermediate between L 's and R 's ones.

prefer centralization since regions with a greater "average" reactivity and with more swing voters are more attractive and are ideal recipients of tactical transfers. This is realistic and consistent with what commonly observed: poorer regions and "swing" regions tend to prefer more centralization while richer regions tend to push towards a more decentralized institutional arrangements.

At least three steps may lead to further interesting research. First, the introduction of asymmetries in the parties' ability to deliver transfers towards different social groups or different regions may lead to some kind of core support result, with different redistribution implications depending on the institutional setting adopted. Second, the theoretical model may be expanded so to include more developed mechanism of electoral campaign and policy determination rules. Finally, empirical analysis will permit to test the main theoretical results and allow to compare the main predictions of the literature on tactical transfers in a federal institutional setting.

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Appendix I - Proof of proposition III

Federal government with transfers among regions

Solving backward, in stage 4 a voter of region i and group j , with ideological position X , votes for party L iff $U_j(c_{ij}^L) - U_j(c_{ij}^R) \geq (1 - \gamma) \times X$.

We can determine the cutoff point for citizens of region i and group j , such that all citizens to the left will vote for L and all citizens to the right will vote for R in the local elections: $\hat{X}_{ij} = \frac{U_j(c_{ij}^L) - U_j(c_{ij}^R)}{(1 - \gamma)}$

During the local electoral campaign (stage 3) parties propose a set of (tactical) transfers so to maximize the share of votes in local election.

Party L solves the following maximization problem:

$$\text{Max}_{t_{i1}^L, t_{i2}^L} \sum_{j=1}^2 N_{ij} \times \Phi_j(\hat{X}_{ij})$$

subject to

$$\sum_{j=1}^2 N_{ij} \times t_{ij}^L = B_i + M_i$$

where M_i is the amount of transfers received by region i by central government and B_i is an exogenous given monetary amount available to the government of region i .

Party R problem is symmetric to party L one.

The context is similar to the one solved in the case of a fully localized government, except for the fact that the total sum available for transfers among citizens of region i is $B_i + M_i$. Therefore, following the same steps, it is possible to derive the optimal transfers and the final consumption schedule of a citizens living in region i and belonging to social group j . The consumption schedule, in particular, will be $C_{ij} = \frac{[k_j \times \phi_j(0)]^{1/\epsilon}}{\sum_{j=1}^2 N_{ij} [k_j \times \phi_j(0)]^{1/\epsilon}} \times (Y + B_i + M_i)$, where M_i is the transfer received by region i by the central government.

It is now possible to proceed with the analysis of the national electoral cycle (stage 1 and stage 2).

The cutoff point for citizens of region i and group j in stage 2 (national elections) is $\hat{X}_{ij} = \frac{U_j(c_{ij}^L) - U_j(c_{ij}^R)}{\gamma}$.

In stage 1 (national electoral campaign), parties aim at maximizing the nation wide share of votes (they cannot influence local elections results). Party L solves, then, the following problem:

$$\text{Max}_{M_1, M_2} \sum_{i=1}^2 \sum_{j=1}^2 N_{ij} \times \Phi_j(\hat{X}_{ij})$$

subject to

$$\sum_{i=1}^2 M_i \leq 0$$

Solving F.O.C.S. and looking for symmetric solutions we find that $M_i = \frac{\sum_{j=1}^2 [k_j \times \phi_j(0)]^{1/\epsilon}}{\sum_{i=1}^2 \left[\sum_{j=1}^2 N_{ij} [k_j \times \phi_j(0)]^{1/\epsilon} \right]} \times (Y + \sum_{i=1}^2 B_i) - \sum_{j=1}^2 N_{ij} Y_j - B_i$, where Y represents the income of all the citizens of the country.

This result, together with the equilibrium consumption derived in stage 3, proves the proposition.

Federal government with transfers among social groups

Solving backward, in stage 4 (local elections) a voter of region i and group j , with ideological position X , votes for party L iff $U_j(c_{ij}^L) - U_j(c_{ij}^R) \geq (1 - \gamma) \times X$.

We can determine the cutoff point for citizens of region i and group j , such that all citizens to the left will vote for L and all citizens to the right will vote for R : $\hat{X}_{ij} = \frac{U_j(c_{ij}^L) - U_j(c_{ij}^R)}{(1 - \gamma)}$.

In the local electoral campaign (stage 3) parties propose a set of (tactical) transfers so to maximize the share of votes in the local election.

Party L solves the following maximization problem:

$$\text{Max}_{T_{i1}^L, T_{i2}^L} \sum_{j=1}^2 N_{ij} \times \Phi_j(\widehat{X}_{ij})$$

subject to

$$\sum_{j=1}^2 N_{ij} \times t_{ij}^L \leq B_i + \sum_{j=1}^2 N_{ij} \times t_{ij}^{N,L}$$

Party R solves a similar maximization problem.

The problem is similar to the one solved in the case of fully localized government, except that the income of each citizen before local elections take place is $y_j + t_j^{N,k}$ where $t_j^{N,k}$ is the transfer performed by central government towards citizens of group j . Therefore, following the same steps, it is possible to derive the optimal transfers and the final consumption schedule of a citizen living in region i and belonging to social group j . The consumption schedule, in particular, will be

$$C_{ij} = \frac{[k_j \times \phi_j(0)]^{1/\varepsilon}}{\sum_{j=1}^G N_{ij} [k_j \times \phi_j(0)]^{1/\varepsilon}} \times (Y_i + B_i + \sum_{j=1}^G N_{ij} \times t_{ij}^N)$$

Proceeding with the analysis of the national electoral cycle (stage 1 and stage 2), the cutoff point for citizens of region i and group j in stage 2 (national elections) is $\widehat{X}_{ij} = \frac{U_j(C_{ij}^L) - U_j(C_{ij}^R)}{\gamma}$

In stage 1 (national electoral campaign, parties aim at maximizing the nation wide share of votes (they cannot influence local elections results). Party L solves the following problem:

$$\text{Max}_{T_1^L, T_2^L} \sum_{i=1}^2 \sum_{j=1}^2 N_{ij} \times \Phi_j(X_{ij})$$

subject to

$$\sum_{i=1}^2 \sum_{j=1}^2 N_{ij} \times T_j^L \leq 0$$

an party R solves

$$\text{Max}_{T_1^R, T_2^R} \sum_{i=1}^2 \sum_{j=1}^2 N_{ij} \times [1 - \Phi_j(X_{ij})]$$

subject to

$$\sum_{i=1}^2 \sum_{j=1}^2 N_{ij} \times t_j^{N,R} \leq 0$$

The solution of this maximization problem leads to the result. Since it involves some long algebraic steps we do not summarize it here, but it is available upon request.