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Cournot competition in spatial markets: a complementary result on complementarity

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Abstract

We study location equilibria for a symmetrical two-store duopoly selling differentiated products on the circle market. In contrast to the existing literature, we assume that each store sells a different product. We consider both complementarity among all varieties on the market and intra-firm complementarity but substitutability between rival varieties, and obtain multiple equilibria in each case. We thus remind that the assumptions on intra-firm as well as on inter-firm competition may be relevant for determining the equilibrium pattern in a spatial model with location choice and various types of product differentiation.

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1. Introduction

The earliest contributions to the spatial competition analysis endogenized location choice for single-store firms selling a homogenous product and competing in prices. It turned out that the location-price model cannot sustain spatial agglomeration, either on the linear (see d'Aspremont et al. (1979)) or the circular (see Kats (1995)) markets. Later, models studying location choice for Cournot rivals proved that the intensity of competition was determinant for this result. Anderson and Neven (1991) and Hamilton et al. (1989) established thus the central agglomeration on the segment. More recently, Pal (1998), Matsushima (2001) and Gupta et al. (2004) showed in turn that the shape of the market was equally important for the location outcome, since Cournot competitors cannot completely agglomerate on the circular market, but instead disperse, although they may cluster at several distinct locations.

Given the appealing properties of the Cournot spatial models, such as overlapping firm areas or agglomeration at discrete points, several papers have soon modified various assumptions of the initial framework. Pal and Sarkar (2002) proved for instance that competition among multi-store firms yields clustering of different firms' outlets on the segment[1]. Shimizu (2002) relaxed the product homogeneity assumption in a single-outlet duopoly framework and confirmed the central agglomeration result on the segment, but showed that the outcome on the circle depends on whether goods are complements or substitutes. Yu and Lai (2003) extended the analysis to a two-store duopoly, and found that on the circle firms agglomerate but stores disperse when rival products are complementary and own products are substitutes.

Building on Yu and Lai (2003), we further explore the implications of the product complementarity assumption for the location equilibrium of a spatial oligopoly on the circular city[2]. However, we consider one particular hypothesis which has not been yet used in Cournot shipping models, namely the one variety per plant assumption. Modelling firms selling two complementary varieties is realistic to the extent that often real-life firms simultaneously produce complementary goods, such as brick and cement, or operating systems and internet navigators. Moreover, intra-firm complementarity is the natural outcome of any vertical merger. Based on this assumption, we study first the case of complementarity among all varieties on the market, which is not only an 'extreme' extension of Yu and Lai (2003) complementarity assumption, but also the complete reversal of the usual hypothesis of homogenous

[1]For the analysis of the circular market see Chamorro-Rivas (2000), Cosnita (2005) and Pal and Sarkar (2006).

[2]The circular representation is appropriate for a number of real-life situations, such as circular towns spreading around lakes, for which consumerscannot afford to cross the lake when going shopping, and therefore department stores take up their locations around the lake. More generally speaking, this occurs for every traffic-jammed city: large shopping malls are located on the outskirts, on the circular belt-way, so as to avoid consumers the downtown traffic. Furthermore, the dial of a clock being a circle, the circular market can be equally used for competing television networks choosing time slots for their shows, or airlines choosing arrival and departure times for their flights.

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product in the spatial Cournot literature. Secondly, we tackle the opposite of Yu and Lai (2003) framework, i.e. the case of a duopoly characterized by intra-firm complementarity but inter-firm substitutability[3].

In the first case, the intuition that complementarity between all varieties induces total agglomeration is obvious. However, we equally identify an equilibrium pattern with intra-firm agglomeration and equidistant firm dispersion. Multiple equilibria obtain in the second case as well. More precisely, the spatial Cournot duopoly with two-product firms selling substitutable system goods exhibits both intra-firm agglomeration with equidistant firm dispersion, but also inter-firm clustering with intra-firm diametrical dispersion. Each time the alternative equilibrium is sustainable only for low product complementarity, consistent with the inter-firm competition effect 'dominating' (weighing more in the location choice than) the intra-firm one. Moreover, these alternative, less intuitive equilibria are each time only local maxima, whereas the predictable, intuitive equilibria are global maxima.

The main findings are thus the following. First, we check that the multiple equilibria property of the circular market extends to the case of multi-store competition with intra-firm complementarity, regardless of the assumption on inter-firm competition (i.e. substitutability or complementarity). This outcome underlines that results obtained for single-plant/product competition do not necessarily extend to multi-plant/product settings, and reminds the relevance of assumptions on plant/product-level (rather than firm-level) competition.

The remainder of the paper is organized as follows. Section 2 outlines the model and presents the complete complementarity case, whereas section 3 deals with the intra-firm complementarity and inter-firm substitutability case. Both are studied under standard linearity assumptions on cost and demand functions, which are used throughout the paper. The conclusion summarizes the results and compares them with those available so far in the literature. All detailed computations are grouped in a Technical Appendix available upon request.

2. Complete Complementarity

Let there be two firms competing in quantities on the unit circular market. Each firm owns two stores and each store delivers only one good. Assume that all goods (denoted 1, 2, 3 and 4) are symmetrically complementary among them[4]. Individual market demands at any

[3] The same setting of affiliates producing complementary goods, and rival stores producing subtitutes was retained by Tan and Yuan (2003) to examine the incentives to divisionalize of rival conglomerate firms, competing though in prices and in a non-spatial market.

[4] Arguably, this type of demand function is rarely considered in the literature, and one may wonder whether there may be any real world examples for it. One type of example is that of firms producing couples of intermediate goods, which are all used later on for the production of a more complex final good. Another example is that of a four-course meal that a customer may obtain by 'buying' two couples of dishes from two different restaurants, each proposing only two-dish combinations.

location x are linear and symmetric, and since each plant sells a different product, the inverse demand[5] will typically be given by: $P_i(x) = a - q_i(x) + bq_j(x) + bq_k(x) + bq_h(x)$, where $i, j, k, h \in \{1, 2, 3, 4\}$ and $i \neq j \neq k \neq h$ with a, $b > 0$ and independent of x. Consumers are uniformly distributed along the unitary perimeter of the market and consume all products.

We assume that each good is produced with the same technology exhibiting constant marginal costs, normalized to zero. Each firm incurs the same linear transport cost to ship the product to consumers' locations: $t|x - z|$, where z is the location from which the product is shipped[6]. For simplicity, let $t = 1$, or equivalently, let a be the transport-cost adjusted reservation price. Consumers have a prohibitive transport cost, preventing arbitrage, therefore firms can and will price discriminate across the set of spatially differentiated markets[7]. Given constant marginal delivery costs, a set of independent Cournot equilibria obtains for each location x. There are no set-up or location costs. The game we consider is two-stage, with firms choosing first locations (denoted x_i) and then competing in quantities. We look for the SPNE by backwards induction.

At the second stage, equilibrium quantities supplied at each market point by each firm are determined. Let the first firm (denoted "12") sell varieties 1 and 2, and the second firm (denoted "34") sell varieties 3 and 4. Firms' profits at each market point x write

$$\Pi_{12}(x) = (P_1(x) - c_1(x)) \cdot q_1(x) + (P_2(x) - c_2(x)) \cdot q_2(x) \tag{1}$$
$$\Pi_{34}(x) = (P_3(x) - c_3(x)) \cdot q_3(x) + (P_4(x) - c_4(x)) \cdot q_4(x) \tag{2}$$

where $c_i(x), i = 1, 2, 3, 4$ stands for the constant marginal delivery cost of product i to location x. Solving the simultaneous system of FOCs gives the equilibrium quantities supplied at each market point:

$$q_1^*(x) = \frac{(2c_1 - 2ab - 2a - 2bc_1 + 2bc_2 + bc_3 + bc_4 - b^2c_1 - b^2c_2 + b^2c_3 + b^2c_4)}{4b + 8b^2 - 4} \tag{3}$$
$$q_2^*(x) = \frac{(2c_2 - 2ab - 2a + 2bc_1 - 2bc_2 + bc_3 + bc_4 - b^2c_1 - b^2c_2 + b^2c_3 + b^2c_4)}{4b + 8b^2 - 4} \tag{4}$$
$$q_3^*(x) = \frac{(2c_3 - 2ab - 2a + bc_1 + bc_2 - 2bc_3 + 2bc_4 + b^2c_1 + b^2c_2 - b^2c_3 - b^2c_4)}{4b + 8b^2 - 4} \tag{5}$$
$$q_4^*(x) = \frac{(2c_4 - 2ab - 2a + bc_1 + bc_2 + 2bc_3 - 2bc_4 + b^2c_1 + b^2c_2 - b^2c_3 - b^2c_4)}{4b + 8b^2 - 4} \tag{6}$$

To ensure positive quantities for each store throughout the market, let $a > 2$ and $b < 0.5$.

[5] Such linear inverse demands obtain from the linear-quadratic utility function.

[6] The norm stands for the shorter distance of the two possible ways to ship goods along the circumference.

[7] This assumption basically defines the shipping model of spatial competition: it is an approximation of the case where transport cost for firms is far more important than that of consumers - in real life, this is what justifies distribution networks. In addition, this is compatible with the flexible manufacturing production systems (see Eaton and Schmitt (1994)), where the firm's basic product (its location) is customized at a cost (transport cost) to make it appropriate for a consumer.

At the first stage, in order to optimally locate their outlets, the duopolists maximize their overall profits with respect to store/product locations denoted x_1 and x_2, and x_3 and x_4 respectively:

$$\max_{x_1,x_2} \Pi_{12}(x_1, x_2; x_3, x_4) = \max_{x_1,x_2} \left(\int_0^1 (q_1^*(x_1, x_2; x_3, x_4; x))^2 \, dx + \int_0^1 (q_2^*(x_1, x_2; x_3, x_4; x))^2 \, dx \right) \tag{7}$$

$$\max_{x_3,x_4} \Pi_{34}(x_3, x_4; x_1, x_2) = \max_{x_3,x_4} \left(\int_0^1 (q_3^*(x_3, x_4; x_1, x_2; x))^2 \, dx + \int_0^1 (q_4^*(x_3, x_4; x_1, x_2; x))^2 \, dx \right) \tag{8}$$

In what follows, we will show that both total store agglomeration as well as intra-firm store agglomeration with maximal inter-firm dispersion may come out as equilibrium location patterns. Note that we neither intend to identify all equilibrium locations, nor show the non-existence of other equilibrium patterns. Our purpose is to show that multiple equilibria may arise, and we only test the optimality of two types of location patterns.

Assume first that $x_3 = x_4 = 0$ and let us show that firm 12 will cluster its stores at the same point that firm 34 does. Thanks to the symmetry of our setting, this is enough to ensure that $(0, 0, 0, 0)$ is an equilibrium. For this, we take the FOCs and SOCs w.r.t. x_1 and x_2 on the profit of firm 12, then check that the locations $x_1 = 0, x_2 = 0$ (or 1) satisfy them both. Then, still assuming that $x_3 = x_4 = 0$, we show by the same token that $x_1 = x_2 = 1/2$ may equally be a location equilibrium. It is intuitive to consider such an equilibrium candidate due to the fact that the quantity median is not unique on the circle market[8]. Two cases had to be discussed, depending on the relative position of x_1 and x_2: case 1) $x_2 \in [x_1, x_1 + 1/2]$, and case 2) $x_2 \in [x_1 + 1/2, 1]$. Indeed, given the intra-firm complementarity assumption, one cannot assume that firm 12 will locate its two stores within distinct disjoint half-circles, as is typically done when own products are substitutes[9]. All corresponding (and space-consuming) computations are displayed in the Technical Appendix available upon request, but we provide in Appendix 1 at the end of this research note the expressions of firm's 12 profit and the FOCs and SOCs in the first case mentioned above.

[8] In spatial models with location choice it has long been established that the FOCs simply translate what is called the quantity median property, i.e. total quantity sold by a plant to the left of its location needs to equal that to the right, if this location is to be optimal. The crucial difference between the linear and the circular frameworks is that on the segment, a firm's quantity median is unique, whereas on the circle it is not. Actually, for given competitors' locations, if a point μ on the circle satisfies the quantity median property for a firm, it is straightforward to see that the diametrically opposite location $\mu + 1/2$ does it too.

[9] With homogenous products, to minimize transport costs, firms supply to each location from the closest store only.

The results we obtain are the following: total agglomeration of all four stores is an equilibrium location pattern whatever the degree of product complementarity, whereas diametrical firm dispersion with intra-firm store agglomeration is also an equilibrium location pattern for low product complementarity, i.e. $b < 0.275$. Given the possibility of multiple equilibria, our analysis would be incomplete without a global optimality test: the payoff functions are not globally concave in our setting, therefore we need to check whether both equilibrium patterns represent global maxima. In the case of total store agglomeration, the profit of the firm 12 equals $\frac{1}{24(2b-1)^2}\left(12a^2 - 6a + 1\right)$, whereas in the second type of equilibrium it amounts to $\left(\frac{1}{24(2b-1)^2}\left(12a^2 - b - 6a + b^2 + 1\right)\right)$. Simple computations yield that total store agglomeration is a global maximum whereas intra-firm clustering with inter-firm dispersion is only a local maximum. To sum up, we obtain the following:

Result 1: *On the circular market, a two-product duopoly producing symmetrically complementary varieties and competing in quantities will exhibit a global-maximum location equilibrium consisting of the total agglomeration of stores regardless of the value of the differentiation parameter $b \in (0, 0.5)$. A local-maximum location equilibrium also exists if $b < 0.275$, involving intra-firm agglomeration with diametrical firm dispersion.*

Proof: See the Technical Appendix.

The first type of location pattern identified is clearly the extension of Yu and Lai (2003), since complementarity between a firm's products induces its own stores to cluster. Indeed, intra-firm complementarity means it is optimal to match a higher quantity of the other affiliate by a higher quantity of its own, so the best two stores owned by the same firm can do is share the same location. As a consequence, firms behave as single-store producers, and by the same token, given the complementarity between their respective outputs, they equally cluster.

Thus we are left to question why the alternative, diametrical pattern involves inter-firm dispersion, and especially why this is no longer possible for the whole range of the complementarity parameter. The answer is provided by the Best Reply functions. At plant level, it is straightforward to notice that own output increases more with the other affiliate's output than with the quantity of a rival outlet: for instance, $BR^1 = \frac{a-c_1}{2} + bq_2 + \frac{b}{2}q_3 + \frac{b}{2}q_4$. In other words, a firm's outlets/product lines value more the intra-firm complementarity than the inter-firm one. The latter can only be 'neglected' for low values of b, which can also be seen at firm level, by considering a firm's aggregate Best Reply: $BR^{12} = \frac{a-\frac{c_1+c_2}{2}}{1-b} + \frac{b}{1-b}\left(q_3 + q_4\right)$. Inter-firm complementarity can be 'neglected' only if this complementarity is not too strong ($b < 0.275$), since the coefficient $\frac{b}{1-b}$ is increasing with b, and thus can only be approximated with 0 for low enough values of b. To put it differently, there is no dispersion force in our setting, but two agglomerations forces, the intra-firm complementarity and inter-firm complementarity. Total store agglomeration necessarily obtains when both agglomeration forces are at work, whereas the second type of location pattern involving firm dispersion is explained by the lack of the second agglomeration force (and not by the presence of a dispersion one).

It is nevertheless obvious that for any positive value of b in the relevant range, total store agglomeration necessarily yields a higher profit for firms than diametrical dispersion. One might then easily conclude that the second location pattern we have identified, characterized by partial agglomeration with equidistant dispersion in which firms 'forget' about the complementarity with the rival products, is actually irrelevant. The point we make with this respect is the following: albeit a local maximum, this location pattern guarantees optimal locations for the firms. Therefore, if faced with some external constraint preventing them from occupying the globally-optimal locations, the firms will cope by resorting to the alternative, locally-optimal location equilibrium[10].

We go on next to look into the case of complementarity between own varieties and substitutability between rival ones, so as to further explore the implications of the intra-firm complementarity assumption.

3. Intra-firm Complementarity with Inter–firm Substitutability

In this section, we deal with the opposite framework to that considered by Yu and Lai (2003). More precisely, instead of considering two complementary varieties produced each by one rival firm in its own two stores, we assume instead that firms produce two substitutable system goods, meaning that a firm's own plants produce complementary products, and that the firm's couple of varieties is substitute for the rival's ones[11]. Keeping the same linearity assumptions and notations as before, let firm "12" ship varieties 1 and 2, complements, and let firm "34" ship products 3 and 4, also complements. However, the couples 1 and 3, and 2 and 4 are now perfect substitutes respectively. Therefore, the system of linear market demands is the following:

$$P_{13}(x) = a - (q_1(x) + q_3(x)) + b(q_2(x) + q_4(x)) \tag{9}$$
$$P_{24}(x) = a - (q_2(x) + q_4(x)) + b(q_1(x) + q_3(x)) \tag{10}$$

At the second stage of the game, firms' profits at each market point x write now

$$\Pi_{12}(x) = (P_{13}(x) - c_1(x)) \cdot q_1(x) + (P_{24}(x) - c_2(x)) \cdot q_2(x) \tag{11}$$
$$\Pi_{34}(x) = (P_{13}(x) - c_3(x)) \cdot q_3(x) + (P_{24}(x) - c_4(x)) \cdot q_4(x) \tag{12}$$

[10] By external constraint we refer to some existing regulation which prohibits firms from choosing identical locations. For instance, in France, chemist shops cannot agglomerate, but must obey a certain minimum distance between their respective locations, which depends upon the density of the population.

[11] A real-world example of such a situation is that of firms selling rival operation systems and internet navigators compatibe with them. Another example, in the vein of the one provided in the previous section, is that of restaurants providing rival two-course menus, composed of starters and main dish for instance.

where $c_i(x), i = 1,2,3,4$ stands for the constant marginal delivery cost of product i to location x. Solving the simultaneous system of FOCs gives the equilibrium quantities supplied at each market point:

$$q_1^*(x) = \frac{1}{3b^2-3}(2c_1 - ab - a - c_3 + 2bc_2 - bc_4) \quad (13)$$

$$q_2^*(x) = \frac{1}{3b^2-3}(2c_2 - ab - a - c_4 + 2bc_1 - bc_3) \quad (14)$$

$$q_3^*(x) = \frac{1}{3b^2-3}(2c_3 - ab - c_1 - a - bc_2 + 2bc_4) \quad (15)$$

$$q_4^*(x) = \frac{1}{3b^2-3}(2c_4 - ab - c_2 - a - bc_1 + 2bc_3) \quad (16)$$

where $a > 2$ and $b \in (0,1)$ to ensure positive quantities throughout the market.

As before, at the first stage, to optimally locate their outlets, the duopolists maximize their overall profits with respect to store locations denoted x_1 and x_2, and x_3 and x_4 respectively:

$$\max_{x_1,x_2} \Pi_{12}(x_1,x_2;x_3,x_4) = \max_{x_1,x_2}\left(\int_0^1 (q_1^*(x_1,x_2;x_3,x_4;x))^2\,dx + \int_0^1 (q_2^*(x_1,x_2;x_3,x_4;x))^2\,dx\right) \quad (17)$$

$$\max_{x_3,x_4} \Pi_{34}(x_3,x_4;x_1,x_2) = \max_{x_3,x_4}\left(\int_0^1 (q_3^*(x_3,x_4;x_1,x_2;x))^2\,dx + \int_0^1 (q_4^*(x_3,x_4;x_1,x_2;x))^2\,dx\right) \quad (18)$$

As before, our purpose is neither to identify all equilibrium locations, nor show the non-existence of other equilibrium patterns, but merely check that multiple equilibria may arise. Again, we are going to focus on two types of location patterns.

We assume first that $x_3 = x_4 = 0$ and show that firm 12 will cluster its stores at the diametrically opposite point. Thanks to the symmetry of our setting, this is enough to ensure that $(1/2, 1/2, 0, 0)$ is an equilibrium. For this, we take the FOCs and SOCs w.r.t. x_1 and x_2 on the profit of firm 12, then check that the locations $x_1 = x_2 = 1/2$ satisfy them both. We argue that the second type of location pattern will involve intra-firm dispersion and rival store agglomeration. For this, we assume that $x_3 = 0, x_4 = 1/2$ and check that $x_1 = 1/2, x_2 = 1$ may come out as a location equilibrium through the same first order and second order approach. Again, two cases had to be discussed, depending on the relative position of x_1 and x_2: case 1) $x_2 \in [x_1, x_1 + 1/2]$, and case 2) $x_2 \in [x_1 + 1/2, 1]$.

We obtain the following result: intra-firm agglomeration with equidistant firm dispersion is an equilibrium location pattern whatever the value of the intra-firm complementarity parameter b in the relevant range, but intra-firm diametrical dispersion with rival store agglomeration is also an equilibrium location pattern if $b < 0.171$. However, the profit comparison between these two types of equilibria reveals that the former is a global maximum whereas the second is only a local maximum. To sum up, we obtain the following:

Result 2: *A symmetric two-plant duopoly selling substitutable system goods and competing in quantities on the circular market has a global-maximum location equilibrium consisting of intra-firm agglomeration and inter-firm equidistant dispersion for all levels of the intra-firm complementarity parameter* $b \in (0, 1)$. *If if the complementarity parameter is low enough* $(b < 0.171)$, *then a local-maximum location equilibrium also exists, involving intra-firm diametrical dispersion and inter-firm agglomeration, i.e. each firm locating its stores at the opposite ends of the same diameter.*

Proof: See the Technical Appendix for the complete computations and a sketch of the proof in case 2). in Appendix 2 at the end of this note.

Following the basic intuition that firms behave as single-product entities because of the inter-firm complementarity, and given Pal's (1998) result of equidistant dispersion for firms selling substitutes on the circle, intra-firm agglomeration with equidistant dispersion of rival varieties is necessarily obtained in equilibrium. The second type of location pattern is less intuitive, and in order to answer the question why own products can disperse/differentiate and rival varieties can agglomerate/be identical although own products are complements but rival ones are substitutes, we turn again to the Best Reply functions. For instance, $BR^1 = \frac{a-c_1}{2} - \frac{1}{2}q_3 + bq_2 + \frac{b}{2}q_4$. Note that the only dispersion force stems from the direct substitutability between varieties 1 and 3, and that its intensity is constant. That is why, in both equilibrium patterns obtained, direct substitutes (such as 1 and 3, or 2 and 4) are always diametrically opposite. However, when b is very low (i.e. sufficiently close to 0), the direct and indirect complementarity effects (such as between varieties 1 and 2 and 1 and 4) are roughly equal. This explains the two alternative equilibria, exhibiting either intra-firm agglomeration (complementary varieties 1 and 2 sharing the same location) or inter-firm agglomeration (indirectly complementary varieties 1 and 4 sharing the same location).

As in the previous case of complete complementarity throughout the market, the more intuitive location pattern ensures a higher profit for each firm, because the direct, intra-firm complementarity weighs more in the profit function than the indirect complementarity w.r.t. one of the rival products. But as before, our point concerning the local maximum location pattern is that whenever the global one may be unattainable, due for instance to some industry-specific regulation, then the firms will be able to resort to an alternative profit-maximizing location pattern for their production/distribution outlets.

Note finally that although both our framework and Yu and Lai's (2003) similarly yield agglomeration between complementary varieties and equidistant dispersion of substitutable ones, the difference lies with the ownership pattern of the various stores. Indeed, a quick look at the Best Reply function in Yu and Lai (2003) case - take variety 1 for the sake of an easy comparison, $BR^1 = \frac{a-c_1}{2} - q_3 + \frac{b}{2}q_2 + \frac{b}{2}q_4$, reveals that the direct intra-firm substitutability is always dominant, so the equilibrium pattern necessarily involves intra-firm dispersion of own affiliates (differentiation of own products). Both the direct and the indirect complementarity (between varieties 1 and 2 or 1 and 4) weigh equally within the Best Reply function, but the two possible patterns resulting are basically the same, since they boil down to rival product lines agglomerating (either clustering between varieties 1 and 2, or between 1 and 4). In contrast, in our setting, once substitutable varieties disperse, the two alternatives left are not equivalent, to the extent that the first pattern involves clustering of strongly complementary products, whereas the second involves clustering of weakly complementary rival varieties.

4. Conclusion

Most of the papers on spatial competition with location choice have dealt with homogenous products, one of the reasons being that the spatial framework naturally yields product differentiation. Homogeneity is particularly important for Cournot spatial competition, because together with the strategic substitutability it typically generates dispersion. We contribute to the literature by tackling the issue of location choice on the circular market for two-store firms, and allowing each store to deliver a different product. We assume intra-firm complementarity and 'combine' it alternatively with complementarity between rival varieties or substitutability between them. In both settings, we show the existence of two types of location equilibria, albeit one being a global maximum and the other a local maximum. The latter is sustainable only for low product complementarity, but its existence is enough to extend the property of multiple equilibria of the circular market to this multi-product setting. Below we present a table summarizing our results and comparing them with those obtained so far in the literature. To sum up, this research note reminds that the assumptions on intra-firm as well as on inter-firm competition may be relevant for determining the equilibrium pattern in a spatial model with location choice and various types of product differentiation.

Table 1

Comparison of results – two-store duopoly

Assumptions	$(x^*_1,\ x^*_2,\ x^*_3,\ x^*_4)$
Intra-firm substitutability + inter-firm complementarity (Yu and Lai (2003))	(0,1/2,0,1/2)
Intra-firm + inter-firm complementarity (this research note)	(0,0,0,0) - global maximum (0,0,1/2,1/2) – local maximum
Intra-firm complementarity + inter-firm substitutability (this research note)	(0,0,1/2,1/2) – global maximum (0,1/2,1/2,1) – local maximum
Intra-firm + inter-firm substitutability (Chamorro-Rivas (2000))	(0,1/2,1/4,3/4)

9

References

Anderson, S.P. and Neven, D.J. (1991) "Cournot competition yields spatial agglomeration", *International Economic Review* 32, 793-808

Chamorro-Rivas, J.M. (2000) "Plant proliferation in a spatial model of Cournot competition", *Regional Science and Urban Economics* 30, 507-518

Cosnita, A. (2005) "Horizontal mergers in the circular city: a note", *Economics Bulletin* 12(7), 1-10

d'Aspremont, C., Gabszewicz, J. and Thisse, J.F. (1979) "On Hotelling's stability in competition" *Econometrica* 47, 1145-1150

Eaton, C.B. and Schmitt, N. (1994) "Flexible manufacturing and market structure", *American Economic Review* 84, 875–888

Gupta, B., Lai, F.C., Pal, D., Sarkar, J. and Yu, C.M. (2004) "Where to locate in a circular city", *International Journal of Industrial Organization* 22, 759-782

Hamilton, J.H., Thisse, J.F. and Weskamp, A. (1989) "Spatial discrimination: Bertrand versus Cournot in a model of location choice", *Regional Science and Urban Economics* 19, 87-102

Kats, A. (1995) "More on Hotelling's stability in competition", *International Journal of Industrial Organization* 13, 89-93

Matsushima, N. (2001) "Cournot competition and spatial agglomeration revisited", *Economics Letters* 73, 175-177

Pal, D. (1998) "Does Cournot competition yield spatial agglomeration?", *Economics Letters* 60, 49-53

Pal, D. and Sarkar, J. (2002) "Spatial competition among multi-store firms", *International Journal of Industrial Organization* 20, 163-190

Pal, D. and Sarkar, J. (2006) "Spatial Cournot Competition among Multi-Plant Firms in a Circular City", *Southern Economic Journal* 73(1), 246-258

Shimizu, D (2002) "Product differentiation in spatial Cournot markets", *Economics Letters* 76, 317-322

Tan, G. and Yuan, L. (2003) "Strategic incentives of divestitures of competing conglomerates", *International Journal of Industrial Organization* 21, 673-697

Yu, C.M. and Lai, F.C. (2003) "Cournot competition in spatial markets: some further results", *Papers in Regional Science* 82, 569-580

Appendix 1

Proof of Result 1 - Firm's 12 profit expression and the FOCs and SOCs in case 1).$x_2 \in [x_1, x_1 + 1/2]$:

$$\Pi_{12} = \Pi_1 + \Pi_2 = \int_0^{x_1} \left((q_1^*)^2 + (q_2^*)^2\right) dx + \int_{x_1}^{x_2} \left((q_1^*)^2 + (q_2^*)^2\right) dx$$
$$+ \int_{x_2}^{1/2} \left((q_1^*)^2 + (q_2^*)^2\right) dx + \int_{1/2}^{1/2+x_1} \left((q_1^*)^2 + (q_2^*)^2\right) dx$$
$$+ \int_{1/2+x_1}^{1/2+x_2} \left((q_1^*)^2 + (q_2^*)^2\right) dx + \int_{1/2+x_2}^{1} \left((q_1^*)^2 + (q_2^*)^2\right) dx$$

$$= \int_0^{x_1} \left(\begin{array}{c} \left(\frac{\left(2(x_1-x)-2ab-2a-2b(x_1-x)+2b(x_2-x)+bx+bx-b^2(x_1-x)-b^2(x_2-x)+b^2x+b^2x\right)}{4b+8b^2-4} \right)^2 \\ + \left(\frac{\left(2(x_2-x)-2ab-2a+2b(x_1-x)-2b(x_2-x)+bx+bx-b^2(x_1-x)-b^2(x_2-x)+b^2x+b^2x\right)}{4b+8b^2-4} \right)^2 \end{array} \right) dx$$

$$+ \int_{x_1}^{x_2} \left(\begin{array}{c} \left(\frac{\left(2(x-x_1)-2ab-2a-2b(x-x_1)+2b(x_2-x)+bx+bx-b^2(x-x_1)-b^2(x_2-x)+b^2x+b^2x\right)}{4b+8b^2-4} \right)^2 \\ + \left(\frac{\left(2(x_2-x)-2ab-2a+2b(x-x_1)-2b(x_2-x)+bx+bx-b^2(x-x_1)-b^2(x_2-x)+b^2x+b^2x\right)}{4b+8b^2-4} \right)^2 \end{array} \right) dx$$

$$+ \int_{x_2}^{1/2} \left(\begin{array}{c} \left(\frac{\left(2(x-x_1)-2ab-2a-2b(x-x_1)+2b(x-x_2)+bx+bx-b^2(x-x_1)-b^2(x-x_2)+b^2x+b^2x\right)}{4b+8b^2-4} \right)^2 \\ + \left(\frac{\left(2(x-x_2)-2ab-2a+2b(x-x_1)-2b(x-x_2)+bx+bx-b^2(x-x_1)-b^2(x-x_2)+b^2x+b^2x\right)}{4b+8b^2-4} \right)^2 \end{array} \right) dx$$

$$+ \int_{1/2}^{1/2+x_1} \left(\begin{array}{c} \left(\frac{\left(2(x-x_1)-2ab-2a-2b(x-x_1)+2b(x-x_2)+b(1-x)+b(1-x)-b^2(x-x_1)-b^2(x-x_2)+b^2(1-x)+b^2(1-x)\right)}{4b+8b^2-4} \right)^2 \\ + \left(\frac{\left(2(x-x_2)-2ab-2a+2b(x-x_1)-2b(x-x_2)+b(1-x)+b(1-x)-b^2(x-x_1)-b^2(x-x_2)+b^2(1-x)+b^2(1-x)\right)}{4b+8b^2-4} \right)^2 \end{array} \right) dx$$

$$+ \int_{1/2+x_1}^{1/2+x_2} \left(\begin{array}{c} \left(\frac{1}{4b+8b^2-4}\right)^2 \left(\begin{array}{c} 2(1-x+x_1)-2ab-2a \\ -2b(1-x+x_1)+2b(x-x_2)+b(1-x)+b(1-x) \\ -b^2(1-x+x_1)-b^2(x-x_2)+b^2(1-x)+b^2(1-x) \end{array} \right)^2 \\ + \left(\frac{1}{4b+8b^2-4}\right)^2 \left(\begin{array}{c} 2(x-x_2)-2ab-2a \\ +2b(1-x+x_1)-2b(x-x_2)+b(1-x)+b(1-x) \\ -b^2(1-x+x_1)-b^2(x-x_2)+b^2(1-x)+b^2(1-x) \end{array} \right)^2 \end{array} \right) dx$$

$$+ \int_{1/2+x_2}^{1} \left(\begin{array}{c} \left(\frac{1}{4b+8b^2-4}\right)^2 \left(\begin{array}{c} 2(1-x+x_1)-2ab-2a \\ -2b(1-x+x_1)+2b(1-x+x_2)+b(1-x)+b(1-x) \\ -b^2(1-x+x_1)-b^2(1-x+x_2)+b^2(1-x)+b^2(1-x) \end{array} \right)^2 \\ + \left(\frac{1}{4b+8b^2-4}\right)^2 \left(\left(\begin{array}{c} 2(1-x+x_2)-2ab-2a \\ +2b(1-x+x_1)-2b(1-x+x_2)+b(1-x)+b(1-x) \\ -b^2(1-x+x_1)-b^2(1-x+x_2)+b^2(1-x)+b^2(1-x) \end{array} \right) \right)^2 \end{array} \right) dx$$

$$= \left(-\frac{1}{24}\right)(2b-1)^{-2}(b+1)^{-2} \left(\begin{array}{c} 6a-2b+12ab-24bx_1x_2-12a^2-b^2 \\ +6ab^2-24a^2b+18bx_1^2+8bx_1^3+18bx_2^2-24bx_2^3 \\ +48bx_1x_2^2-48bx_1^2x_2+36b^2x_1x_2-6b^4x_1x_2-12a^2b^2-12b^2x_1^2 \\ -32b^2x_1^3-12b^2x_2^2-6b^3x_1^2+16b^2x_2^3+8b^3x_1^3-6b^3x_2^2 \\ -3b^4x_1^2+8b^3x_2^3+12b^4x_1^3-3b^4x_2^2+4b^4x_2^3-72b^2x_1x_2^2 \\ +72b^2x_1^2x_2+12b^4x_1x_2^2-12b^4x_1^2x_2-1 \end{array} \right)$$

$$\frac{\partial \Pi_{12}}{\partial x_1} = \left(-\frac{1}{4}\right)(2b-1)^{-2}(b+1)^{-2} \left(\begin{array}{c} 6x_1-4x_2-4bx_1+6bx_2-16x_1x_2+24bx_1x_2 \\ +4x_1^2+8x_2^2-16bx_1^2-2b^2x_1-12bx_2^2-b^3x_1 \\ -b^3x_2-4b^3x_1x_2+4b^2x_1^2+6b^3x_1^2+2b^3x_2^2 \end{array} \right) b$$

$$\frac{\partial \Pi_{12}}{\partial x_2} = \left(-\frac{1}{4}\right)(2b-1)^{-2}(b+1)^{-2}\begin{pmatrix} 6x_2 - 4x_1 + 6bx_1 - 4bx_2 + 16x_1x_2 - 24bx_1x_2 \\ -8x_1^2 - 12x_2^2 + 12bx_1^2 + 8bx_2^2 - 2b^2x_2 - b^3x_1 \\ -b^3x_2 + 4b^3x_1x_2 + 4b^2x_2^2 - 2b^3x_1^2 + 2b^3x_2^2 \end{pmatrix} b$$

$$\frac{\partial^2 \Pi_{12}}{\partial x_1^2} = \frac{1}{4}(2b-1)^{-2}(b+1)^{-2}\begin{pmatrix} 4b - 8x_1 + 16x_2 + 32bx_1 - 24bx_2 + 2b^2 \\ +b^3 - 8b^2x_1 - 12b^3x_1 + 4b^3x_2 - 6 \end{pmatrix} b$$

$$\frac{\partial^2 \Pi_{12}}{\partial x_2^2} = \left(-\frac{1}{4}\right)(2b-1)^{-2}(b+1)^{-2}\begin{pmatrix} 16x_1 - 4b - 24x_2 - 24bx_1 + 16bx_2 - 2b^2 \\ -b^3 + 8b^2x_2 + 4b^3x_1 + 4b^3x_2 + 6 \end{pmatrix} b$$

Appendix 2

Sketch of Proof of Result 2 in case 2).$x_2 \in [x_1 + 1/2, 1]$:

Assume $x_3 = 0, x_4 = 1/2$ and let us show that $x_1 = 1/2, x_2 = 0$ may come out as a location equilibrium:

$$\Pi_{12} = \Pi_1 + \Pi_2 = \int_0^{x_1}\left((q_1^*)^2 + (q_2^*)^2\right)dx + \int_{x_1}^{x_2-1/2}\left((q_1^*)^2 + (q_2^*)^2\right)dx + \int_{x_2-1/2}^{1/2}\left((q_1^*)^2 + (q_2^*)^2\right)dx$$
$$+ \int_{1/2}^{x_1+1/2}\left((q_1^*)^2 + (q_2^*)^2\right)dx + \int_{x_1+1/2}^{x_2}\left((q_1^*)^2 + (q_2^*)^2\right)dx + \int_{x_2}^{1}\left((q_1^*)^2 + (q_2^*)^2\right)dx$$
$$= \int_0^{x_1}\begin{pmatrix} \left(\frac{1}{3b^2-3}\left(2(x_1 - x) - ab - a - x + 2b(1 - x_2 + x) - b(1/2 - x)\right)\right)^2 \\ + \left(\frac{1}{3b^2-3}\left(2(1 - x_2 + x) - ab - a - (1/2 - x) + 2b(x_1 - x) - bx\right)\right)^2 \end{pmatrix} dx$$
$$+ \int_{x_1}^{x_2-1/2}\begin{pmatrix} \left(\frac{1}{3b^2-3}\left(2(x - x_1) - ab - a - x + 2b(1 - x_2 + x) - b(1/2 - x)\right)\right)^2 \\ + \left(\frac{1}{3b^2-3}\left(2(1 - x_2 + x) - ab - a - (1/2 - x) + 2b(x - x_1) - bx\right)\right)^2 \end{pmatrix} dx$$
$$+ \int_{x_2-1/2}^{1/2}\begin{pmatrix} \left(\frac{1}{3b^2-3}\left(2(x - x_1) - ab - a - x + 2b(x_2 - x) - b(1/2 - x)\right)\right)^2 \\ + \left(\frac{1}{3b^2-3}\left(2(x_2 - x) - ab - a - (1/2 - x) + 2b(x - x_1) - bx\right)\right)^2 \end{pmatrix} dx$$
$$+ \int_{1/2}^{x_1+1/2}\begin{pmatrix} \left(\frac{1}{3b^2-3}\left(2(x - x_1) - ab - a - (1 - x) + 2b(x_2 - x) - b(x - 1/2)\right)\right)^2 \\ + \left(\frac{1}{3b^2-3}\left(2(x_2 - x) - ab - a - (x - 1/2) + 2b(x - x_1) - b(1 - x)\right)\right)^2 \end{pmatrix} dx$$
$$+ \int_{x_1+1/2}^{x_2}\begin{pmatrix} \left(\frac{1}{3b^2-3}\left(2(1 - x + x_1) - ab - a - (1 - x) + 2b(x_2 - x) - b(x - 1/2)\right)\right)^2 \\ + \left(\frac{1}{3b^2-3}\left(2(x_2 - x) - ab - a - (x - 1/2) + 2b(1 - x + x_1) - b(1 - x)\right)\right)^2 \end{pmatrix} dx$$
$$+ \int_{x_2}^{1}\begin{pmatrix} \left(\frac{1}{3b^2-3}\left(2(1 - x + x_1) - ab - a - (1 - x) + 2b(x - x_2) - b(x - 1/2)\right)\right)^2 \\ + \left(\frac{1}{3b^2-3}\left(2(x - x_2) - ab - a - (x - 1/2) + 2b(1 - x + x_1) - b(1 - x)\right)\right)^2 \end{pmatrix} dx$$
$$= \left(-\frac{1}{54}\right)(b+1)^{-2}(b-1)^{-2}\begin{pmatrix} 6a - 11b + 12ab + 24x_2 - 96bx_1 + 48bx_2 + 288bx_1x_2 \\ -12a^2 - 6b^2 + 6ab^2 - 24a^2b - 12x_1^2 + 16x_1^3 - 36x_2^2 + 16x_2^3 \\ -120bx_1^2 - 96bx_1^3 - 72bx_2^2 + 24b^2x_2 + 32bx_2^3 - 192bx_1x_2^2 + 192bx_1^2x_2 \\ -12a^2b^2 - 12b^2x_1^2 + 16b^2x_1^3 - 36b^2x_2^2 + 16b^2x_2^3 - 6 \end{pmatrix}$$

The FOCs write:

$$\frac{\partial \Pi_{12}}{\partial x_1} = \frac{4}{9}(b+1)^{-2}(b-1)^{-2}\begin{pmatrix} 4b + x_1 + 10bx_1 - 12bx_2 - 16bx_1x_2 \\ -2x_1^2 + 12bx_1^2 + b^2x_1 + 8bx_2^2 - 2b^2x_1^2 \end{pmatrix}$$

$$\frac{\partial \Pi_{12}}{\partial x_2} = \left(-\frac{4}{9}\right)(b+1)^{-2}(b-1)^{-2}\begin{pmatrix} 2b - 3x_2 + 12bx_1 - 6bx_2 - 16bx_1x_2 \\ +b^2 + 2x_2^2 + 8bx_1^2 + 4bx_2^2 - 3b^2x_2 + 2b^2x_2^2 + 1 \end{pmatrix}.$$

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It is straightforward to check that both FOCs are satisfied for $x_1 = 1/2, x_2 = 1$.
The SOCs:
$\frac{\partial^2 \Pi_{12}}{\partial x_1^2} = \left(-\frac{4}{9}\right)(b+1)^{-2}(b-1)^{-2}(4x_1 - 10b - 24bx_1 + 16bx_2 - b^2 + 4b^2x_1 - 1)$
$\frac{\partial^2 \Pi_{12}}{\partial x_2^2} = \frac{4}{9}(b+1)^{-2}(b-1)^{-2}(6b - 4x_2 + 16bx_1 - 8bx_2 + 3b^2 - 4b^2x_2 + 3)$,
and yield, respectively, for $x_1 = 1/2, x_2 = 1$: $\begin{bmatrix} -\frac{4}{9(b-1)^2(b+1)^2}(-6b + b^2 + 1) \\ \frac{4}{9(b-1)^2(b+1)^2}(6b - b^2 - 1) \end{bmatrix}$.
It is easy to check that $\frac{4}{9(b-1)^2(b+1)^2}(6b - b^2 - 1) < 0$ for any $b < 0.171\,57$.