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Impact of Returns Time Dependency on the Estimation of Extreme Market Risk

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Abstract

The estimation of Value-at-Risk generally used models assuming independence. However, fi nancial returns tend to occur in clusters with time dependency. In this paper we study the impact of negligence of returns dependency in market risk assessment. The main methods which take into account returns dependency to assess market risk are: Declustering, Extremal index and Time series-Extreme Value The- ory combination. Results shows an important reduction of the estimation error under dependency assumption. For real data, methods which take into account returns dependency have generally the best performances.

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1. Introduction

The extreme value theory (EVT) and Pickands approximation give interesting results concerning modeling and estimating financial extreme risk under the assumption of independence of financial returns. In reality, financial research shows that returns tend to occur in clusters. Hence, it is interesting to identify the impact of negligence of returns dependence on market risk estimation. Therefore, the purpose of this work is to use techniques from EVT to estimate market risk under both independent (i.i.d) and dependent financial returns assumptions. The methods based on EVT which measure the market risk with the VaR (*Value-at-Risk*) are the following: Extremal index (Longin 2000), Declustering (Hsing 1987) and Time series and EVT combination (Mc Neil and Frey 2000). In all what follows; it is assumed that a financial asset is studied, its daily returns on a period of n days are denoted $R_{t_1}; R_{t_2}; ...; R_{t_n}$. VaR($\alpha$) denote the Value at Risk of this asset at confidence level $1-\alpha$ with a predictible horizon of one day. VaR(α) is defined by $\Pr(R_{t+1} < -VaR_t(\alpha)/\mathcal{H}_t) \leq \alpha$. where $\mathcal{H}_t$ denotes all the information available at time t.

In this paper, we try to answer the following questions:

1- What is the impact of negligence of returns dependence on risk measurement?

2- How does wrong specification of the model affect the measure of risk?

3- How do the main methods for assessing risk for dependent returns compare?

This paper is organized as follows; section 2 describes the methods taking into account dependence of extreme returns; in section 3 the methodology is presented with an application to simulated and real data. In the last section we summarize our findings and conclude.

2. Dependence modeling

In reality, financial returns often tend to occur in clusters, the i.i.d hypothesis in financial data was widely criticized. The main methods taking into account the time dependence of extreme financial returns are briefly presented in this section. Those methods are Extremal index, Declustering and Time series-EVT combination. In this paper, we model extremal risk in finance with a special focus on portfolio losses. Therefore we consider only observations lower than a given threshold. VaR is a used tool for assessing financial market risk characterized by a level α and an horizon 1 day.

2.1 Extremal index

The extremal index is a quantity, presented in Leadbetter et al. (1983), characterizing the relationship between the dependence structure of data and their extremal behavior.

Definition 1. *: Let $(R_n)_{n\geq 0}$ be a strictly stationary sequence and F is the marginal distribution function, M_n is the maximum of $R_1, ..., R_n$ and θ a non negative number. Assume that for every $\tau > 0$ there exists a sequence (u_n) such that:*
$$\begin{cases} \lim\limits_{n\to+\infty} [n(1-F(u_n))] = \tau. \\ \lim\limits_{n\to+\infty} [\Pr(M_n \leq u_n] = \exp(-\theta\tau). \end{cases}$$
Then θ is called the extremal index of the sequence $(R_n)_{n\geq 0}$.

□

The extremal index should be in [0,1]: $0 \leq \theta \leq 1$. If $\theta = 1$ financial returns are independent, if $\theta = 0$ financial returns are strongly dependent.

The extremal index was interpreted according to different viewpoints: the extremal index was introduced by Longin (2000) in financial risk, Hsing, Husler and Leadbetter (1988) showed that extremal index is a reciprocal of the mean cluster size. The Extremal index estimator used in this article is the one proposed by Ferro and Segers (2003). The estimation of the extremal index is introduced in the VaR in order to estimate the VaR under dependence assumption with the following formula:

$$VaR(\alpha) = \mu + \frac{\hat{\zeta}}{\hat{\sigma}}[1 - (-\ln \alpha^{\hat{\theta} * n}))^{\hat{\zeta}} \qquad (1)$$

where $(\hat{\zeta}, \hat{\sigma})$ are the estimates of GPD parameters.

2.2 Declustering

In order to estimate the market risk of extreme value, it is necessary to model the minima of returns under a specific threshold. Declustering method consists on dividing data into clusters. Theorem 4.5 of Hsing (1987) shows that clusters of exceedance may be considered asymptotically independent. Hence, the minimums of cluster are approximately i.i.d. The two classical methods used to identify clusters are the Blocks and Runs methods (for details see Leadbetter et al. 1989).

2.3 Time series-EVT combination

Time series-EVT combination method was developed by Mc Neil and Frey (2000), they suggest that the appropriate model of financial returns is a stationary one with stochastic volatility, this implies the dependence in data is modeled by:

$$R_t = \hat{\mu}_t + \hat{\sigma}_t Z_t \qquad (2)$$

where: $\hat{\mu}_t$ and $\hat{\sigma}_t$ are deducted respectively from AR-ARMA and ARCH-GARCH appropriate model and Z_t is the residual of the time series.

The idea behind this method consists in eliminating dependence in data by time series. Under i.i.d assumption of residuals we estimate VaR of residuals then the returns VaR is computed (for more details see Mc Neil and Frey 2000).

VaR for real data was estimated by the model AR(1)-GARCH(1,1)-GPD with the following formula:

$$\begin{cases} VaR_\alpha = \hat{\mu}_{t+1} + \hat{\sigma}_{t+1} VaR_\alpha(Z) & (3) \\ VaR_\alpha(Z) = \vartheta + \frac{\hat{\xi}}{\hat{\sigma}} \left[\left(\frac{1-\alpha}{\frac{m}{n}} \right)^{-\hat{\xi}} - 1 \right] & (4) \end{cases}$$

$$\begin{cases} \hat{\mu}_{t+1} \text{ deducted by AR(1) model} \\ \hat{\sigma}_{t+1} \text{ deducted by GARCH(1, 1) model} \\ Z \text{ residual i.i.d.} \end{cases}$$

AR(1)-GARCH(1, 1) parameters are estimated using “QML” (Quasi-Maximum likelihood) estimators. The GPD parameters are estimated with “PWM” (Probability Weighted Moment) estimators proposed by Hosking et Wallis (1987).

3. An application to simulated and real data

The main purpose of this empirical part is to detect the impact of negligence of returns time dependence in market risk. Diebold (1996) suggested that simulation should be used to test the pertinence of risk assessment methods based on extreme value theory. Declustering, Extremal index and Time series-EVT combination are applied on simulated data from random i.i.d variables for normal and Student distributions, dependent data with AR(1) and ARMA(1,1) processes, Brownian geometrical motion with constant volatility (BMCV) and Brownian geometrical motion with stochastic volatility (BMSV). Those methods are also applied on real data.

3.1 Simulated data

In this subsection we present the procedure of VaR estimation and the calculation of approximate real VaR for the simulated data according the following steps.

- **First step**: Data are simulated according to the following models: normal, Student i.i.d observations, AR(1) and ARMA(1,1) process, Brownian motion with constant and stochastic volatility. We simulate N data sets ($N = 1000$), every one is composed by n ($n = 1000$) observations.
- **Second step**: Estimate the VaR(α) where $(1 - \alpha)$ is equal to 99.9%; 99%; 97.5%; 95%; 90% under i.i.d and dependence assumptions.
- **Third step**: calculate the real VaR (or approximated real) denoted $VaRr(\alpha)$ as follows:

1- Normal return's $VaRr(\alpha) = \Phi^{-1}(\alpha)$, where Φ^{-1} denotes reverse Normal distribution function.

2- Student return's $VaRr(\alpha) = F_{\nu}^{-1}(\alpha)\sqrt{\frac{\nu-2}{\nu}}$, where F_{ν}^{-1} denotes reverse Student distribution function and ν freedom degrees ($\nu = 6$ is chosen here).

3- $AR(1)$ return's such as $AR(1)$: $R_t = \varphi R_{t-1} + \xi_t$, the $VaRr(\alpha) \simeq \Phi^{-1}(\alpha) \sum_{i=1}^{m-1} \varphi^{2i}$.

4- For $ARMA(1,1)$, BMCV and BMSV return's $VaRr(\alpha)$ are approximated according simulation method by the following steps:

* We simulate $N = 1000$ data set containing $n = 1000$ observations.

* For every data set we consider only the last observation R_{1000}, orders statistics of these last observations are denoted: $R_{(1000)}^{(1)} \leq R_{(1000)}^{(2)} \leq ... \leq R_{(1000)}^{(1000)}$.

VaR(α) is then approximated by the empirical quantile of the previous order statistics.

$$VaRr(\alpha) = R_{(1000)}^{(1000*\alpha)}.$$

- **Fourth step**: In order to determine the performance of each method we propose the use of Percentage Absolute Relative Error (PARE) as a measure of difference between an approximation of real VaR ($VaRr$) and estimated VaR ($VaRe$).

$$PARE = 100 * \frac{1}{N} \sum_{i=1}^{N} \mid \frac{VaRe_i - VaRr_i}{VaRr_i} \mid \qquad (5)$$

where $VaRe_i$ is the estimation of $VaR(\alpha)$ and $VaRr_i$ is an approximation of the real $VaR(\alpha)$ related to simulated data set number i.

3.1.1 Results and interpretation

Table I of PARE shows that the i.i.d assumption gives a low PARE level for data simulated according to i.i.d normal and student distributions. This result is predictable because data here are simulated under i.i.d assumption. For example, for $N(0.1)$ process the amplitude of error is about 22.7% and 20% for $t(6)$ process for $\alpha = 0.1\%$. When data

α	0.1%	1%	2.5%	5%	10%
i.i.d N(0.1)	22.7%	31%	23.3%	48.9%	46.3%
i.i.d t(6)	20%	23.2%	27.5%	29.6%	23.5%
BMCV	96.6%	95.2%	93.5%	90.86%	84.4%
BMSV	98%	97%	96%	93%	88%
AR(1)	96.4%	81.49%	88.6%	102.5%	163.7%
ARMA(1,1)	95.5%	83.5%	83%	119%	185%

Table I: Estimated VaR inder independence for simulated data.

α	0.1%	1%	2.5%	5%	10%
i.i.d N(0.1)	129%	105.9%	112.6%	136.2%	109.9%
i.i.d t(6)	93%	97%	98.5%	100.8%	104.5%
BMCV	25%	23%	21%	27%	29%
BMSV	11%	18%	15%	20%	19%
AR(1)	26%	16%	10%	11%	15%
ARMA(1,1)	7.7%	13%	20.7%	10.4%	16.6%

Table II: Estimated VaR inder dependence assumption with declustring method for simulated data.

are simulated from BMCV, BMSV, AR(1) and ARMA(1,1) processes, the differences between real and approximated VaR become very important when i.i.d based method is used to estimate VaR. Relative errors may reach 185%. This results showed that i.i.d hypothesis used for estimating VaR from dependent returns give generally huge errors.

Under the dependence assumption with Declustering method (see table 2), PARE gives a high level for $N(0.1)$ and $t(6)$ process. PARE is respectively about 112.6% and 98.5% for $\alpha = 2.5\%$ compared with 23.3% and 27.5% obtained with i.i.d method for VaR estimation we remark that the dependence assumption gives an important value of errors. However for dependent observations BMCV, BMSV, AR and ARMA process, for $\alpha = 2.5\%$ PARE is respectively about 11%, 15%, 10.1% and 20.7%. In general, PARE is under 26% for all α levels. This result shows that dependence assumption must be considered when we estimate extreme financial risk.

Under the dependence assumption with Time series-EVT combination method (see table III), we remark the same results. PARE is lower than 50% for dependent data and higher than 92% for independent ones.

Under the dependence assumption with Extremal index method (see table IV), we remark the same results. PARE is lower than 30% for dependent data and higher than 80% for independent ones.

The i.i.d assumption gives a best VaR estimation according to i.i.d simulated data, the dependence assumption gives a best VaR estimation with Declustering and extremal index method. Therefore we must use a test of independance like Sprearman test in order to use the appropriate method and allocate the appropriate capital to cover the financial risk.

Table V summarizes the best method. We remark that i.i.d assumption is the best method for i.i.d data, for BMCV and BMSV extremal index and Declustring are the best one and for AR(1) and ARMA(1,1) process Time series-EVT combination was absent in

α	0.1%	1%	2.5%	5%	10%
i.i.d N(0.1)	99.3%	105.3%	98.3%	92.2%	93%
i.i.d t(6)	97.2%	95.2%	102.3%	98.2%	95.3%
BMCV	36.2%	35%	31.2%	34.9%	33.4%
BMSV	39%	37.5%	33.4%	34.2%	35.1%
AR(1)	41.2%	22.9%	28.5%	28.9%	39%
ARMA(1,1)	53.7%	52.9%	44.3%	27.9%	32%

Table III: Estimated VaR inder dependence assumption with Time series-EVT combination method for simulated data.

α	0.1%	1%	2.5%	5%	10%
i.i.d N(0.1)	105.6%	125.4%	148.8%	107.3%	127.6%
i.i.d t(6)	87%	83%	81%	80%	84%
BMCV	25%	22%	21%	18%	12%
BMSV	23%	21%	20%	18%	15%
AR(1)	17.2%	9.8%	30.4%	25.4%	39%
ARMA(1,1)	22%	28%	18%	20.5%	30%

Table IV: Estimated VaR inder dependence assumption with extremal index method for simulated data.

α	0.1%	1%	2.5%	5%	10%
i.i.d N(0.1)	i.i.d	i.i.d	i.i.d	i.i.d	i.i.d
i.i.d t(6)	i.i.d	i.i.d	i.i.d	i.i.d	i.i.d
BMCV	decl/ex-ind	ex-ind	decl/ex-ind	ex-ind	ex-ind
BMSV	decl	decl	decl	decl	ex-ind
AR(1)	ex-ind	ex-ind	decl	ex-ind	decl
ARMA(1,1)	decl	decl	ex-ind	ex-ind	decl

Table V: Best VaR method for simulated data.

this table.

3.2 Real data

In order to estimate market risk we chose to apply these methods to Tunisian Stock Exchange index (Tunindex) data during the period from January first, 1998 to September 31, 2002, French CAC40 index during the period from January 22, 2000 to July 2, 2006, SP500 index during the period from January 06, 2000 to February, 25, 2006 and NASDAQ index during the period from july 03, 1999 to January, 7, 2006. The daily returns are defined as $R_i = \ln(\frac{X_i}{X_{i-1}}) * 100$, where X_i is the daily closing value on day i.

According to these index, Jarque Berra test shows that the daily financial returns are far from normality. Based on Kurtosis estimates we argue that returns distribution are fat-tailed.

For real data we are faced to a problem of estimating real VaR. In order to measure the performance of each method. Therefore we adopt a sliding window with a size of 1000 observations. The relative performance of each model is summarized by a "Violation Ratio" denoted by (VR). A Violation occurs if the realized return is smaller than the estimated VaR in a giving day. The Violation Ratio is defined as the total number of violations, divided by the total number of one step a head forecasts (Gençay and Selçuk 2002).

- If $VR > \alpha \Longrightarrow$underestimation of the risk.
- If $VR < \alpha \Longrightarrow$ overestimation of the risk.
- If $VR = \alpha \Longrightarrow$ appropriate model for the risk estimation.

3.2.1 Results and interpretation

Violation Ratio of extreme financial risk for Tunindex, CAC40, SP500 and NASDAQ index are presented respectively in tables VI, VII, VIII and IX. Violation Ratio values should be compared to α. For Tunindex when VaR is estimated under i.i.d assumption Violation Ratio (VR) is Higher than α, for example for $\alpha = 1\%, VR = 5.2\%$. However, under dependence assumption VR is approximately equal to α for example with Declustering method when $\alpha = 1\%$, $VR = 1.6\%$. When comparing α with VR we remark that Time series-EVT combination is the best method.

For CAC40 index when VaR is estimated under i.i.d assumption Violation Ratio (VR) is Higher than α level, for example for $\alpha = 0.1\%, VR = 2.3\%$. However, under dependence assumption VR is approximately equal to α . When comparing α with VR we remark that Declustering method is the best method.

For SP500 NASDAQ index when VaR is estimated under i.i.d assumption Violation Ratio (VR) is higher than α level, for all α values. However, under dependence assumption VR is approximately equal to α. When comparing α with VR we remark that Time series-EVT combination is the best method.

The most important result (presented in table X) is that the dependence assumption reduces the difference between real and estimated VaR. This shows that the negligence of dependence on extreme risk market estimation affects financial risk estimation. This work showed that the model AR(1)-GARCH(1,1)-GPD is the most appropriate pattern for modeling extreme market risk. This method combines econometric models of volatility and extreme value models; it takes into account the time dependence and detects extreme events.

α	*0.1%*	*1%*	*2.5%*	*5%*	*10%*
i.i.d	*7.4%*	*5.2%*	*7.3%*	*9.9%*	*13.4%*
Declustering	*0.6%*	*1.6%*	**3.2%**	**5.7%**	*10.9%*
Extremal index	*0.8%*	*2.8%*	*3.4%*	*7.8%*	*11.1%*
Time-series EVT combination	**0.12%**	**1.5%**	*4.1%*	*6.1%*	**10.6%**

Table VI: Violation Ratio for Tunindex data. *Note : The bold number present the best VR value according these methods for every* α

α	*0.1%*	*1%*	*2.5%*	*5%*	*10%*
i.i.d	2.3%	*7.2%*	*5.9%*	*8.7%*	*14.1%*
Declustering	**0.3%**	**1.3%**	*3.9%*	**5.3%**	*10.4%*
Extremal index	*0.4%*	*2.2%*	**3.2%**	*6.2%*	**10.2%**
Time-series EVT combination	**0.3%**	*1.4%*	*3.3%*	**5.3%**	*10.3%*

Table VII: Violation Ratio for CAC40 data. *Note : The bold number present the best VR value according these methods for every* α

α	*0.1%*	*1%*	*2.5%*	*5%*	*10%*
i.i.d	*12.2%*	*10.5%*	*9.4%*	*10.7%*	*14.6%*
Declustering	*4.6%*	*6.5%*	**3.2%**	*7.5%*	*9.1%*
Extremal index	**1.8%**	*2%*	*4.2%*	*9.7%*	**10.1%**
Time-series EVT combination	2.12%	**1.5%**	**3.2%**	**5.1%**	*8.65%*

Table VIII: Violation Ratio for SP500 data. *Note : The bold number present the best VR value according these methods for every* α

α	*0.1%*	*1%*	*2.5%*	*5%*	*10%*
i.i.d	*10.7%*	*15.2%*	*16.23%*	*19%*	*14.1%*
Declustering	*0.6%*	*6.1%*	**2.2%**	**4.1%**	**9.6%**
Extremal index	*0.5%*	*1.4%*	**2.2%**	*3.1%*	*9.1%*
Time-series EVT combination	**0.10%**	**1.2%**	*3.1%*	*3.4%*	**9.6%**

Table IX: Violation Ratio for NASDAQ data. *Note : The bold number present the best VR value according these methods for every* α

α	*0.1%*	*1%*	*2.5%*	*5%*	*10%*
Tunindex	*Time-series*	*Time-series*	*decl*	*decl*	*Time-series*
CAC40	*decl/Time-series*	*decl*	*ex-ind*	*decl/Time-series*	*ex-ind*
SP500	*ex-ind*	*Time-series*	*decl/Time-series*	*Time-series*	*ex-ind*
NASDAQ	*Time-series*	*Time-series*	*ex-ind/decl*	*decl*	*decl/Time-series*

Table X: Best VaR method for real data.

4. Conclusion

In this paper, it has been shown that methods which take into account dependency have an impact in market risk estimation by VaR. Under i.i.d assumption there is a huge difference between real and estimated VaR. However, under dependence assumption Time-series EVT combination, Declustering and extremal index methods are the best. The AR(1)-GARCH (1,1)-GPD is the most appropriate pattern for modeling extreme market risk.

For the simulated data, the amplitude of error is about 280% for BMCV with $\alpha = 0.1\%$, this error induces an allocation of high amount of capital to cover a low level of risk. According to real data VR shows that in general time series-EVT combination method is the best. There fore wesuggest that practitioners must test the independence of extreme returns to have an idea of the used hypothesis. Under dependence assumption there is not a clear method to adopt but when we use a Time series-EVT combination we must take care when chosing of the adequate model. This error induces an allocation of high amount of capital to cover a low level of risk

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