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### Optimal fiscal policy with social status and productive government expenditure

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#### Abstract

We examine the optimal tax rule in an endogenous growth model with public capital and wealth-enhanced social status. When government expenditure is productive, the equality of marginal productivities of private and public capital holds. Wealth-induced preference can violate this equality, since the marginal utility from private capital is also a value of private capital. We obtain the optimal fiscal policy in which the positive income tax rate is higher than the subsidy rate for saving but is lower than the tax rate in the case without social-status preference.

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## 1 Introduction

We examine the optimal tax rule in an endogenous growth model with public capital as in Barro (1990) and wealth-enhanced social status.

An economy in this paper is similar to Tamai's (2008) base model in which labor is inelastic. He shows that the positive values of the income tax rate and tax deduction rate can be set such that the growth rate in the decentralized economy equals the socially optimal growth rate.<sup>1</sup> We check the robustness of this result by assuming the social-status preference as in Chang (2006), Clemens (2004) and Zou (1998). This implies that individuals accumulate wealth not only for consumption, but also for its own sake. Consequently, the agent's preference should depend on one's wealth (private capital) holdings as well as one's consumption.<sup>2</sup>

Although many literature about the optimal tax exists as Turnovsky (1997), Gomez (2004) and so on, there are few papers discussing the relation between social-status preference and the optimal fiscal policy. Chang (2006) investigates optimal fiscal policy in the economy with social status, but he focuses on consumption tax, additive-separable utility and the production function with only private capital and without government spending. He shows that an optimal consumption tax policy provides full subsidies to consumption so as to induce the economy to achieve the social optimum and the optimal growth rate, and that an increase in the consumption tax always raises the long-term growth rate of the endogenous growth model with wealth-enhanced status preferences.<sup>3</sup>

As shown in Tamai (2008), when government expenditure is productive, the equality of marginal productivities of private and public capital is critical for the result that the positive tax rate of income equals to the subsidy rate for saving. We present that wealth-induced preference in which consumption and private capital are not necessarily additive separable can violate this equality, since the marginal utility from private capital is also a value of private capital. It generates the higher economic growth by promoting capital accumulation,

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<sup>1</sup>He additionally shows that this result is robust in the economy with endogenous labor.

<sup>2</sup>There has been a growing literature that investigates macroeconomic effects of agents' wealth-induced preferences for social status within monetary models of capital accumulation and economic growth, such as Chang et. al. (2000), Chen and Guo (2011), Gong and Zou (2001), Hosoya (2002) and so on. It may be interesting and important to examine monetary models with social-status in the future.

<sup>3</sup>Itaya et. al. (2010) revises his model by endogenizing labor and assuming non-separable utility with capital. In contrast to Chang (2006), they show that a rise in consumption taxation may or may not stimulate long-run growth, depending on whether the balanced growth path (BGP) is determinate or indeterminate and/or on the magnitude of social status concern.

and thus we obtain the optimal fiscal policy in which the positive income tax rate is higher than the subsidy rate for saving but is lower than the tax rate in the case without social-status preference.

## 2 Social Optimum

We set up an economy based on Tamai (2008) with social status of private capital. Social planner maximizes representative household's life-time utility

$$\int_0^{\infty} \frac{[c(k/K^\theta)^\beta]^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \quad 0 \leq \theta \leq 1, \quad \beta > 0, \quad \sigma > 0, \quad (1)$$

subject to

$$\dot{k} = y - c - i_g, \quad (2)$$

$$\dot{g} = i_g, \quad (3)$$

where  $c$  is consumption,  $\rho$  the time discount rate,  $\sigma$  the inverse of the elasticity of intertemporal substitution,  $\beta$  the weight on utility toward the holdings of capital stock,  $\theta$  the importance of society's average wealth relative to an individual's wealth,  $k$  individual's private capital,  $K$  average stock of private capital, and  $i_g$  the investment in public capital  $g$ . Additionally, the function of production  $y$  is

$$y = k^{1-\eta} g^\eta, \quad 0 < \eta < 1. \quad (4)$$

The first-ordered conditions are

$$c^{-\sigma} \left( \frac{k}{K^\theta} \right)^{\beta(1-\sigma)} = \lambda_k, \quad (5)$$

$$\lambda_k = \lambda_g, \quad (6)$$

$$\dot{\lambda}_k = \left[ \rho - \frac{(1-\eta)y}{k} \right] \lambda_k - \beta \frac{c^{1-\sigma}}{k} \left( \frac{k}{K^\theta} \right)^{\beta(1-\sigma)}, \quad (7)$$

$$\dot{\lambda}_g = \left[ \rho - \frac{\eta y}{g} \right] \lambda_g, \quad (8)$$

together with the transversality conditions

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_{kt} k_t = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_{gt} g_t = 0,$$

where  $\lambda_k$  and  $\lambda_g$  respectively denote the implicit values of private capital and public one. Combining these conditions and imposing symmetry ( $k = K$ ), we derive

$$\beta z = \eta x^{-(1-\eta)} - (1-\eta)x^\eta, \quad (9)$$

where  $z \equiv \frac{c}{k}$  and  $x \equiv \frac{g}{k}$ . Equation (9) represents the non-arbitrage condition between private and public capital. When  $\beta = 0$ , that is, there is no social-status preference, the real rates of return on these capital are equivalent and thus  $x = \frac{\eta}{1-\eta}$  is constant as in Tamai(2008). Otherwise, the condition implies that the marginal productivity of public capital should be higher and thus  $x \in \left(0, \frac{\eta}{1-\eta}\right)$  is decreasing in  $z$ , because marginal utility from the social status is also a value of private capital.

In the following, we focus on the balanced growth path  $(z^*, x^*)$  such that  $\frac{\dot{z}}{z} = \frac{\dot{x}}{x} = 0$ . Denoting  $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = u^*$  as the balanced growth rate, we describe

$$u^* = \frac{(1-\eta)(x^*)^\eta + \beta z^* - \rho}{\sigma - \beta(1-\sigma)(1-\theta)} = \frac{\eta(x^*)^{-(1-\eta)} - \rho}{\sigma - \beta(1-\sigma)(1-\theta)} \equiv F(x^*) \quad (10)$$

from (5)-(8). Moreover, using (2), (3), and (9), we obtain

$$u^* = \frac{(x^*)^\eta - z^*}{1+x^*} = \frac{\left(\frac{\beta+1-\eta}{\beta}\right)(x^*)^\eta - \frac{\eta}{\beta}(x^*)^{-(1-\eta)}}{1+x^*} \equiv G(x^*). \quad (11)$$

In order to satisfy  $u^* > 0$ , we assume that

$$\sigma > \frac{\beta(1-\theta)}{1+\beta(1-\theta)} \quad \text{and} \quad \frac{\eta}{\beta+1-\eta} < x^* < \min\left[\frac{\eta}{1-\eta}, \left(\frac{\eta}{\rho}\right)^{\frac{1}{1-\eta}}\right].$$

On the  $(x^*, u^*)$  plane,  $F(x^*)$  in (10) has a downward slope and  $G(x^*)$  (11) is increasing in  $x^*$ . Therefore, the value set of  $(x^*, z^*, u^*)$  is uniquely determined although it is difficult to obtain the specific form of these values. Now, we investigate the effect of status parameters.

If  $(1-\theta)$ , the importance of individual's private capital relative to aggregate private capital, becomes higher, the curve  $F(x^*)$  shifts clockwise (resp. counter-clockwise) if  $\sigma < 1$  (resp.  $\sigma > 1$ ), but the curve  $G(x^*)$  does not move. Therefore,  $x^*$  and  $u^*$  increase and  $z^*$  decreases when  $\sigma < 1$ , and vice versa under  $\sigma > 1$ . In the case of  $\sigma = 1$  or  $\beta = 0$ ,  $\theta$  does not affect on the balanced-growth rate.

When  $\beta$  rises,  $F(x^*)$  moves the same direction as  $\theta$  becomes smaller. On the other hand,  $G(x^*)$  becomes steeper. We find that the balanced-growth rate  $u^*$  is higher under  $\sigma \leq 1$ , but that the effect on  $u^*$  is not clear when  $\sigma > 1$ .

Table 1: The Effects of Social Status on Economic Growth

an increase in $(1 - \theta)$	$x^*$	$z^*$	$u^*$	an increase in $\beta$	$x^*$	$z^*$	$u^*$
$0 < \sigma < 1$ and $\beta > 0$	+	-	+	$0 < \sigma < 1$ and $1 < \theta$	?	?	+
$\sigma = 1$ or $\beta = 0$	0	0	0	$\sigma = 1$ or $\theta = 1$	-	+	+
$\sigma > 1$ and $\beta > 0$	-	+	-	$\sigma > 1$ and $1 < \theta$	-	+	?

+: higher, 0: zero effect, -: lower, ?: ambiguous

We can intuitively understand these results summarized in Table 1 and Figure 1 as follows. Under  $\sigma < 1$ , a rise in  $\beta$  or  $(1 - \theta)$  means higher implicit values of private and public capital. Therefore, capital accumulation and thus economic growth are accelerated. Additionally,  $x^*$  decreases with  $\beta$  from the non-arbitrage condition (9) if  $z^*$  is fixed. Then, the marginal productivity of public capital rises and that of private capital falls. However, the former effect is superior if  $\beta > 0$ , since the real rate of return on public capital is higher than the one on private capital when social-status exists. Therefore, the growth rate of income becomes much higher.

### 3 A Decentralized Economy

The budget constraint of the representative household is

$$\dot{k} = (1 - \tau)y - c + s\dot{k}, \tag{12}$$

where  $\tau$  and  $s$  respectively denote the income tax rate and the subsidy rate for saving. This can be rewritten as

$$\dot{k} = \frac{(1 - \tau)y - c}{1 - s}. \tag{13}$$

The household maximizes his lifetime utility (1) subject to (13). Optimal conditions are

$$c^{-\sigma} \left( \frac{k}{K^\theta} \right)^{\beta(1-\sigma)} = \frac{\lambda}{1 - s}, \tag{14}$$

$$\dot{\lambda} = \left[ \rho - \frac{1 - \tau(1 - \eta)y}{1 - s} \frac{1}{k} \right] \lambda - \beta \frac{c^{1-\sigma}}{1 - s} \left( \frac{k}{K^\theta} \right)^{\beta(1-\sigma)}, \tag{15}$$

together with the transversality conditions

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t k_t = 0,$$

where  $\lambda$  represents the implicit values of private capital in the decentralized economy.

Defining  $u^{**}$  as the balanced growth rate such that  $\frac{\dot{z}}{z} = \frac{\dot{x}}{x} = 0$  in this decentralized economy and assuming the symmetry ( $k = K$ ) again, we acquire the relation

$$u^{**} = \frac{1}{\sigma - \beta(1 - \sigma)(1 - \theta)} \left[ \frac{1 - \tau}{1 - s} (1 - \eta)(x^{**})^\eta + \frac{\beta z^{**}}{1 - s} - \rho \right], \quad (16)$$

where  $x^{**}$  and  $z^{**}$  are the corresponding values under the balanced growth. Additionally, the budget constraint of the government is

$$\dot{g} = \tau y - s\dot{k}. \quad (17)$$

Along the balanced-growth path where  $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = u^{**}$ , the income tax rate is given as

$$\tau = \frac{((x^{**})^\eta - z^{**})(s + x^{**})}{(1 + x^{**})(x^{**})^\eta} \quad (18)$$

from (13) and (17).

Using (10) and (16), we can derive

$$(1 - \eta) \left[ \frac{1 - \tau}{1 - s} (x^{**})^\eta - (x^*)^\eta \right] + \beta \left[ \frac{z^{**}}{1 - s} - z^* \right] = 0, \quad (19)$$

and we obtain the condition for optimal fiscal policy such that  $x^* = x^{**}$  and  $z^* = z^{**}$  from (9), (18) and (19):<sup>4</sup>

$$\tau = \left[ \frac{\eta/(1 - \eta)}{x^*} \right] s. \quad (20)$$

If  $\beta = 0$ ,  $x^* = \frac{\eta}{1 - \eta}$  holds and thus  $\tau = s > 0$  as in Tamai (2008). Otherwise, the following propositions summarize the properties of optimal fiscal policy:

**Proposition 1** *Combining (18) and (20), we obtain the optimal fiscal policy such that*

$$\begin{aligned} \tau(x^*) &= \frac{\eta[(\beta + 1 - \eta)x^* - \eta]}{\eta(\beta + 1 - \eta) + [\eta\beta - (1 - \eta)(\beta + 1 - \eta)]x^*} \\ &= \frac{\eta}{1 - \eta} \frac{s(x^*)}{x^*} > s(x^*) > 0. \end{aligned} \quad (21)$$

<sup>4</sup>A detailed process to attain the social optimum via the fiscal policy is described in some literature as in Gomez (2004) and Tamai (2008). Moreover, note that  $\tau = s = 0$  is not optimal although  $x^* = x^{**}$ ,  $z^* = z^{**}$  and (19) can be satisfied, since the balanced-growth rate becomes zero against the assumption  $u^* > 0$ .

**Proposition 2** Since  $\frac{\partial \tau(x^*)}{\partial x^*} > 0$  and  $\frac{\partial s(x^*)}{\partial x^*} > 0$ ,

$$\text{sign}\left[\frac{\partial \tau(x^*)}{\partial \theta}\right] = \text{sign}\left[\frac{\partial s(x^*)}{\partial \theta}\right] = \text{sign}\left[\frac{\partial x^*}{\partial \theta}\right], \quad (22)$$

but the effect of  $\beta$  on fiscal policy is analytically ambiguous. Additionally,  $\frac{\partial \tau(x^*)}{\partial x^*} > 0$  implies that  $\tau(x^*)$  is lower than the optimal tax rate in the case without social-status preference.

Accumulation of private capital is promoted by the wealth-enhanced utility. Therefore, the tax rate of income should be higher than the subsidy rate for saving in order to balance the government budget constraint. On the other hand, accelerated capital accumulation makes economic growth higher, and thus the tax rate of income needed for ensuring the government revenue is lower than the tax rate in the case without social-status preference.

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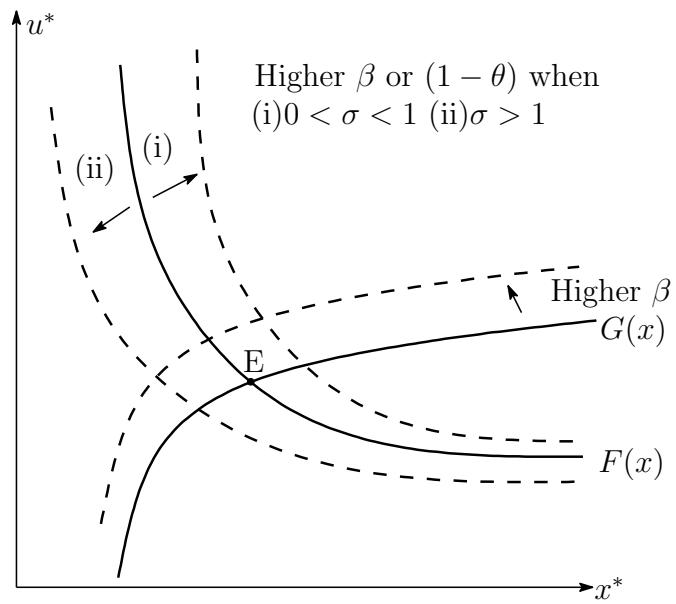


Figure 1: Comparative Statics of Social Status under Social Optimum

An intersection point E displays an unique balanced growth rate.