Economics Bulletin

Volume 32, Issue 1

Stock index Value-at-Risk forecasting: A realized volatility extreme value theory approach

Dimitrios P. Louzis
Bank of Greece / Athens University of Economics and Business

Spyros Xanthopoulos - Sissinis
Athens University of Economics and Business

Apostolos P. Refenes
Athens University of Economics and Business

Abstract

In this study, we propose the use of Heterogeneous Autoregressive (HAR) type realized volatility models in combination with the Extreme Value Theory (EVT) method for Value-at-Risk (VaR) forecasting. The proposed model accounts for the long memory property of the realized volatility and the fat tails of the returns distribution. The out-of-sample forecasting results, based on the S&P 500 stock index, indicate that the HAR-type-EVT models outperform their GARCH-type counterparts in terms of statistical and regulatory accuracy as well as capital efficiency. The HAR-GARCH-EVT model, which also accounts for the conditional heteroscedasticity of the HAR errors, is the overall best performing model as it generates accurate VaR estimates that minimize the Basel II regulatory capital during both the full out-of-sample period and the 2007-2009 crisis period.

Citation: Dimitrios P. Louzis and Spyros Xanthopoulos - Sissinis and Apostolos P. Refenes, (2012) "Stock index Value-at-Risk forecasting: A realized volatility extreme value theory approach", *Economics Bulletin*, Vol. 32 No. 1 pp. 981-991.
Contact: Dimitrios P. Louzis - dlouzis@aueb.gr, Spyros Xanthopoulos - Sissinis - xanthopo@aueb.gr, Apostolos P. Refenes - arefenes@aueb.gr.
Submitted: December 05, 2011. **Published:** March 26, 2012.

1. Introduction

Financial assets' market risk is commonly measured by the so called Value-at-Risk (VaR), which is defined as the potential asset's value loss over a prespecified holding period and for a predetermined confidence level. Since financial volatility is a key input in measuring and forecasting VaR, a plethora of volatility models have been proposed and tested in the VaR literature.

Recently, the realized volatility (RV hereafter), i.e. the sum of squared intra-daily returns, which is an efficient and consistent estimator of the latent volatility (Andersen and Bollerslev, 1998; Andersen *et al*. 2001), was implemented in VaR studies. The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model, which accounts for the long memory property of the RV, is the most frequently used RV model in the VaR literature (e.g. see Giot and Laurent, 2004; Angelidis and Degiannakis, 2008 amongst others). An alternative approximate long memory RV model, is the Heterogeneous Autoregressive (HAR) model of Corsi (2009). Despite its good volatility forecasting performance, the HAR model has received limited attention in VaR applications (Martens *et al*., 2009; Clements *et al*., 2008).

Moreover, most of the authors in the realized volatility - VaR literature account for the fat tails and the asymmetry of the assets returns' density by utilizing either the fat tailed t-student distribution (Clements et al., 2008) or the skewed student (skst) distribution, which considers both of the abovementioned characteristics (Giot and Laurent, 2004; Angelidis and Degiannakis, 2008).

In this study, we differentiate form previous works in the VaR field and we propose the use of HAR–type RV models in conjunction with the conditional Extreme Value Theory (EVT) (McNeil and Frey, 2000). To our knowledge, this is the first time that the EVT VaR method is combined with the informational content of high frequency intra-daily data incorporated in a RV model. Our empirical analysis, using thirteen years of the S&P 500 stock index from 1997 to 2009, indicates that the proposed specification can provide statistical and regulatory accurate VaR estimates that minimize the Basel II capital reserves. We also show that the proposed model outperform its GARCH-type counterparts. These results are also confirmed during the highly volatile 2007-2009 period.

The remaining of the paper is organized as follows. Section 2 describes the HAR-type-EVT VaR models, while Section 3 briefly presents the VaR evaluation metrics. In Section 4, we present the empirical results. Section 5 concludes this article.

2. The Realized Volatility Extreme Value Theory VaR Model

2.1. The AR(1)-HAR-GARCH(1,1) model

The HAR model of Corsi (2009) uses daily, weekly and monthly realized volatility components in an autoregressive structure in order to approximate the persistence in realized volatility. Moreover, Corsi et al. (2008) account for the conditional heteroscedasticity of the HAR errors, by implementing a GARCH error process. Assuming that $r_t = \log\left(P_t / P_{t-1}\right)$ are the daily returns, the AR(1)-logarithmic HAR-GARCH(1,1) model is defined as:

Conditional mean: $r_t = c + \phi_1 r_{t-1} + \sqrt{h_t} z_t$, with $z_t \sim$ i.i.d N(0,1) (1)

Conditional variance: $h_t = g\widehat{RV}_{t|t-1}$ (2)

HAR-GARCH model: $lrv_t^{(d)} = a_0 + a_{(d)} lrv_{t-1}^{(d)} + a_{(w)} lrv_{t-1}^{(w)} + a_{(m)} lrv_{t-1}^{(m)} + u_t$ (3)

$u_t = \sigma_{u,t}\upsilon_t$ and $\sigma_{u,t}^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{u,t-1}^2$ (4)

where $\upsilon_t \sim N(0,1)$, $lrv_t^{(d)} = \log\left(RV_t\right)$, $RV_t = \left[\left(\sigma_{oc}^2 + \sigma_{co}^2\right)/\sigma_{oc}^2\right]\sum_{j=1}^{M}\left(r_{t,j}\right)^2$ is the realized variance with $r_{t,j}$ being the j-th intraday return, σ_{oc}^2 and σ_{co}^2 are the "open-to-close" and "close-to-open" sample variances respectively that account for the overnight returns bias (Martens, 2002) and $lrv_t^{(h)} = \left(1/h\right)\left(lrv_t + lrv_{t,t-1} + lrv_{t,t-2} + \ldots + lrv_{t,t-h+1}\right)$ with $h = w = 5$ and $h = m = 22$ are the weekly and monthly volatility components respectively. We model the conditional mean as an AR(1) process in order to account for any autocorrelation in the returns series, while the conditional variance is modeled as a fraction of the estimated conditional realized variances, i.e. $\widehat{RV}_{t|t-1}$.

The parameters in Equations (1)-(4) are estimated in two steps using maximum likelihood techniques (Giot and Laurent, 2004). In the first step, we estimate the HAR-GARCH model parameters and we obtain the conditional realized variance estimates as $\widehat{RV}_{t|t-1} = \exp\left(lrv_t - \hat{u}_t + 0.5\hat{\sigma}_{u,t}^2\right)$. In the second step, we use the $\widehat{RV}_{t|t-1}$ and we estimate the conditional variance parameter, g, and the AR(1) parameters. This implementation ensures that z_t is a unit variance process.

2.2. The Extreme Value Theory VaR method

In statistical terms next day's VaR is defined as $\Pr\left(r_{t+1} \le VaR_{t+1}^{\alpha} \middle| \mathcal{F}_t\right) = 1 - \alpha$ or $VaR_{t+1}^{\alpha} = F_{t+1}^{-1}\left(\alpha\right)$, where α is the significance level and F^{-1} denotes the inverse cumulative distribution function of the returns. For the returns process described in Equations (1)-(4) tomorrow's VaR is given by:

$$\widehat{VaR}_{t+1}^{\alpha} = \hat{c} + \hat{\phi}_1 r_t + \sqrt{\hat{g}\widehat{RV}_{t+1|t}}\, F_z^{-1}\left(\alpha\right) \tag{5}$$

Here, we estimate the α^{th} quantile of the innovations, i.e. $\hat{z}_t = \left(r_t - \hat{c} - \hat{\phi}_1 r_{t-1}\right)/\sqrt{\hat{g}\widehat{RV}_{t|t-1}}$, using the conditional EVT method of McNeil and Frey (2000). The method models the $\hat{z}_t$ which exceed a prespecified threshold U. If the magnitude of exceedence of z over U is defined as $y_i = z_i - U$, where $i = 1,\ldots,T_U$ and T_U being the total number of exceedences, then the distribution of z is given by:[1]

[1] For the choice of U we follow Chan and Gray (2006)

$$F_z(z) = 1 + \frac{T_U}{T}\left(F_U(y) - 1\right) \tag{6}$$

where $F_U(y) = \Pr\{z - U \le y \mid z > U\}$. A key result in EVT is that for a sufficiently high threshold U, the $F_U(y)$ converges to the Generalized Pareto Distribution (GPD) which is defined as:

$$G_{\zeta,\beta}(y) = \begin{cases} 1 - \left(1 + (\zeta/\beta)\, y\right)^{-1/\zeta} & \text{if } \zeta \neq 0 \\ 1 - \exp(-y/\beta) & \text{if } \zeta = 0 \end{cases}, \text{ for } 0 \le y \le z - U \tag{7}$$

where ζ and $\beta > 0$ are the shape and scale parameters respectively. Heavy tailed distributions correspond to $\zeta > 0$ which implies that we expect a positive ζ, since most of the financial time series exhibit fat tails. Therefore, for $\zeta \neq 0$ and $1 + \zeta \frac{y}{\beta} > 0$ the α^{th} quantile of $F_z(z)$ is given by:

$$F_z^{-1}(\alpha) = U + (\beta/\zeta)\left[\left((T/T_U)\,\alpha\right)^{-\zeta} - 1\right] \tag{8}$$

We obtain the estimates of ζ and β by maximizing the GPD log-likelihood function:

$$L_g(\zeta, \beta) = -T_U \log(\beta) - \left(1 + (1/\zeta)\right) \sum_{i=1}^{T_U} \log\left(1 + (\zeta/\beta)\, y_i\right) \tag{9}$$

3. VaR Evaluation Measures

The VaR evaluation measures implemented here build on the "failure process" described by the following indicator function $I_t = \mathcal{I}_{\left\{r_{t+1:t+h} < VaR^{a}(t+h|t)\right\}}$, which takes the value of 1 if $r_{t+1:t+h} < VaR^{\alpha}(t+h|t)$ and zero otherwise. We expect that an accurate VaR model will generate a failure rate (FR) i.e. $\hat{\alpha} = n_1/n$, where n_1 and n are the number of exceptions and the sample size respectively, close to the predetermined coverage level, α.

Christoffersen's (1998) unconditional coverage test examines statistically if $\hat{\alpha} = \alpha$. Under the null hypothesis of accurate unconditional coverage, i.e. $E(I_t) = \alpha$ and given the assumption of independence between the exceptions, the likelihood ratio (LR) test is:

$$LR^{uc} = 2\log\left((1-\hat{\alpha})^{n_0}\, \hat{\alpha}^{n_1} / (1-\alpha)^{n_0}\, \alpha^{n_1}\right) \sim \chi^2(1) \tag{10}$$

The complementary conditional coverage test proposed by Christoffersen (1998) is a joint test of correct unconditional coverage and first order independence of the failure process against a first order Markov failure. The corresponding LR test is:

$$LR^{cc} = 2\log\left(\left(1-\hat{p}_{01}\right)^{n_{00}}\ \hat{p}_{01}^{n_{01}}\left(1-\hat{p}_{11}\right)^{n_{10}}\ \hat{p}_{11}^{n_{11}} / \left(1-\alpha\right)^{n_0}\ \alpha^{n_1}\right) \sim \chi^2\left(2\right) \tag{11}$$

where $p_{ij} = \Pr\left(I_t = i \middle| I_{t-1} = j\right)$ estimated as $\hat{p}_{ij} = n_{ij} / \sum_{j=0}^{1} n_{ij}$, with $i, j = 0,1$ and n_{ij} is the number of transitions from state i to state j. Note that, for both tests, the null hypothesis is rejected if the VaR model generates too many or too few exceptions while for the conditional coverage a VaR model may also be rejected if it generates too clustered exceptions.

Engle and Manganelli (2004) argued that given the sequence of returns, r_t, it is straightforward to generate an i.i.d. failure process I_t and they proposed a more powerful test. Specifically, they defined $Hit_t = I_t - \alpha$, where α is the significance level and they suggested a regression based approach to test whether $E\left(Hit_t\right) = 0$ and also if Hit_t is uncorrelated with the variables included in the information set, Ω_{t-1}. In matrix notation the regression equation can be written as: $\mathbf{Hit} = \mathbf{X\beta}$ where $\mathbf{X}$ is the explanatory variables vector and $\boldsymbol{\beta}$ is the coefficients vector. The authors emphasized on the use of the contemporaneous value of VaR, VaR_t^{α}, in the explanatory variables set, as well as the use of lagged values of Hit_t, i.e. $Hit_{t-1}, Hit_{t-2}, ...Hit_{t-q}$, with $q = 5$ in our case. Under the null hypothesis $H_0 : \boldsymbol{\beta} = \mathbf{0}$, the regressors, i.e. the five lags of Hit_t and the VaR_t^{α}, should have no explanatory power. The corresponding test statistic is:

$$DQ = \frac{\boldsymbol{\beta}_{LS}^{'}\mathbf{X}^{'}\mathbf{X}\boldsymbol{\beta}_{LS}^{'}}{\alpha(1-\alpha)} \tag{12}$$

which follows an asymptotic $\chi^2\left(p+1\right)$ distribution, where p is the total number of explanatory variables used in the regression.

Finally, we use the formula for the market risk capital (MRC) requirements prescribed by the Banking Committee on Banking Supervision (BCBS, 2006) and it is a widely accepted method for evaluating alternative VaR models (e.g. see Ferreira and Lopez, 2005):

$$MRC_t = \max\left[VaR_{t-1}^{0.01}\left(10\right), \frac{k}{60}\sum_{i=1}^{60} VaR_{t-i}^{0.01}\left(10\right)\right] \tag{13}$$

where $VaR_t^{0.01}\left(10\right)$ denotes the 1% VaR estimate of day t for a holding period of ten days, while k is a multiplier set by the MRA's traffic light system.[2] Specifically, the value of k is based on the number of 1% daily VaR exceptions over the previous 250 trading days. If the model produces 4 or less violations, then it is considered sufficiently accurate and the multiplier k takes its minimum value of 3. These are the so-called green zone or green light models. If the model generates between 5 and 9 violations over the previous trading year then it is placed in the yellow zone, or it is given a yellow light. It is also considered acceptable for regulatory purposes, with k being set to 3.4, 3.5, 3.65, 3.75 or 3.85, for the corresponding

[2] For the calculation of the MRC, daily VaR is expressed in dollars: $VaR\$_t^{0.01} = P_{t-1}\left[1-\exp\left(VaR_t^{0.01}\right)\right]$, where P is the asset's price and is multiplied by $\sqrt{10}$ to get the 10 day VaR estimates as in Ferreira and Lopez (2005) .

exceptions in the interval $[5,9]$. A red zone or red light model is one which generates 10 or more exceptions and then k takes its maximum value of 4. In this case, the regulators can reject the VaR model and put a request to the financial institution to revise their risk management systems.

4. Empirical Analysis

We use five minutes previous tick interpolated prices for the S&P 500 stock index from 1.1.1997 to 09.30.2009, obtained from Tick Data. For liquid assets, the five minutes sampling frequency is found to be the highest sampling frequency with acceptable market microstructure noise bias (Andersen *et al.*, 2001). Fig. 1 presents the S&P 500 daily returns and the logarithmic RV along with their estimated densities. The departure from normality is evident for the daily returns, while the logarithmic RV is approximately normal.

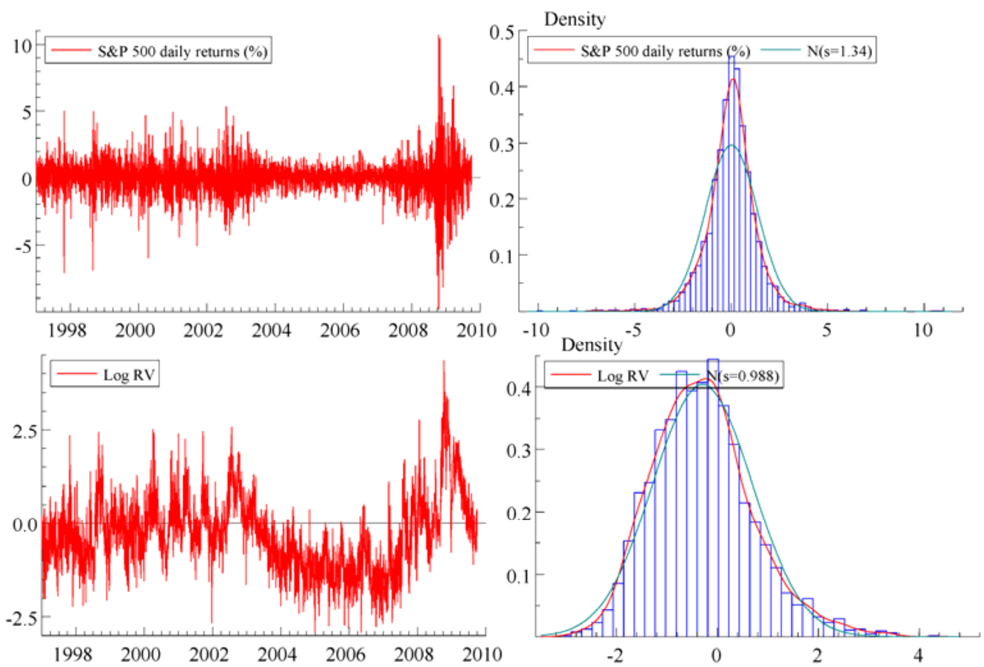

Fig. 1. S&P 500 stock index daily returns, logarithmic realized variance and their corresponding densities

For comparison reasons we also implement the GJR-GARCH[3] model of Glosten *et al.* (1993) combined with the skewed student distribution (Lambert and Laurent, 2001) and the EVT method as well as the HAR model without a GARCH error process and a HAR model with the realized power variation (RPV) (Barndorff-Nielsen and Shephard, 2004) used as regressors. The RPV uses absolute intraday returns that mitigate extreme price movements and it has superior predictive ability in volatility forecasting (e.g. see Louzis *et al.*, 2012). However, its forecasting performance has not been extensively tested in VaR applications (Clements *et al.*, 2008).

[3] For the GJR-GARCH VaR models see the Appendix.

We use a rolling sample of 1,250 observations in order to produce 1,946 out-of-sample day-ahead VaR forecasts from 12.20.2000 to 09.30.2009. Table I presents the FR and *p*-values for the (un)conditional coverage and DQ tests for the 5%, 1% and 0.5% quantiles.

Table I. Failure rates, (un)conditional coverage and dynamic quantile tests

	Failure Rate (%)			Unconditional Coverage test (*p*-values)			Conditional Coverage test (*p*-values)			Dynamic Quantile test (*p*-values)		
Models	5%	1%	0.5%	5%	1%	0.5%	5%	1%	0.5%	5%	1%	0.5%
GJR-skst	5.04*	1.13	0.46	0.94*	0.57	0.81	0.62	0.66	0.93	0.55	0.63	0.97
GJR-EVT	5.29	1.28	0.51*	0.56	0.23	0.93*	0.41	0.35	0.95*	0.56*	**0.04**	1.00*
HAR -EVT	5.09	0.92*	0.46	0.86	0.74*	0.81	0.87	0.80*	0.93	0.39	0.80	0.99
HAR-RPV-EVT	5.04*	0.92*	0.46	0.94*	0.74*	0.81	0.90*	0.80*	0.93	0.39	0.80	0.99
HAR-GARCH-EVT	5.14	0.82	0.46	0.78	0.42	0.81	0.83	0.63	0.93	0.40	0.86*	0.99

Notes: The asterisk (*) indicates the best performing model

The bold faced number indicates rejection of the null hypothesis at a 5% significance level

The results indicate that all models can generate FRs that are closed to the predetermined coverage level. However, a closer examination of the results reveals some interesting points. First, for the 5% quantile the inclusion of the RPV improves slightly the forecasting ability of the HAR model, but this is not evident for the lower quantiles (1% and 0.5%). Second, the HAR-type-EVT models outperform the GJR-EVT model across quantiles, with the exception of the 0.5% quantile. Third, the GJR-skst model has comparable forecasting behaviour with the HAR-type-EVT models. However, for the 1% coverage level the GJR-skst model overestimates the FR, while the HAR-GARCH-EVT model generates the most conservative VaR estimates.

The results for the (un)conditional coverage and the DQ tests are in accordance with the above analysis, since all models reject the hypothesis of incorrect VaR estimates at a 5% significance level. The only exception is the GJR-EVT model for the 1% VaR forecasts, which rejects the DQ test null hypothesis. Nonetheless, it should be noted that the HAR-type-EVT models generate the highest p-values for the 1% quantile, which bears the greatest practical interest.

In Table II, we present the Basel II market risk capital requirements analysis. The most striking feature of Table 3 is that the GJR models generate red zone days, failing to comply with regulators' VaR accuracy mandates. On the contrary, the HAR-type-EVT models have zero red zone days with the HAR-GARCH-EVT maximizing the green zone days. The latter also minimizes the regulatory capital, producing the most efficient VaR estimates. This has significant economic consequences for the financial institutions, as now they can utilize the released capital in more productive and efficient ways.

Table II. Basel II market risk capital requirements

	Basel II zones			Basel II Capital requirements	
Models	Green (%)	Yellow (%)	Red (%)	Average	St. Deviation
GJR-skst	74.9	24.9	0.2	318	178
GJR-EVT	71.5	27.1	1.5	303	182
HAR-EVT	77.6	22.4	0.0	306	169
HAR-RPV-EVT	77.6	22.4	0.0	306	169
HAR-GARCH-EVT	82.3	17.7	0.0	300	164

Note: The table presents the percentage of days during the out of sample forecasting period that the model is placed in the green, yellow and red zone according to the Basel traffic light system, the average and the standard deviation of the daily capital requirements.

The results for the 2007-2009 crisis period and for the 1% VaR, presented in Table III, are unequivocal. The HAR-type-EVT models clearly outperform their GJR counterparts, in both statistical and regulatory accuracy terms, while the HAR-GARCH-EVT model has the overall best forecasting performance as it generates accurate and efficient VaR estimates that minimize the regulatory capital.

Table III. 1% VaR forecasting performance during the 07.01.2009 – 09.30.2009 period

Models	Failure Rate (%)	Unconditional Coverage test (p-values)	Conditional Coverage test (p-values)	Dynamic Quantile test (p-values)	Basel II Red zone days (%)	Average Basel II Capital Requirements
GJR-skst	2.66	**0.01**	**0.02**	**0.00**	0.7	507
GJR-EVT	2.31	**0.00**	**0.00**	**0.00**	4.4	515
HAR-EVT	1.42	0.35	0.57	0.46	0.0	503
HAR-RPV-EVT	1.42	0.35	0.57	0.47	0.0	503
HAR-GARCH-EVT	1.07*	0.88*	0.93*	0.95*	0.0	487*

Notes: The asterisk (*) indicates the best performing model

The bold faced numbers indicates rejection of the null hypothesis at a 5% significance level

5. Concluding remarks

In this study, we propose the use of HAR-type realized volatility models combined with the Extreme Value Theory method. The proposed model is used in order to produce day-ahead out-of-sample VaR forecasts using a forecasting period of eight years for the S&P 500 stock index. The results indicate that HAR-type-EVT model can produce superior VaR forecasts compared to their GARCH-type counterparts in terms of regulatory and statistical accuracy. The HAR-GARCH-EVT model is, however, the best performing model as it generates accurate and efficient VaR forecasts that minimize the regulatory capital especially during the turbulent 2007-2009 period. Moreover, the realized power variation (RPV) does not help improve HAR's VaR forecasting ability. This study can be extended to other stock indices and asset classes in order to gain more insights on the proposed methodology.

Appendix

The AR(1)- CJR-GARCH(1,1) model (GJR in short) of Glosten *et al*. (1993) is given by:

$$r_t = c + \phi_1 r_{t-1} + \varepsilon_t = c + \phi_1 r_{t-1} + \sqrt{h_t} z_t \tag{14}$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I\left(\varepsilon_{t-1} < 0\right)\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{15}$$

where $\mathrm{I}(\cdot)$ is an indicator function which equals one if the condition in the parenthesis is satisfied and zero otherwise. For positive and statistically significant asymmetry parameter γ the impact of past negative returns on conditional variance is greater than the impact of positive returns.

In the GJR-EVT model the innovations quantiles are estimated using the EVT method described in Section 2.2. For the GJR-skst model next day's VaR is given by: $\widehat{VaR}_{T+1,j}^{\alpha} = \hat{\mu}_{T+1,j} + \hat{h}_{T+1,j} c_{\alpha,\nu,\xi}^{skst}$, with

$$c_{\alpha,\nu,\xi}^{skst} = \begin{cases} \left\{ \frac{1}{\xi} c_{\alpha,\nu}^{st} \left[\frac{\alpha}{2}\left(1+\xi^2\right)\right] - m \right\} / \mathrm{s} \ \text{ if } \ \alpha < \dfrac{1}{1+\xi^2} \\ \left\{ -\xi c_{\alpha,\nu}^{st} \left[\frac{1-\alpha}{2}\left(1+\xi^{-2}\right)\right] - m \right\} / \mathrm{s} \ \text{ if } \ \alpha \geq \dfrac{1}{1+\xi^2} \end{cases} \tag{16}$$

where $c_{\alpha,\nu,\xi}^{skst}$ is the α^{th} quantile of the unit variance skewed student distribution with $\nu > 2$ degrees of freedom and asymmetry parameter $\xi > 0$, $c_{\alpha,\nu}^{st}$ denotes the quantile function of the standardized Student-t density function, while $m = \frac{\Gamma\left(\frac{\nu+1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(\xi - \frac{1}{\xi}\right)$ and $s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$ are the mean and the standard deviation of the non-standardized skst distribution respectively.

References

Andersen, T.G. and T. Bollerslev (1998) "Answering the sceptics: yes, standard volatility models do provide accurate forecasts" *International Economic Review* **39**, 885-905.

Andersen, T.G. Bollerslev, T., Diebold, F.X. and H. Ebens (2001) "The distribution of realized stock returns volatility" *Journal of Financial Economics* **6**, 43-76.

Angelidis, T. and S. Degiannakis (2008) "Volatility forecasting: Intra-day versus Inter-day models" *Journal of International Financial Markets, Institutions and Money* **18,** 449-465.

Basel Committee on Banking Supervision. (2006) "International convergence of capital measurement and capital standards" Bank for International Settlements, Basel.

Barndorff-Nielsen, O.E. and N. Shephard (2004) "Power and bipower variation with stochastic volatility and jumps" *Journal of Financial Econometrics* **2**, 1-37.

Chan, K.F. and P. Gray (2006) "Using extreme value theory to measure value-at-risk for daily electricity spot prices" *International Journal of Forecasting* **22**,283-300.

Christoffersen, P. (1998) "Evaluating interval forecasts" *International Economic Review* **39**, 841-862.

Clements, M., Galvao, A. and J. Kim (2008) "Quantile forecasts of daily exchange rate returns from forecasts of realized volatility" *Journal of Empirical Finance* **15**, 729-750.

Corsi, F. (2009) "A simple approximate long memory model of realized volatility" *Journal of Financial Econometrics* **7**, 174-196.

Corsi, F., Mittnik, S., Pigorsch, C. and U. Pigorsch (2008) "The volatility of realized volatility" *Econometric Reviews* **27**, 46-78.

Engle, R.F. and S. Manganelli (2004) "CoViaR: conditional autoregressive value at risk by regression quantiles" *Journal of Business and Economic Statistics* **22**, 367-381.

Ferreira, M.A. and J.A. Lopez (2005) "Evaluating interest rate covariance models within a Value-at-Risk framework" *Journal of Financial Econometrics*, **3**, 126-168.

Giot, P. and S. Laurent (2004) "Modelling daily value-at-risk using realized volatility and ARCH type models" *Journal of Empirical Finance* **11**, 379-398.

Glosten, L., Jagannathan, R. and D. Runke (1993) "Relationship between the expected value and the volatility of the nominal excess return on stocks" *Journal of Finance* **48**, 1779-1801.

Lambert, P. and S. Laurent (2001) "Modelling financial time series using GARCH-type models and skewed Student density" Mimeo, Universite de Liege.

Louzis, D.P., Xanthopoulos-Sisinis, S. and A.P. Refenes (2012) “Stock index realized volatility forecasting in the presence of heterogeneous leverage effects and long range dependence in the volatility of realized volatility” *Applied Economics* **44**, 3533-3550.

Martens, M. (2002) “Measuring and forecasting S&P 500 index-futures volatility using high-frequency data” *Journal of Futures Markets* **22**, 497-518.

Martens, M., Dijk, D. and M. Pooter (2009) “Forecasting S&P 500 volatility: Long memory, level shifts, leverage effects, day of the week seasonality and macroeconomic announcements” *International Journal of forecasting* **25**, 282-303.

McNeil, A.J. and R. Frey (2000) “Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach” *Journal of Empirical Finance* **7**, 271-300.