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Ex-Post Equivalence under Capital Gains Taxation

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Abstract

In this article, we analyze a state-contingent tax on capital gains. We start by focusing on Auerbach's (1991) retrospective capital gains tax device. Although this system is equivalent to an accrual method from an ex-ante perspective, it is not on an ex-post basis. As recognized by Auerbach, this causes a fairness problem. To overcome this limitation, we follow Zhu (1992) and design a state-contingent tax rule. As will be proven, state-contingent taxation can modify the risk profile of assets and also ensure ex-post equivalence.

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1 Introduction

Capital gains can be taxed either at accrual or at realization. Under the former method, fairness is ensured, since capital gains are taxed when they accrue, thereby providing a more precise measure of a taxpayer's ability to pay. However, given its limitations, most countries have opted for a realization-based capital gains tax.[1] This latter system has at least two shortcomings. Firstly, an investor is "locked-in", i.e., is induced to delay selling assets to save taxes (see, e.g., Constantinides, 1983). Secondly, the realization-based system fails to ensure ex-post equivalence.

In order to neutralize the lock-in effect, and at the same time, implement a realization-based system, Auerbach (1991) proposes a retrospective capital gains tax device, which raises the effective tax burden if realization is postponed. He shows that this system is equivalent to an accrual method from an ex-ante perspective (i.e., at the beginning of the holding period), and therefore, an investor is no longer encouraged to delay realization. In doing so, Auerbach overcomes the informational problems arising from Vickrey (1939)'s original proposal, which required the availability of information on past assets' price. Indeed, the retrospective tax formula simply requires an asset's current price and the spot interest rate during the holding period.[2]

As pointed out by Auerbach (1991), however, his proposal ensures equivalence only from an ex-ante perspective, namely before investors know what their income will be. Such an equivalence fails to hold after an investor's return is known. Suppose that, at the end of the holding period, an investor faces a huge gain. Therefore, the effective tax rate under the Auerbach system is less than the one levied under an accrual tax system and vice versa. As Auerbach (1991) recognizes, the absence of ex-post equivalence is a problem in terms of fairness.[3]

To solve this fairness problem, we thus propose a state-contingent fiscal tool (as in Zhu, 1992 and Kaplow, 1994) with a twofold aim. Firstly, we show that through such an in-

[1] The accrual system suffers from two main limitations. Firstly, a taxpayer should pay an income tax return accounting for all changes in the assets' values. This would be easy if securities were sold and the asset price would be known. Otherwise, a taxpayer would have to calculate changes in his/her asset values on the basis of data that may be imprecise. In particular, it is usually hard to evaluate an asset that is not publicly traded (see Green and Sheshinski, 1979). Secondly, a liquidity problem may arise. As the tax liability may occur in a period when the taxpayer accrues but does not realize the capital gains, the taxpayer might be forced to sell part of his/her assets, against his/her own wishes, in order to pay the taxes. Hence, a distortion would arise.

[2] For an extension of this system see Auerbach and Bradford (2004). More recently, Menoncin and Panteghini (2010) have proven that, with no *ad hoc* adjustment, ex-ante equivalence holds even if interest rates are stochastic and asset returns are serially correlated.

[3] Auerbach (1991) recognizes the seriousness of this problem although he says that it is similar to the one caused by a wealth tax, levied after the wealth value is known.

strument the government can adjust the riskiness of after-tax returns.[4] Secondly, we prove that, by means of state-contingent taxation, the government can eliminate the after-tax risk, thereby ensuring ex-post equivalence.

The structure of the article is as follows. In Section 2 we design our ex-post equivalent device. In Section 3 we discuss implementation issues.

2 The state-contingent tax

In line with Auerbach (1991), let us introduce a partial-equilibrium model in continuous time. By assumption, financial markets are arbitrage-free, complete and frictionless.

For simplicity, there are only two assets: a risk-free asset whose (risk-free) return is $r(t)$ and a risky one whose price is $S(t)$. Following Menoncin and Panteghini (2010), we assume that:

Axiom 1. *The value of the risky asset solves the stochastic differential equation:*

$$\frac{dS(t)}{S(t)} = r(t)\,dt + \underset{1\times k}{\sigma(t,S)'}\underset{k\times 1}{dW(t)^{\mathbb{Q}}}, \quad S(t_0) = S_0 > 0, \tag{1}$$

where S_0 is known, $\sigma(t,S)'\sigma(t,S) \in \mathbb{R}^+$ is the variance of the instantaneous asset return, $W^{\mathbb{Q}}(t)$ is a $k-$dimensional Wiener process under the (unique) risk-neutral probability measure $\mathbb{Q}$, and the prime denotes transposition.

Axiom 2. *The spot interest rate $r(t)$ is stochastic.*

Note that the solution of Equation (1) can be represented as an expected value in the following form:

$$S(t) = \mathbb{E}_t^{\mathbb{Q}}\left[S(T)\,e^{-\int_t^T r(s)ds}\right], \quad \forall T \geq t. \tag{2}$$

Let us next introduce taxation. Denoting the tax factor as

$$\theta(t) \equiv 1 - \tau_c(t) \leq 1,$$

where $\tau_c(t)$ is the effective capital gains tax rate at time t, and assuming that a risky asset is bought at time t_0 and sold at $t > t_0$, its after-tax value is equal to $S(t)\,\theta(t)$. Accordingly, the condition

$$\theta(t_0) = 1 \tag{3}$$

[4] Recent bailouts suggest that contingent policies have been implemented by many policymakers. For further details on state-contingent policies see also Chari, Christiano, and Kehoe (1994) and Chari and Kehoe (1999).

must hold: if the asset is bought and immediately after sold, its transaction value is the same and therefore, no tax is due (i.e., $S(t_0)\theta(t_0) = S(t_0)$).

Given these assumptions, the government can implement a state-contingent tax policy and accordingly adjust the risk profile of an asset. Such a fiscal policy can be designed as follows:

$$\frac{d\theta(t)}{\theta(t)} = \mu_\theta(t)\,dt + \sigma_\theta(t)'\,dW(t)^{\mathbb{Q}}, \tag{4}$$

where $\mu_\theta(t)$ is the drift component of taxation. The term $\sigma_\theta(t)$ is a vector of state-contingent policy tools that can offset any shock $dW(t)^{\mathbb{Q}}$. This means that, in line with Zhu (1992), a government can choose $\sigma_\theta(t)$ in order to adjust taxation in each state of the world.

Applying Itô's lemma to the product between (1) and (4) gives

$$\frac{d(\theta(t)S(t))}{\theta(t)S(t)} = \left(r(t) + \mu_\theta(t) + \sigma(t,S)'\sigma_\theta(t)\right)dt + \left(\sigma(t,S)' + \sigma_\theta(t)'\right)dW(t)^{\mathbb{Q}}. \tag{5}$$

Assuming that the risk-free interest rate is taxed with rate τ (with $\tau \lesseqgtr \tau_c$), in the absence of arbitrage, the drift term of (5) must coincide with the after-tax risk-free interest rate, i.e.,

$$r(t) + \mu_\theta(t) + \sigma(t,S)'\sigma_\theta(t) = (1-\tau)\,r(t),$$

or equivalently,

$$\mu_\theta(t) = -\tau r(t) - \sigma(t,S)'\sigma_\theta(t).$$

2.1 Contingent-tax policy

Notice that the government can choose either the drift or the diffusion term of (5) (not both). So, if it aims at adjusting market riskiness and reaching the target volatility $\hat{\sigma}(t)$, then it must set $\theta(t)$ in order for the diffusion term of $\theta(t)S(t)$ to be equal to $\hat{\sigma}(t)$. Therefore, we can prove the following:

Proposition 1. *If the government's target diffusion term is $\hat{\sigma}(t)$, then capital gains taxation must evolve as follows:*

$$\begin{aligned} \frac{d\theta(t)}{\theta(t)} &= \left(-\tau r(t) + \sigma(t,S)'\left(\sigma(t,S) - \hat{\sigma}(t)\right)\right)dt \\ &\quad - \left(\sigma(t,S)' - \hat{\sigma}(t)'\right)dW^{\mathbb{Q}}(t), \\ \theta(t_0) &= 1. \end{aligned} \tag{6}$$

Proof. Using (5), let us set

$$\begin{aligned}(1-\tau)\,r\,(t) &= r\,(t)+\mu_\theta\,(t)+\sigma\,(t,S)'\,\sigma_\theta\,(t)\,,\\ \sigma\,(t,S)+\sigma_\theta\,(t) &= \hat{\sigma}\,(t)\,.\end{aligned}$$

Rearranging gives

$$\begin{aligned}\sigma_\theta\,(t) &= \hat{\sigma}\,(t)-\sigma\,(t,S)\,,\\ \mu_\theta\,(t) &= -\tau r\,(t)-\sigma\,(t,S)'\,(\hat{\sigma}\,(t)-\sigma\,(t,S))\,.\end{aligned}$$

Substituting these results into (4) gives (6). The Proposition is thus proven. □

According to Proposition 1, the dynamics of $\theta\,(t)$ depends on both market volatility $\sigma\,(t,S)$ and the government's target volatility $\hat{\sigma}\,(t)$. In particular, the diffusion term of (4), which measures the portion of volatility absorbed by the government, is equal to the difference between market volatility and target volatility.

Solving (6) gives $\theta\,(t)$. Notice that this solution needs the knowledge of the whole path of asset prices $S\,(s)$ for $s\in[t_0,t]$ if the asset return is heteroschedastic, i.e., when $\sigma\,(t,S)$ depends on S. Thus, $\theta\,(t)$ is not only stochastic but may also be price dependent.

Proposition 1 has a straightforward implication in terms of ex-post equivalence. As we know, ex-post equivalence means that the after-tax return on asset $S\,(t)$, under $\mathbb{Q}$, is equal to the after-tax return on the risk-free asset, *in any state of the world*. This means that ex-post equivalence is met if:

$$\frac{d\,(\theta\,(t)\,S\,(t))}{\theta\,(t)\,S\,(t)}=(1-\tau)\,r\,(t)\,dt. \tag{7}$$

Namely, taxation is such that shocks are always offset and therefore the after-tax asset value evolves according to the deterministic Equation (7). Instead, under the Auerbach device the after-tax return is equal to:

$$\frac{d\,(\theta\,(t)\,S\,(t))}{\theta\,(t)\,S\,(t)}=(1-\tau)\,r\,(t)\,dt+\sigma\,(t,S)'\,dW\,(t)^{\mathbb{Q}}\,, \tag{8}$$

and ex-ante equivalence thus implies the equality

$$\mathbb{E}_t^{\mathbb{Q}}\left[\frac{d\,(\theta\,(t)\,S\,(t))}{\theta\,(t)\,S\,(t)}\right]=(1-\tau)\,r\,(t)\,dt.$$

This means that, on average, the after-tax return on asset $S\,(t)$, under $\mathbb{Q}$, is equal to the after-tax return on the risk-free asset. Of course, this is a milder condition than the one written in (7).

As we have pointed out, rule (6) allows the government to adjust after-tax risk. In particular, if it aims at eliminating risk, ex-post equivalence is reached. Using Proposition 1 and Equation (7) we can write the following:

Proposition 2. *Ex-post equivalence is achieved by setting* $\hat{\sigma}(t) = 0$ *(i.e.,* $\sigma_\theta(t) = -\sigma(t, S)$*).*

According to Proposition 2, ex-post equivalence is reached if the government's state-contingent tax tool $\sigma_\theta(t)$ fully offsets market risk. In this case, the effective tax liability is always equal to $\tau r(t)$. Under the Auerbach system, however, only the expected tax liability is $\tau r(t)$, whereas the (ex-post) effective tax burden depends on the state of nature: as shown in (8), if term $\sigma(t, S)' dW(t)^{\mathbb{Q}}$ were positive, the effective capital gains tax rate would be less than $\tau r(t)$ and vice versa.

2.2 Two numerical examples

Two numerical examples will give an idea of contingent-tax effects. Let us set: i) initial asset price $S(t_0) = 25$ euros; ii) risk-free interest rate $r = 0.05$; iii) asset volatility $\sigma = 0.20$; iv) tax rate $\tau = 0.33$. Daily prices are considered (i.e. $dt = 1/250$) and, of course, condition (3) holds.

Example 1. The government wants to halve market volatility by setting $\hat{\sigma} = 0.1$. As shown in Figure 1 (upper panel), over a five-year period (i.e., $5 \times 250 = 1250$ trading days), the after-tax price (in **bold**) is less volatile than the before-tax price ($\sigma = \frac{1}{2}\hat{\sigma}$) and its growth rate is lower. The bottom panel of Figure 1 draws the dynamics of the tax factor. As expected, it decreases over time. This means that the tax burden is increasing in time. During the two first years, $\theta(t)$ may be higher than unity: this happens when the asset price is less than the initial one (and thus a capital loss emerges). Since taxation is symmetric and fully compensates for losses, the effective tax rates may be negative.

Example 2. The government aims at eliminating volatility and ensuring ex-post equivalence: $\hat{\sigma}(t) = 0$. As shown in the upper panel of Figure 2, the after-tax price follows a deterministic path without volatility and, of course, with a much lower rate of return (equal to the risk-free rate). In this case, the government absorbs volatility and therefore the tax factor has a much larger range of variation.

3 Implementation issues

In this article we have studied a state-contingent capital gains tax which can ensure ex-post equivalence. As we have shown, the implementation of such a tool needs the knowledge

Figure 1: The dynamics of the before- ($\sigma = 0.2$) and after-tax asset price ($\hat{\sigma} = 0.1$) (upper panel). The dynamics of the tax factor $\theta(t)$ (bottom panel).

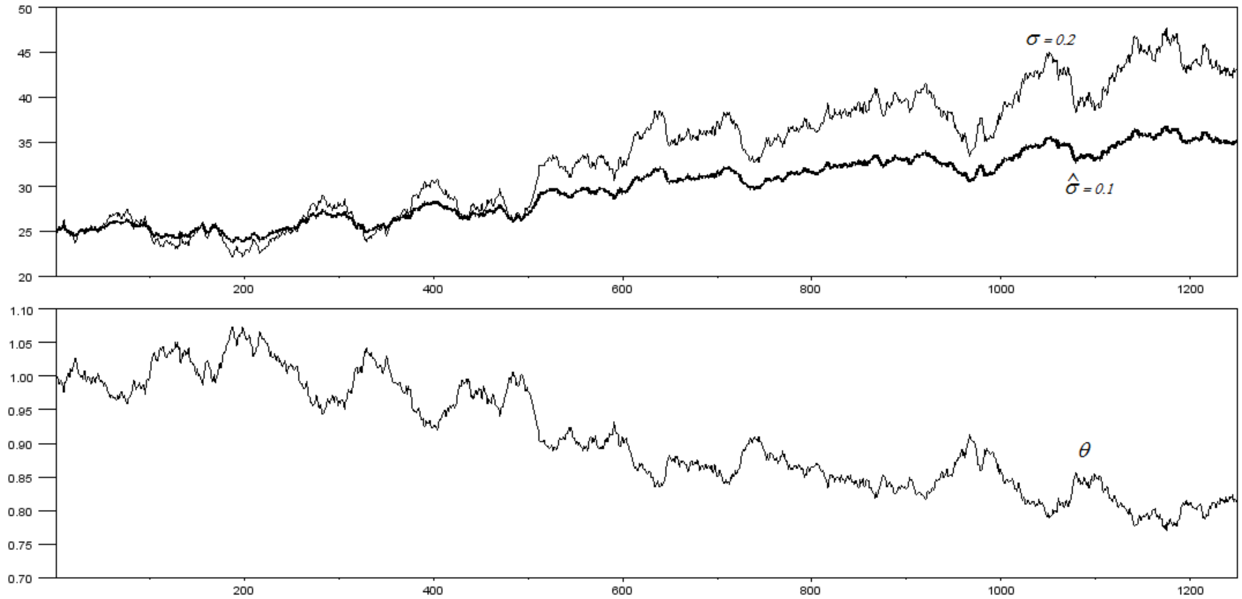
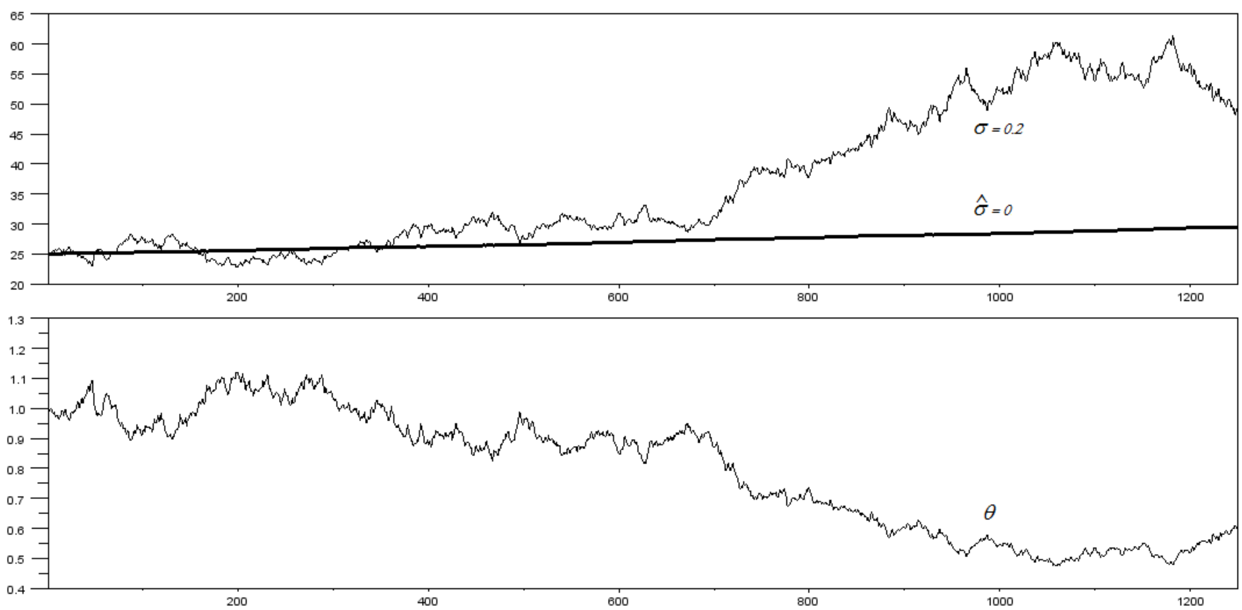


Figure 2: The dynamics of the before- ($\sigma = 0.2$) and after-tax asset price ($\hat{\sigma} = 0$) under ex-post equivalence (upper panel). The dynamics of the tax factor θ (bottom panel).



of the whole path of asset prices if asset returns are heteroschedastic. In particular, we can say that the implementation of the fiscal rule described by (6) needs the application of two appropriate models aimed at describing: 1) the dynamics of the risk-free rate and 2) the evolution of the volatility of S. It is worth noting that when the volatility of an asset price is deterministic, the implementation of this tax model is relatively easy. When however volatility is subject to heteroskedasticity, the implementation of an ex-post equivalent scheme is harder, since we need to know the whole path of asset return volatility.

This means that the implementation of $\theta(t)$ is, in principle, subject to the same criticism as Vickrey's original proposal. However, technological improvements since 1939 make data collection much easier and econometric estimates more precise. This holds, in particular, for listed assets, where past values are known with no cost. Of course difficulties may arise when non-listed assets are considered. In this case, implementation of an ex-post equivalent tax depends on the characteristics of these assets. In some cases (e.g., cars and real estate assets), there is no official listing. However, these assets are almost always registered. Moreover, information on their current value can be gathered among dealers. All these data, as well as the continuous improvement in the understanding asset price dynamics can help to refine volatility estimates.

In other cases (e.g., trading of a picture or of an object that is rarely traded) a periodical evaluation is impossible and our ex-post equivalent method cannot be implemented. For this reason, we believe that such a system can be applied only to some categories of assets but not to all. Of course, a further technological upgrade would enable a government to extend the ex-post equivalent method to other categories.

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