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On the coincidence of the core and the bargaining sets

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Abstract

We prove that for any coalitional game the core coincides with the bargaining set à la Davis and Maschler when we sufficiently raise the worth of the grand coalition (the efficiency level). This coincidence result does not hold for other well-known bargaining sets like the Mas-Colell bargaining set and its variants.

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1. Introduction and terminology

Given a cooperative situation, one of the most conflicting problems is to distribute the profit arising from cooperation among the agents. The core of a coalitional game plays a central role in this analysis. Core-selection is a meaningful relevant property for any solution concept in order to avoid that any coalition might object to it. However, since the publication of the seminal paper of Aumann (1973), there has been an interesting debate in the literature in favor and against to this property. Maschler (1976) illustrates this point by analyzing examples where some allocations in the bargaining set, and not in the core, have more sense than the core ones. We want to contribute with some simple but interesting results in favor of both solution concepts. As a first result we show that if the amount to be distributed among agents (the worth of the grand coalition or efficiency level) is large enough the core and the bargaining set à la Davis and Maschler will always coincide, and this coincidence will remain stable for higher levels of efficiency[1]. This result contrasts to what happens with other bargaining sets like the Mas-Colell bargaining set and its variants. For all these bargaining sets, an example of the non-coincidence with the core at any level of efficiency is provided.

A coalitional *game* with transferable utility is a pair (N, v), where $N = \{1, 2, \ldots, n\}$ is the set of players and v is the *characteristic function*, $v : 2^N \to \mathbb{R}$, assigning to every *coalition* $S \subseteq N$ a real number $v(S)$, the *worth* of S, with $v(\varnothing) = 0$. We denote by $|S|$ the cardinality of the coalition $S \subseteq N$.

A *payoff allocation* is a vector $z = (z_i)_{i \in N} \in \mathbb{R}^N$, where z_i is the payoff to player $i \in N$. We write $z(S) = \sum_{i \in S} z_i$ for all nonempty coalition $S \subseteq N$ and $z(\varnothing) = 0$. A payoff allocation $z \in \mathbb{R}^N$ is a *preimputation* of a game (N, v) when $z(N) = v(N)$ (efficiency). The set of all the preimputations of a game v is denoted by $I^*(N, v)$. Moreover, we say that a preimputation $z \in I^*(N, v)$ is an *imputation* of the game (N, v) when $z_i \geq v(\{i\})$ for all $i \in N$ (individual rationality). The set of all the imputations of a game v is denoted by $I(N, v)$. A game (N, v) is said to be *essential* if $I(N, v) \neq \varnothing$ (i.e., if $\sum_{i \in N} v(\{i\}) \leq v(N)$). The *core* of a game (N, v) is the set $C(N, v) = \{z \in I(N, v) \mid z(S) \geq v(S) \text{ for all } S \subseteq N\}$.

Next we define different concepts of bargaining sets. Let (N, v) be an essential game, let $x \in I(N, v)$ and let $i, j \in N$ be two different players. A pair (S, y) is an *objection* at x of player i against j when

$$i \in S \subseteq N \setminus \{j\}, \; y \in \mathbb{R}^S, \; y(S) = v(S) \text{ and } y_k > x_k \text{ for all } k \in S.$$

A *counter-objection* to the above objection (S, y) is a pair (T, z) where

$$\begin{aligned} & j \in T \subseteq N \setminus \{i\}, \; z \in \mathbb{R}^T, \; z(T) = v(T), \\ & z_k \geq y_k \text{ for all } k \in T \cap S, \; z_k \geq x_k \text{ for all } k \in T \setminus S. \end{aligned}$$

An objection is *justified* if there is no counter-objection to it. The *bargaining set* of a game (N, v) (Davis and Maschler, 1963, 1967) is the set

$$\mathcal{M}_1^i(N, v) = \{x \in I(N, v) \mid \text{there is no justified objection at } x\}.$$

[1]Using a completely different argument, another lower bound is reached in Meertens (2005). We thank to an anonymous referee for this information.

By definition, we have $C(N,v) \subseteq \mathcal{M}_1^i(N,v) \subseteq I(N,v)$ since each core allocation cannot be objected. Moreover, $\mathcal{M}_1^i(N,v)$ is always nonempty provided the game v is essential (Davis and Maschler, 1963).

Let (N,v) be a game, let $x \in I^*(N,v)$ and $S \subseteq N$. A pair (S,y) is a Mas-Colell *objection* at x when $y \in \mathbb{R}^S, y(S) = v(S)$ and $y_k \geq x_k$ for all $k \in S$, being at least one of the inequalities strict. A Mas-Colell *counter-objection* to the objection (S,y) is a pair (T,z) where $z \in \mathbb{R}^T$, $z(T) = v(T)$, $z_k \geq y_k$, for all $k \in T \cap S$, $z_k \geq x_k$ for all $k \in T \backslash S$, being one of the inequalities strict. The Mas-Colell *bargaining set* of a game (N,v) (Mas-Colell, 1989) is the set

$$\mathcal{MB}(N,v) = \{x \in I^*(N,v) \mid \text{there is no justified Mas-Colell objection at } x\}.$$

The *individually rational bargaining set* (Vohra, 1991) $IR\mathcal{MB}(N,v)$ is the set of imputations contained in the Mas-Colell bargaining set. Vohra notes that this bargaining set is nonempty in the class of weakly superadditive coalitional games[2]. Therefore, for any weakly superadditive game we have $\varnothing \neq IR\mathcal{MB}(N,v) \subseteq \mathcal{MB}(N,v)$. Shimomura (1997) considers a modification of the Mas-Colell bargaining set (1989). As usual the bargaining set is defined by means of an interaction of objections and counter-objections. Let $x \in I(N,v)$ be an imputation of the game $I(N,v)$. A Shimomura *objection* at x is a pair (S,y), $\varnothing \neq S \subseteq N$ and $y \in \mathbb{R}^S$ with $y(S) = v(S)$ such that $y_i > x_i$, for all $i \in S$. A Shimomura *counter-objection* to (S,y) is a pair (T,z), $z \in \mathbb{R}^T$ with $z(T) = v(T)$ such that $z_i > y_i$, for all $i \in T \cap S$, and $z_i > x_i$ for all $i \in T \setminus S$. The Mas-Colell bargaining set (*à la Shimomura*) is defined as

$$\mathcal{MB}_{Sh}(v) = \{x \in I(N,v) \mid \text{there is no justified Shimomura objection at } x\}.$$

Shimomura proves that this bargaining set is nonempty for games satisfying grand coalition zero-monotonicity[3] by checking that the individually rational quasi-core is a nonempty subset of it (see Theorem 1 in Shimomura, 1997).

2. The bargaining sets and the core

We explore the eventual coincidence of some bargaining sets with the core of a coalitional game with transferable utility when we raise appropriately the worth of the grand coalition(the efficiency level). In general, the core is included in each of the different bargaining sets, and at his turn the bargaining sets are included in the imputation set, except for the original Mas-Colell bargaining set which is defined on the preimputation set. All of theses sets change when we raise the efficiency level. For some of these bargaining sets, the coincidence with the core never occurs in general, that is, we can found games where the inclusion between them is strict irrespective of how large the efficiency level is (see Example 4). By contrast, we prove that for the Davis and Maschler bargaining set, it always exists an efficiency threshold from where this coincidence holds.

[2] A game (N,v) is weakly superadditive if for all $i \in N$, $v(S) + v(\{i\}) \leq v(S \cup \{i\})$, for all $S \subseteq N \setminus \{i\}$.
[3] A game (N,v) is grand-coalition zero-monotonic if, for all $S \subseteq N$, $v(S) + \sum_{i \in N \setminus S} v(\{i\}) \leq v(N)$.

For any game, we determine a bound, depending on the characteristic function of the game, from which the coincidence of the core and the bargaining set holds. From now on we denote by $\hat{v}$ the vector $(v(S))_{\substack{S\subseteq N \\ S\neq\varnothing,N}}$.

Theorem 1 *For any coalitional game (N,v) there exists a real number $k^*(\hat{v})\in\mathbb{R}$ such that*

$$\mathcal{M}_1^i(N,v)=C(N,v), \textit{ if } v(N)\geq k^*(\hat{v}).$$

Proof: For $|N|=2$ we simply take $k^*(\hat{v})=v(\{1\})+v(\{2\})$ and the proof is done. For the case $|N|\geq 3$ it holds that for any game (N,v) we have $C(N,v)\subseteq\mathcal{M}_1^i(N,v)$. Therefore, it only remains to check that, for any game (N,v) with $|N|\geq 3$, there exists a lower bound for the worth of the grand coalition such that $\mathcal{M}_1^i(N,v)\subseteq C(N,v)$. For this compute the solution of the following min-max optimization program

$$k^*(\hat{v})=\min_{\substack{a\in\mathbb{R}^N \text{ s.t.} \\ a_i\leq v(\{i\}), \\ i=1,\dots,n}}\left\{\max_{S\neq\varnothing,N}\{n\cdot[v(S)-a(S)]+a(N)\}\right\}. \tag{1}$$

The above optimization program has at least one optimal solution[4], say $a^*=(a_1^*,a_2^*,\dots,a_n^*)$ with a corresponding optimal value, say $k^*(\hat{v})$. In particular, if $v(N)\geq k^*(\hat{v})$, then we have

$$v(N)\geq n\cdot(v(\{i\})-a_i^*)+a^*(N) \text{ or } \frac{v(N)-a^*(N)}{|N|}\geq v(\{i\})-a_i^*\geq 0, \text{ for any } i\in N. \tag{2}$$

Moreover, we claim that

$$\text{if } v(N)\geq k^*(\hat{v})\geq n\cdot(v(S)-a^*(S))+a^*(N), \text{ for all } S\neq\varnothing,N, \tag{3}$$

then the allocation $u\in\mathbb{R}^N$, defined as,

$$u_i:=\frac{v(N)-a^*(N)}{|N|}+a_i^*, \text{ for all } i\in N,$$

is in the core of the game (N,v). To check it, first notice that $u(N)=v(N)$. Moreover, for all $S\neq\varnothing,N$,

$$u(S)=\frac{|S|\cdot(v(N)-a^*(N))}{|N|}+a^*(S)\geq\frac{v(N)-a^*(N)}{|N|}+a^*(S)\geq v(S)-a^*(S)+a^*(S)=v(S),$$

where the first inequality holds by (2) and the last inequality by (3).

[4]Let $\bar{a}\in\mathbb{R}^N$ with $\bar{a}_i\leq v(\{i\})$, for all $i\in N$. Denote by $\bar{\ell}=\max_{S\neq\varnothing,N}\{n\cdot[v(S)-\bar{a}(S)]+\bar{a}(N)\}$ and define $\bar{D}=\{a\in\mathbb{R}^N \mid a_i\leq v(\{i\}),$ for all $i\in N$ and $n\cdot[v(S)-a(S)]+a(N)\leq\bar{\ell}$, for all $S\neq\varnothing,N\ \}$. Then, $\bar{D}$ is non-empty and compact and $\min_{a\in\bar{D}}\left\{\max_{S\neq\varnothing,N}\{n\cdot[v(S)-a(S)]+a(N)\}\right\}$ has the same solution than the program given in (1).

Now, let us suppose that we take $x \in I(N,v) \setminus C(N,v)$. Hence, since $u \in C(N,v)$, there exists a player in N – w.l.o.g. say player n – such that, for all nonempty coalition $S \subsetneq N$,

$$x_n > u_n = \frac{v(N) - a^*(N)}{|N|} + a_n^* \geq v(S) - a^*(S) + a_n^*. \tag{4}$$

Since $x \notin C(N,v)$, let $\tilde{S} \subseteq N$ be an arbitrary coalition such that $x(\tilde{S}) < v(\tilde{S})$. Note that player $n \notin \tilde{S}$; otherwise $x(\tilde{S}) = x(\tilde{S} \setminus \{n\}) + x_n > x(\tilde{S} \setminus \{n\}) + v(\tilde{S}) - a^*(\tilde{S} \setminus \{n\}) \geq v(\tilde{S})$, reaching a contradiction, where the strict inequality comes from (4) and the last inequality holds since $x_i \geq v(\{i\}) \geq a_i^*$, for all $i \in N$.

Now, let $i \in \tilde{S}$ be an arbitrary player of coalition $\tilde{S}$. We show that any objection $(\tilde{S}, y)$ at x of player $i \in \tilde{S}$ to player $n \notin \tilde{S}$ cannot be countered. To see it, for any eventual counter-objection (T, z) to the objection $(\tilde{S}, y)$ we have $n \in T$, $i \notin T$ and

$$\begin{aligned} z(T) &\geq y(\tilde{S} \cap T) + x(T \setminus \tilde{S}) \geq x(T) = x_n + x(T \setminus \{n\}) \\ &> v(T) - a^*(T \setminus \{n\}) + x(T \setminus \{n\}) \geq v(T), \end{aligned}$$

where the strict inequality holds by (4) and the last one again since $x_i \geq v(\{i\}) \geq a_i^*$, for all $i \in N$. Therefore, we obtain $z(T) > v(T)$, which invalidates the counter-objection. We conclude $x \notin \mathcal{M}_1^i(N,v)$ and so $\mathcal{M}_1^i(N,v) = C(N,v)$. □

A direct consequence of Theorem 1 is that for the reactive and the semireactive bargaining set (see Granot (2010), Sudhölter and Potters (2001) respectively) the coincidence of the core and these bargaining sets also holds, since they are subsets of the Davis and Maschler bargaining set. Furthermore, and by the same reason, we also get that the kernel of an arbitrary game (Davis and Maschler, 1965) tends to be included in its core when we raise the efficiency level.

We next apply Theorem 1 to the example given by Maschler (1976), showing an easy method to calculate the bound $k^*(\hat{v})$. Later, we will use this example to add some comments and remarks.

Example 1 (Maschler, 1976) *Consider a two-sided market where $P = \{1,2\}$ is the set of players of one side of the market, $Q = \{3,4,5\}$ is the set of players of the other side and $N = P \cup Q$ is the set of agents. Agents in P are complementary to agents in Q such that the corresponding coalitional game is defined by*

$$v(S) = \min\{|S \cap P|, \frac{1}{2}|S \cap Q|\}, \text{ for all } S \subseteq N.$$

The worth of the grand coalition is $v(N) = \frac{3}{2}$, the core shrinks into a singleton, $C(N,v) = \{(0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2})\}$ and the Davis and Maschler bargaining set strictly includes the core since

$$\mathcal{M}_1^i(N,v) = \{(\alpha,\alpha,\beta,\beta,\beta) \mid 0 \leq \alpha \leq \frac{3}{4}, 2\alpha + 3\beta = \frac{3}{2}\}.$$

By Theorem 1 we know that at some level of efficiency $k^*(\hat{v})$ both sets coincide. As $k^*(\hat{v})$ is the result of the min-max optimization problem (1) with linear objective functions *and linear constraints we can solve the problem (1) directly by solving a linear programming problem. To this aim we introduce a new variable $\varepsilon \in \mathbb{R}$ and the program (1) is equivalent to solve*

$$\begin{aligned} &\min \varepsilon \\ &(a,\, \varepsilon)\ s.t. \\ &\varepsilon + (n-1)\cdot a(S) - a(N\setminus S) \geq n\cdot v(S), && \textit{for all } S\subseteq N,\ S\neq\varnothing, N \\ &a_i \leq v(\{i\}), && \textit{for all } i\in N. \end{aligned} \tag{5}$$

Using a software to solve this problem we obtain that $a^* = (0,0,0,0,0)$ and $\varepsilon^* = 5$ is the optimal solution of (5). Hence, we conclude that if we raise the efficiency level above $\varepsilon^* = 5$, while the worth of the other coalitions remain fixed, the bargaining set and the core coincide. This is, for any game (N, v') such that $v'(S) = v(S)$, for all $S \subsetneq N$, and $v'(N) \geq 5$ we have $\mathcal{M}_1^i(N, v') = C(N, v')$.

Let us point out that Theorem 1 states that there always exists a lower bound for the worth of the grand coalition of a game that ensures that its core and its bargaining set do coincide. There are examples where the bound $k^*(\hat{v})$ arises as the best bound (the minimal one) to guarantee the coincidence. As an example, take $N = \{1,2,3\}$ and $v(S) = 1$, for all $S \subsetneq N, \varnothing$. In this case $k^*(\hat{v}) = 3$ and it is attained at $a^* = (1,1,1)$. Notice the game (N, v) is balanced if $v(N) \geq 3$ and we know $\mathcal{M}_1^i(N, v) = C(N, v)$ for any three-person balanced game.

There are other cases where $k^*(\hat{v})$ could be improved since, in fact, the coincidence between the corresponding core and the bargaining set starts before the bound $k^*(\hat{v})$ defined in (1). This is the case of the classical glove market game defined by $N = P \cup Q$, $P = \{1,2\}$, $Q = \{3,4\}$ and $v(S) = \min\{|S\cap P|, |S\cap Q|\}$, for all $S \subseteq N$; in particular $v(N) = 2$. This game is a four-person balanced superadditive game and it is well known that for such games it holds $\mathcal{M}_1^i(N, v) = C(N, v)$ (see Solymosi, 1999). Hence, for any game (N, v') such that $v'(S) = v(S)$, for all $S \subsetneq N$ and $v'(N) \geq 2$ the game is still superadditive and balanced and so $\mathcal{M}_1^i(N, v') = C(N, v')$. However, it is easy to check that $k^*(\hat{v}) = 4$, attained at $a^* = (0,0,0,0)$.

From the above comments, an immediate question arises: why don't we try to get the "best bound "? The answer is that we cannot obtain it since, in general, we cannot guarantee its existence. To see it, just take the example given by Meertens et al. (2007). This example illustrates that the coincidence of the bargaining set and the core holds in all the range of the worth of the grand coalition for which the game is balanced except one value just in the middle of this range. This fact makes the study of the coincidence of the two aforementioned sets far away to be a trivial problem, even when we only move the worth of the grand coalition.

Example 2 (Meertens et al., 2007) *Let $\delta \geq 0$, $|N| = 13$ and $f_\delta : \{0,1,\ldots,13\} \to \mathbb{R}$ be defined as follows:*

s	0	1	2	3	4	5	6	7	8	9	10	11	12	13
f_δ	0	0	0	0	0	22	22	22	22	22	47	47	47	$61.1+\delta$

We define $v_\delta(S) = f_\delta(|S|)$, *for all* $S \subseteq N$. *The authors prove that* $\mathcal{M}_1^i(N, v_\delta) = C(N, v_\delta)$, *for* $\delta \geq 0$, $\delta \neq 0.9$ *and that* $C(N, v_{0.9}) \subsetneq \mathcal{M}_1^i(N, v_{0.9})$.

This interesting example also shows that the coincidence between the bargaining set and the core is not a prosperity property (van Gellekom et al., 1999). Let us recall that the coincidence of the bargaining set and the core would be a prosperity property if, given an arbitrary coalitional game, by increasing only the value $v(N)$, the coincidence will arise at some given moment and will be kept if we go on with increasing this value, and vice versa. Theorem 1 states that there always exists an efficiency level from which the coincidence holds. Example 2 shows that the converse does not hold and illustrates the impossibility to check, in general, the "best bound".

A second remark comes from Example 1. The optimal solution was attained at $\varepsilon^* = 5$ and $a^* = (0, 0, 0, 0, 0)$, that is $a_i^* = v(\{i\}) = 0$, for all $i \in N$. However, we would like to remark that it is not always the case as next example shows.

Example 3 *Let us review Example 1 but now taking* $v(\{1, 2, 3, 4\}) = 2$. *Solving the linear program given in (5) we obtain* $\varepsilon^* = 9$ *and* $a^* = (0, 0, 0, 0, -1)$, *where* $a_5^* \neq v(\{5\}) = 0$. *Notice that if we take* $a = (v(\{i\}))_{i \in N} = (0, 0, 0, 0, 0)$ *we obtain*

$$\max_{S \neq \varnothing, N} \{n \cdot (v(S) - a(S)) + a(N)\} = 5 \cdot \max_{S \neq \varnothing, N} \{(v(S)\} = 5 \cdot 2 = 10 > k^*(\hat{v}) = 9.$$

Finally, let us see that other bargaining sets fail to have the coincidence result showed in Theorem 1. This is the case of the original Mas-Colell bargaining set $\mathcal{MB}(N, v)$, the individual rational Mas-Colell modification bargaining set introduced by Vohra (1991) $IR\mathcal{MB}(N, v)$ and the Mas-Colell bargaining set *à la* Shimomura $\mathcal{MB}_{Sh}(N, v)$ introduced by Shimomura (1997). Next example proves this fact.

Example 4 *Let* (N, v) *be the following family of 5-person games where* $v(\{1, 2\}) = v(\{3, 4\}) = 1$, $v(\{1, 2, 3, 4, 5\}) = 2 + \delta$, $\delta \geq 0$, *and* $v(S) = 0$, *otherwise. It is easy to see that* (N, v_δ), $\delta \geq 0$, *is a family of balanced coalitional games and so* $\delta = 0$ *is when the core starts to be nonempty. In fact,*

$$C(N, v_\delta) = \left\{ (x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5_+ \mid \begin{array}{l} x_1 + x_2 \geq 1, \ x_3 + x_4 \geq 1 \\ x_1 + x_2 + x_3 + x_4 + x_5 = 2 + \delta \end{array} \right\}.$$

Let us see that, independently of the parameter $\delta \geq 0$, *the core and the Mas-Colell bargaining set and its variants do not coincide. To this aim, let us take the allocation* $x^\delta = (0.3, 0.3, 0.3, 0.3, 0.8 + \delta)$, $\delta \geq 0$. *Notice* $x_1^\delta + x_2^\delta = 0.6 < v(\{1, 2\}) = 1$ *which says that* $x^\delta \notin C(N, v_\delta)$, *for any* $\delta \geq 0$.

Now, we check that $x^\delta \in \mathcal{MB}(N, v)$, *for any* $\delta \geq 0$. *Note that only coalitions* $\{1, 2\}$ *and* $\{3, 4\}$ *can make objections at* x^δ, $\delta \geq 0$, *since they are the unique ones where* $x_1^\delta + x_2^\delta < v(\{1, 2\}) = 1$ *and* $x_3^\delta + x_4^\delta < v(\{3, 4\}) = 1$. *Moreover, since* $v(\{1, 2\}) - x_1^\delta - x_2^\delta = v(\{3, 4\}) - x_3^\delta - x_4^\delta = 0.4$, *any objection (in the sense of Mas-Colell) made by one of these coalitions can be countered by the other and vice versa. Therefore, the coincidence result does not hold for the Mas-Colell bargaining set. Moreover, since* $x^\delta \in I(N, v^\delta)$, *for any*

$\delta \geq 0$, the coincidence result also does not hold for the individual rational bargaining set of Vohra. Finally, it can be argued that $x^\delta \in \mathcal{MB}_{Sh}(N, v_\delta) \setminus C(N, v_\delta)$, for all $\delta \geq 0$, since strict objections (à la Shimomura) made via $S = \{1, 2\}$ and $S = \{3, 4\}$ can be countered in a similar way.

Curiously, the Davis and Maschler bargaining set of any game of this family equals its core, that is

$$\mathcal{M}_1^i(N, v_\delta) = C(N, v_\delta), \text{ for any } \delta \geq 0.$$

Roughly speaking, the reason is that if $x \notin C(N, v_\delta)$, because $x_5 > 0$ and $x_1 + x_2 < 1$, then any objection through $S = \{1, 2\}$ of player $i = 1$ to player $j = 5$ cannot be countered. If we are in case where $x_5 = 0$ and $x_1 + x_2 < 1$, then by efficiency $x_3 + x_4 > 1$, and any objection of player $i = 1$ against player $j = 3$ via coalition $S = \{1, 2\}$ cannot be countered. The analysis of eventual objections made through $S = \{3, 4\}$ is analogous to the previous case. Therefore any $x \in \mathcal{M}_1^i(N, v_\delta)$ must be in the core $C(N, v_\delta)$, which reveals a completely different behavior between these two families of bargaining sets.

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