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Quantiles autocorrelation in stock markets returns

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Abstract

Knowledge of dependence pattern in stock market has paramount importance for both theoretical and practical in financial markets. Their usefulness is wide, can be used in portfolio predictability (of portfolio) and risk management. The aim of this paper is to investigate the autoregressive dependence under the alternative perspective of quantile regression. Our study investigates a period from 2001 until 2012 daily returns of twenty stock markets in Latin America, Europe, USA and Asia-Pacific. Our results emphasize that the estimates obtained by quantile regression are different and more consistent than those by AR-GARCH. We conclude also that there is an asymmetric behavior of the investor, in association the quantiles with bear and bull markets.

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1. Introduction

One of the most important stylized facts reported in empirical finance is the autocorrelation coefficient of stock returns. The term autocorrelation is used to denote the variable dependence on its previous price. The ability to predict future behavior of changes in asset prices has been a constant target of studies developed by researchers in finance. This fact is leveraged by the possibility of abnormal gains. However, through various methodologies developed over times, there have been assets with predictable behavior to some degree, as well with no predictable, in different economy segments.

According to Fama (1970), when talking about stock market predictability, one must emphasize the random walk, which brought important contributions to the empirical literature, referring to the fact that future returns are independent of past information. Thus, the random walk hypothesis carries implications as the possibility of prediction, in some way, based on past returns, future returns taking advantage of it to earn extra income.

Campbell, Lo and MacKinlay (1997) distinguish three types of random walks according to the dependence structure of the increment series. Random walk 1 corresponds to independent and identically distributed, random walk 2 to independent but not identically distributed, and random walk 3 to uncorrelated increments (martingale difference sequence). Thus, the efficient markets is more directly associated (realistically) with the random walk 3 and implies the impossibility of using information passed to the definition of abnormal earnings with strategic.

In this paper we used the approach of quantile regression to investigate the predictability of the various conditional return distribution parts of the in a linear autoregressive framework. The use of the idea that predictability is linked to the quantiles has yet been incipient in finance. The contribution of this work is to verify how much is the appropriate AR-GARCH estimates for the identification of autoregressive dependence forward quantiles estimates. Campbell et al. (2008) found that cross-correlations between stock return indices vary systematically across quantiles.

In a recent study Baur, Dimpfl and Jung (2012) found that the autoregressive parameters in the first order autoregressive quantile model in general follow a decreasing pattern over the conditional return distribution quantiles: negative returns (lower quantiles) are generally marked by positive dependence while positive returns (upper quantiles) in general exhibit negative dependence on past returns.

2. Autoregressive Models: Ordinary Least Squares and Quantile Regresion

When the linear dependence between r_t and its historical nearest r_{t-1} is of interest, the concept of correlation is generalized to autocorrelation. The linear regression model $r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t$, often estimated by Ordinary Least Squares – OLS, is obtained minimizing $L = \sum (r_t - \beta_0 - \beta_1 r_{t-1})^2$ for $t = 1, \ldots N$ in relation to the model parameters. There is extensive literature on stock return autocorrelation and great quantitative detail is presented on pages 27 to 80 of Campbell, Lo, and MacKinlay (1997). The basic assumptions of this estimation are homoscedastic, symmetry and mesokurtic pattern. These assumptions are not supported by many empirical asset returns, which tend to have a positive excess kurtosis, skewness and heteroskedasticity (Tsay, 2010).

There are two implications when these basic hypotheses are violated: i) The OLS estimative is still a linear and unbiased estimator, but is not efficient, i.e., there is another estimative with a smaller variance; ii) The standard errors computed for OLS estimative are incorrect, so confidence intervals and hypothesis tests that use these standard errors may be misleading.

Quantile regression introduced for Koenker and Bassett (1978) has recently gaining ground as an alternative to OLS in econometrics applications; see Chuang, Kuan and Lin (2009); Galvão Jr. (2011), Baur, Dimpfl and Jung (2012), Cai and Xiao (2012). The quantile autoregression model is given by the conditional quantile functions. The τ the conditional quantile of r_t conditional on the past information set F_{t-1}, can be expressed as a linear function of r_{t-1} as follows:

$$Q_{r_t}(\tau \left| F_{t-1}\right.) = \beta_0(\tau) + \beta_1(\tau) r_{t-1} + \varepsilon_t \, , \qquad [1]$$

where $Q_{r_t}(\tau \left| F_{t-1}\right.)$ denotes the $\tau -$quantile of r_{t-1} conditional on F_{t-1}. Estimates are obtained by minimizing L in relation to the model parameters as follows:

$$L = \sum_{t: r_t \geq \beta_0 + \beta_1(\tau) r_{t-1}}^{N} \tau \left| r_t - \beta_0(\tau) - \beta_1(\tau) r_{t-1} \right| + \sum_{t: r_t \leq \beta_0 + \beta_1(\tau) r_{t-1}}^{N} (1-\tau) \left| r_t - \beta_0(\tau) - \beta_1(\tau) r_{t-1} \right|, \qquad [2]$$

where $0 < \tau < 1$ and $\beta_1(\tau)$ is the coefficient quantile autoregressive, see Koenker (2005).

As described in Baur, Dimpfl and Jung (2012), we extend the Equation [1] to capture the possible skewness in the autocorrelation caused by signal the previous return. This new model is given by [3].

$$Q_{r_t}(\tau \left| F_{t-1}\right.) = \beta_0(\tau) + \beta_1(\tau) r_{t-1} + \beta_2(\tau) r_{t-1} D(r_{t-1}) + \varepsilon_t \, , \qquad [3]$$

where $Q_{r_t}(\tau \left| F_{t-1}\right.)$, r_{t-1} F_{t-1}, are as in [2] and D is a dummy variable that takes value one if $r_{t-1} < 0$ and zero otherwise.

3. Empirical Results and Discussion

The data used in this study consist of daily closing prices of four Latin America markets – Argentina (Merval), Brazil (Bovespa), Mexico (IPC), Chile (IPSA); seven European markets – France (CAC40), Germany (DAX), United Kingdom (FTSE), Norway (OSEAX), Switzerland (SMI), Austria (ATX), Belgium (BEL20); eight Asia-Pacific emerging markets – China (SSEC), Hong Kong (HSI), India (BSESN), Indonesia (JKSE), Malaysia (KLSE), Japan (NIKKEI), Singapore (STI), Taiwan (TWII) and the United States (S&P500). These markets are the largest and longest operating in the world.

The sample period is from February 7th, 2001 to May 30th, 2012, covering 11 years. In Table 1 we report the sample basic statistics. In order to avoid non-stationarity issues we calculated the market indices log-returns by formulation $r_t = \ln P_t - \ln P_{t-1}$ where r_t is the log-return at period *t*; P_t is the price at period *t*. The maximum number of observations is N = 2887 for France and Belgium and the minimum number of observations N = 2177 for Chile.

Analyzing the values in Table 1, it seems that the return mean across all markets is close to zero, however Latin American and Asia-Pacific markets tend to be positive, in contrast to the European markets which tend to be negative. The standard deviation values show that the developed markets are no less sensitive to fluctuations than emerging ones. Minimum returns are not larger in absolute value compared to positive ones; the same can be said for extreme quantiles (0.1 and 0.9). This may not be surprising, due to the fact that our study covers financial crisis periods which started in 2007-2008 (Baba and Packer, 2009).

Further, with exception of Taiwan, all markets had leptokurtic log-returns, and predominance of negative skewness, stronger than the positive skewness (Righi and Ceretta, 2012). Leptokurtic distributions have higher peaks around the mean if compared to normal distributions, which leads to thick tails on both sides. These peaks result from the data being highly concentrated around the mean, due to lowest variations within observations. When analyzing historical returns, kurtosis helps gauge the level of determined stock market risk. If the past return data yields to a leptokurtic distribution, the stock market will have a relatively low amount of variance, because return values are usually close to the mean. Investors who wish to avoid large, erratic swings in portfolio returns may wish to structure their investments to produce a leptokurtic distribution.

To verify if error variance is constant, we applied test of White (1980). The White test is particularly useful because it makes few assumptions about the likely form of the heteroscedasticity. The p-values for the White test showed in Table 1 indicate for the rejection of the null hypothesis of homoscedasticity for all stock markets. All results associated with kurtosis, skewness and heteroscedasticity contribute to the adequacy of the model quantile autoregressive. One of the important properties of quantile regression is that it is relatively robust to outliers. Such a property is especially attractive in financial applications since many financial data such as stock and portfolio returns are usually heavy-tailed and asymmetrically distributed. Another important aspect of quantile regression is heteroscedastic effects are best accommodated by fit the conditional quantiles, Koenker (2005).

Table 1. Descriptive Statistics for daily log- returns from February 7, 2001 to May 30, 2012.

	Mean (%)	Standard Deviation	Kurtosis	Skew	Min.	Max.	White* p-value	0.1 Quantile	0.9 Quantile
Latin America Markets									
Argentina	0.052	0.022	4.976	-0.141	-0.130	0.161	0.000	-0.024	0.023
Brazil	0.041	0.019	4.141	-0.124	-0.121	0.137	0.000	-0.022	0.022
Mexico	0.063	0.014	4.938	0.016	-0.073	0.104	0.000	-0.015	0.015
Chile	0.065	0.013	115.909	4.942	-0.072	0.279	0.000	-0.011	0.012
European Markets and E.U.A.									
France	-0.023	0.016	4.847	0.051	-0.095	0.106	0.000	-0.018	0.016
Germany	-0.002	0.017	4.081	0.036	-0.074	0.108	0.000	-0.018	0.017
United Kingdom	-0.006	0.013	5.870	-0.115	-0.093	0.094	0.000	-0.015	0.014
Norway	0.029	0.016	5.456	-0.613	-0.097	0.092	0.000	-0.017	0.016
Switzerland	-0.010	0.013	5.752	0.038	-0.081	0.108	0.000	-0.014	0.013
Austria	0.017	0.016	6.777	-0.317	-0.103	0.120	0.000	-0.016	0.016
Belgium	-0.013	0.014	5.642	0.026	-0.083	0.093	0.000	-0.016	0.014
USA	-0.001	0.014	7.809	-0.181	-0.095	0.110	0.000	-0.015	0.014
Asia-Pacific Markets									
China	0.007	0.016	4.162	-0.122	-0.093	0.094	0.000	-0.019	0.018
Hong Kong	0.006	0.016	8.324	0.002	-0.136	0.134	0.000	-0.018	0.017
India	0.047	0.016	7.059	-0.105	-0.118	0.160	0.000	-0.018	0.018
Indonesia	0.080	0.015	6.400	-0.720	-0.110	0.076	0.000	-0.016	0.017

Malaysia	0.028	0.011	100.245	-0.228	-0.192	0.199	0.000	-0.009	0.010
Japan	-0.015	0.016	6.828	-0.391	-0.121	0.132	0.000	-0.018	0.017
Singapore	0.012	0.013	5.666	-0.331	-0.092	0.075	0.000	-0.014	0.013
Taiwan	0.008	0.015	2.061	-0.197	-0.069	0.065	0.000	-0.017	0.017

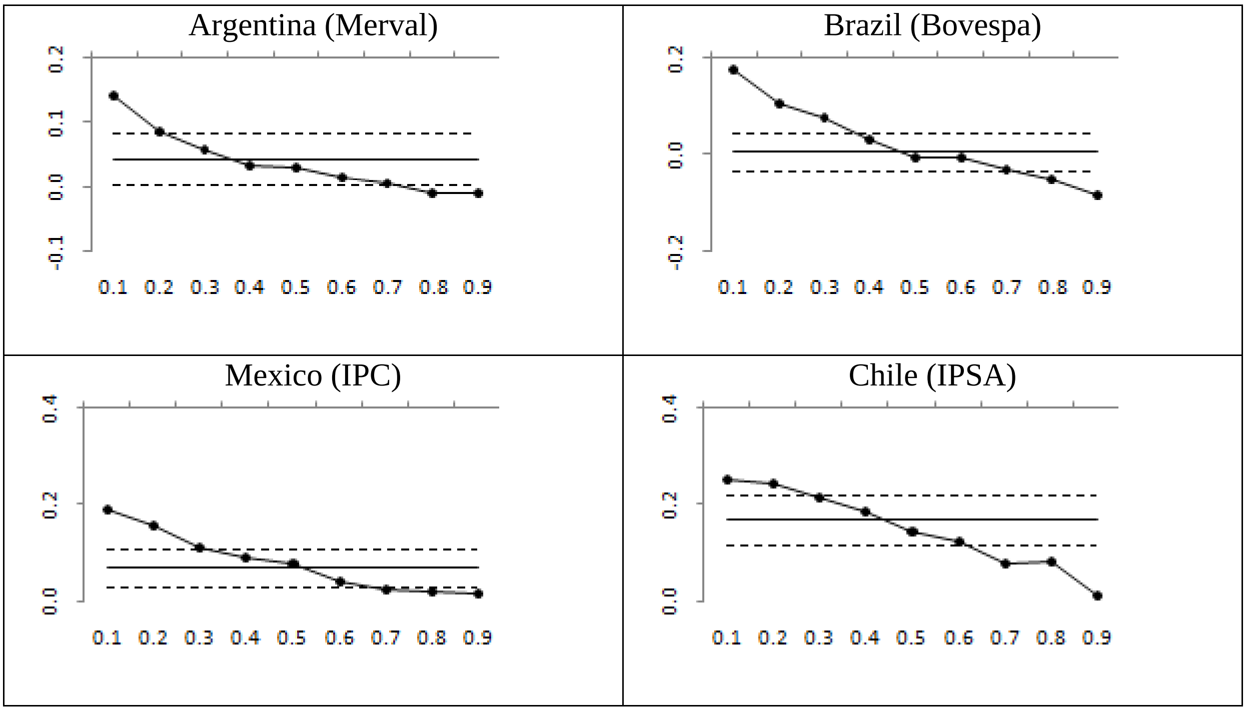
* We also applied ARCH test and p-values indicated rejection of the null hypothesis of homoscedasticity for all stock markets.

We first calculated the autocorrelation coefficient of each index market. The Figures 1, 2 and 3 exhibit the plots of these autocorrelation coefficients [$\beta_1(\tau)$ in equation1] for Latin America, European and Asia-Pacific markets, respectively.

Regarding to Latin American markets, for lowest quantiles, we find high and positive autoregressive coefficient estimates whereas for the upper quantiles coefficient estimates tend to be low. Coefficient estimates for central quantiles are limited by the AR(1)-GARCH(1,1) coefficients and are close to zero.

We also found that only the lowest quantiles have autoregressive coefficient estimated significantly different from zero while the central and upper quantiles do not. Only Brazil has coefficient estimated significantly different from zero for upper quantiles.

Figure 1. Quantile Autocorrelation for selected Latin America markets

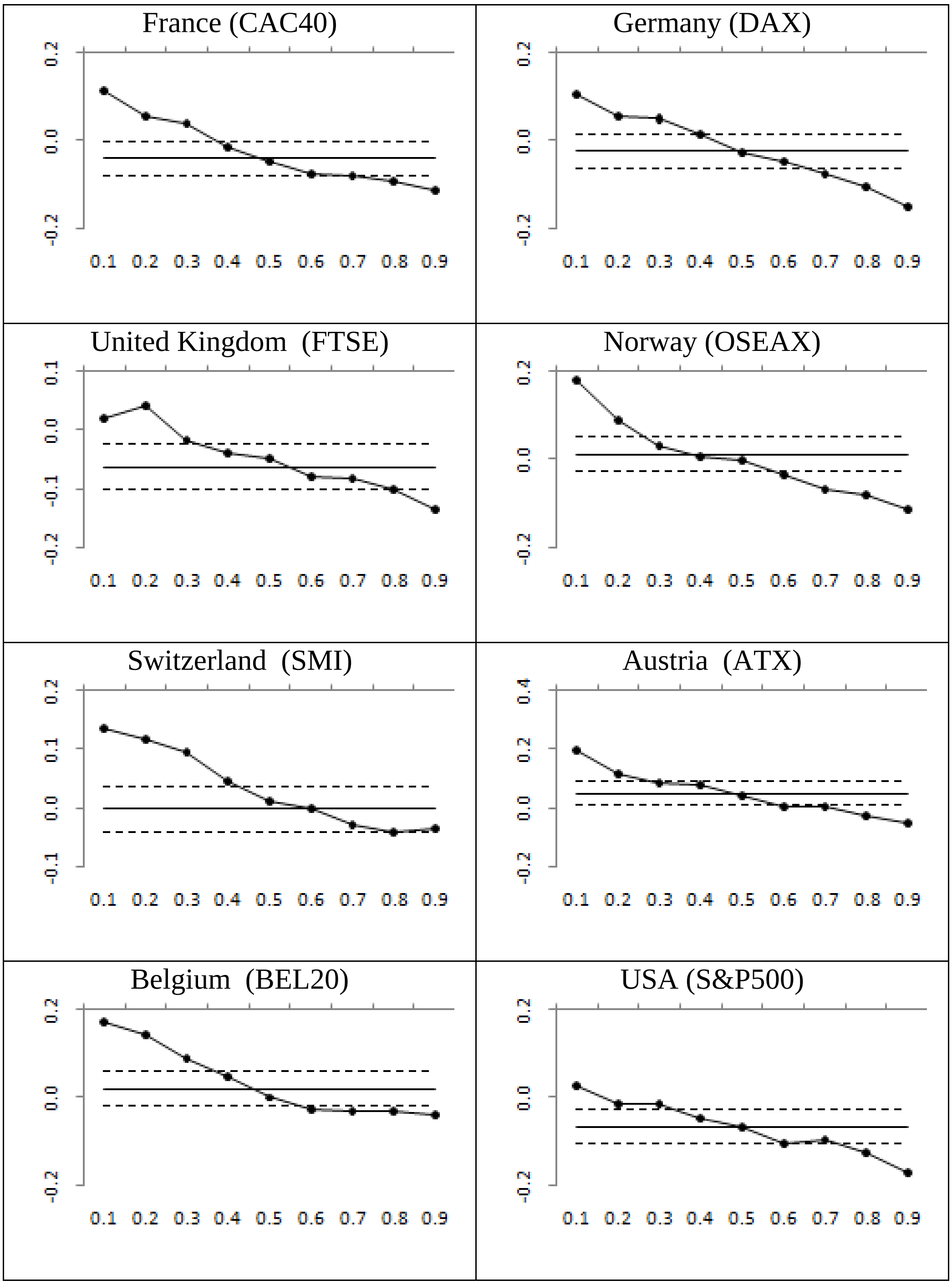


Plot of the estimated $\beta_1(\tau)$ coefficient for 0.1 to 0.9 quantiles of the Equation [1] $Q_{r_t}(\tau|F_{t-1}) = \beta_0(\tau) + \beta_1(\tau) r_{t-1} + \varepsilon_t$. The solid and dashed lines represents, respectively, the AR(1)-GARCH(1,1) coefficients and boundaries for 95% confiance.

Concerning about Figure 2, which represents European and U.S. markets, we find similar pattern to the Latin America markets, i.e. for lowest quantiles, we find high and positive coefficient autoregressive whereas estimates for the upper quantiles coefficients tend to be low.

Coefficient estimates for central quantiles are limited by the AR-GARCH coefficients and are close to zero. We also found that only in United Kingdom and United States markets are not significant coefficient for the lowest quantiles. Only for Austria and Belgium markets the estimated coefficients are not significant for upper quantiles.

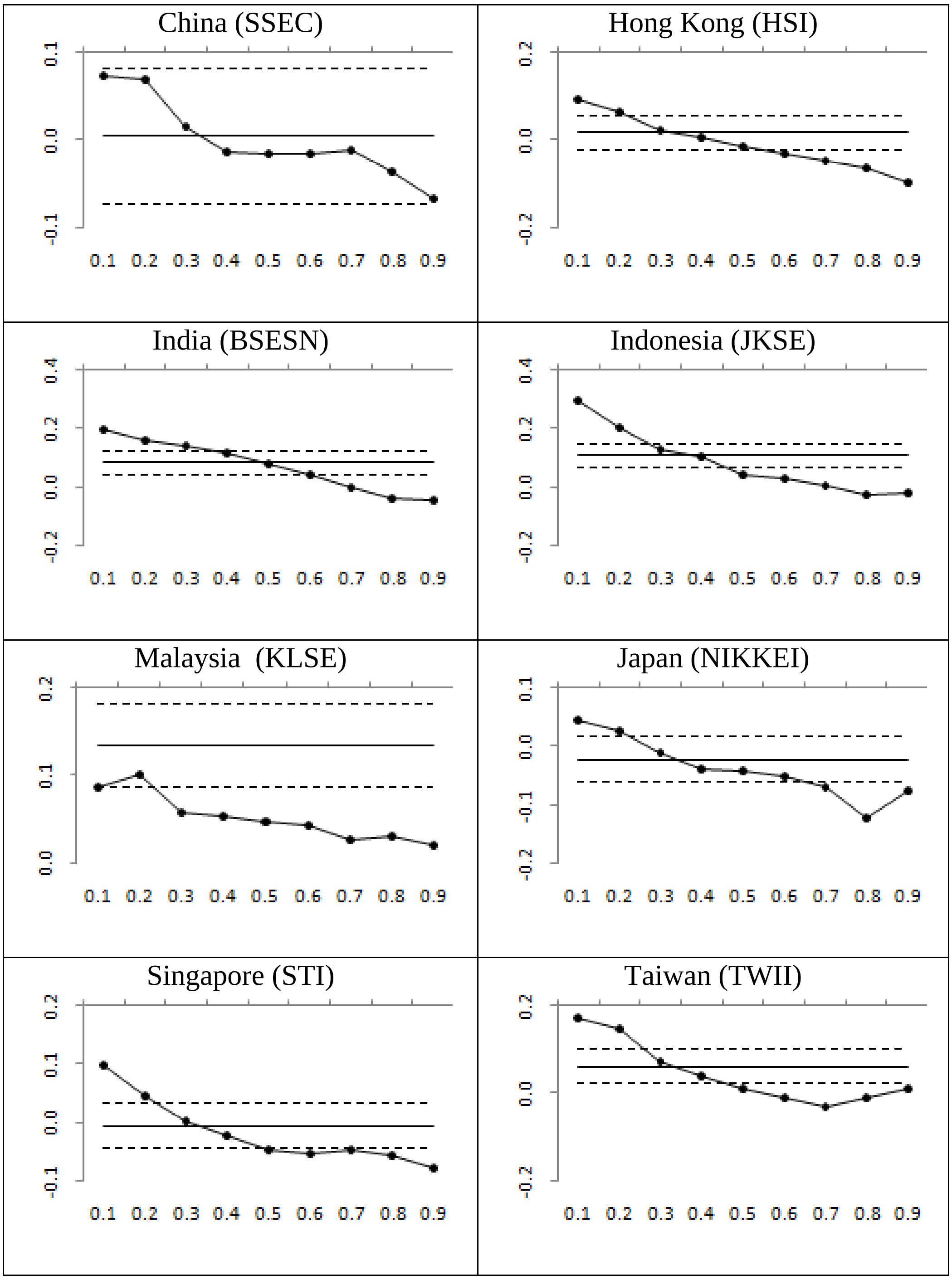
Figure 2. Quantile Autocorrelation for selected European markets and E.U.A.



Equation [1] $Q_{r_t}(\tau | F_{t-1}) = \beta_0(\tau) + \beta_1(\tau) r_{t-1} + \varepsilon_t$.

The coefficients estimated for Asia-Pacific also have similar pattern, as exhibited in Figure 3. We find that only the Chinese and Japanese markets had not significant coefficients for the lowest quantiles. Only for China, Indonesia, Malaysia and Taiwan markets the estimated coefficients were not significant for upper quantiles. Malaysia has a completely different standard autoregressive parameter obtained by AR-GARCH, i.e., for all the quantiles the estimated coefficients are high and significantly different from zero.

Figure 3. Quantile Autocorrelation for selected Asia-Pacific markets



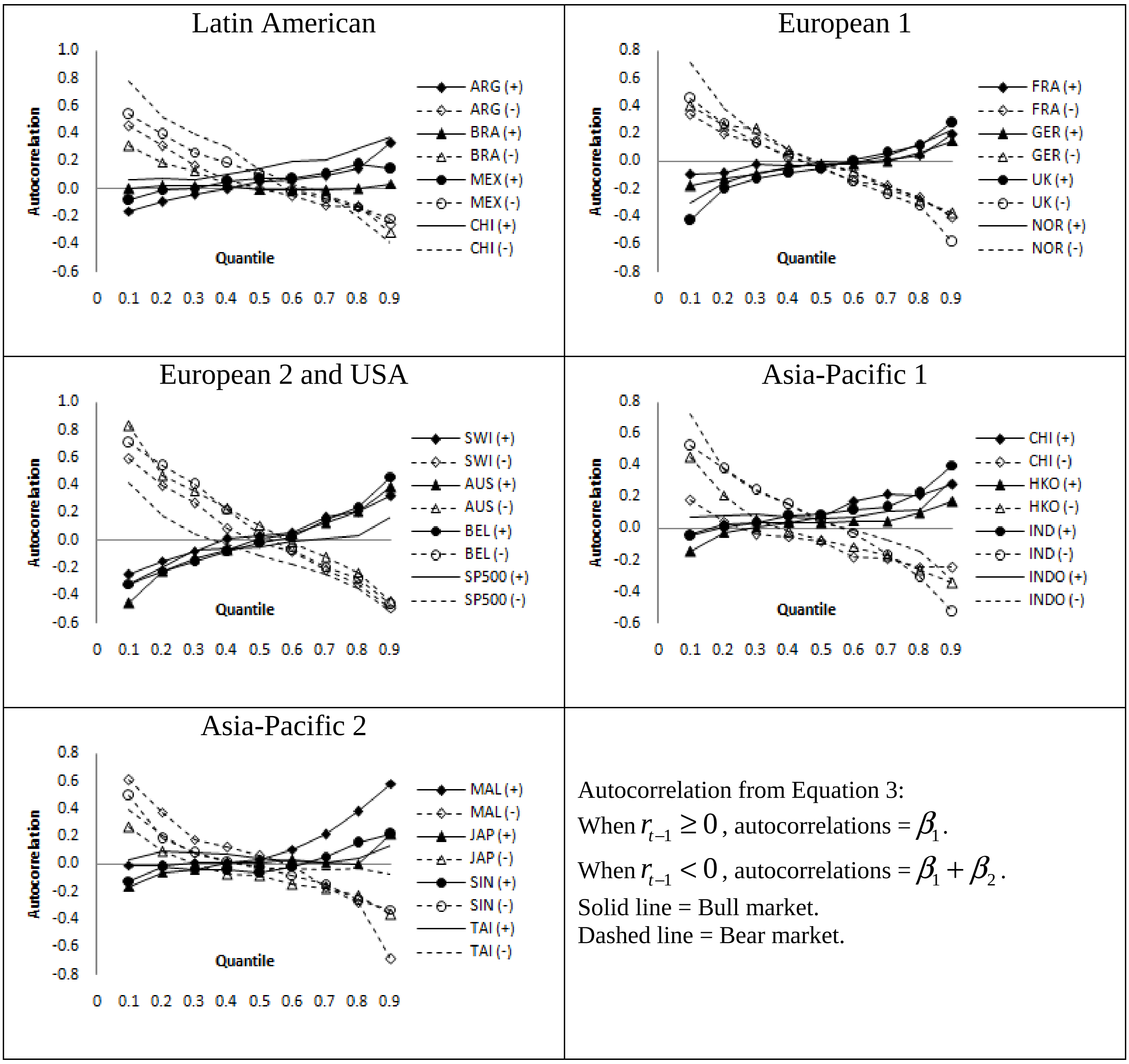
Equation [1] $Q_{r_t}(\tau | F_{t-1}) = \beta_0(\tau) + \beta_1(\tau) r_{t-1} + \varepsilon_t$.

The Figures 1, 2 and 3, show that in general, for all stock markets, positive autoregressive coefficient estimates for lowest quantiles whereas for the upper quantiles, coefficient estimates tend to be negative. Although, central quantiles as well as the AR-GARCH coefficients estimate are close to zero. In contrast to the quantile autocorrelation method, the autoregressive coefficient estimates for AR-GARCH implies, in average, that (on) yesterday's average returns have no impact (or small impact) on today's returns.

Figure 4 illustrates the quantiles autocorrelations coefficients when $r_{t-1} \geq 0$ (solid line) and when $r_{t-1} < 0$ (dashed line). These results are obtained with the application of Equation [3]. If $r_{t-1} < 0$, the quantile autocorrelations coefficient is associated with the bear market. If $r_{t-1} \geq 0$ the quantile autocorrelation is associated with bull market. Independent of the market index there is a similar pattern in previous day dependence.

The Equation 3 considers the possibility that lagged positive and negative returns have asymmetric impact on the conditional return distribution. Figure 4 illustrates that quantile autocorrelation coefficient with the inclusion of the negative returns as explanatory variable completely alters the pattern of autoregressive estimates.

Figure 4. Quantile Autocorrelation when $r_{t-1} \geq 0$ and $r_{t-1} < 0$



The results indicate that investors have an asymmetric reaction to situations of bull and bear markets. Thus, the reactions of investors depend on quantile and market conditions.

In the lowest quantiles and in situations of bear market there is positive dependence. Positive and negative returns tend to persist. However, in situations of bull market, the opposite happens. If the return yesterday was negative, the influence today is positive, on the other hand, if the return yesterday was positive, today will be negative.

In the upper quantiles and in situations of bear market there is negative autocorrelation, but it is not as strong as the positive autocorrelation that occurs in lowest quantiles. On the other hand, in situations of bull market dependence becomes positive but it is not as strong as the positive dependence in the lowest quantiles in situations of bear market. This suggests that the combination of lowest quantile and bear market is stronger and more persistent than any other combination scenario.

We sought to verify the difference that occurs between the autocorrelation coefficients in the various quantiles. In Table 2 we present the difference test p-values (equation 2) between the coefficients autocorrelations for bear and bull markets.

Table 2. P-values of the difference between the autocorrelation coefficients for situations of bull and bear markets

Markets		Quantiles								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Latin America Markets										
Argentina	p-value	**0.000**	**0.000**	**0.013**	0.411	0.445	0.074	**0.002**	**0.003**	**0.000**
Brazil	p-value	**0.022**	**0.028**	0.198	0.875	0.990	0.869	0.553	0.192	**0.000**
Mexico	p-value	**0.000**	**0.000**	**0.005**	0.064	0.664	0.186	**0.004**	**0.000**	**0.001**
Chile	p-value	**0.000**	**0.000**	**0.000**	0.060	0.888	0.055	**0.002**	**0.002**	**0.000**
European Markets and E.U.A.										
France	p-value	**0.003**	**0.006**	**0.011**	0.316	0.683	**0.048**	**0.002**	**0.001**	**0.000**
Germany	p-value	**0.000**	**0.000**	**0.000**	0.074	0.957	0.348	**0.012**	**0.000**	**0.000**
United Kingdom	p-value	**0.000**	**0.000**	**0.002**	0.100	0.807	**0.028**	**0.000**	**0.000**	**0.000**
Norway	p-value	**0.000**	**0.000**	**0.005**	0.071	0.793	0.362	**0.011**	**0.000**	**0.000**
Switzerland	p-value	**0.000**	**0.000**	**0.000**	0.426	0.307	0.070	**0.000**	**0.000**	**0.000**
Austria	p-value	**0.000**	**0.000**	**0.000**	**0.000**	0.207	0.426	**0.003**	**0.000**	**0.000**
Belgium	p-value	**0.000**	**0.000**	**0.000**	**0.000**	0.411	0.187	**0.000**	**0.000**	**0.000**
USA	p-value	**0.000**	**0.003**	0.171	0.639	0.346	**0.003**	**0.000**	**0.000**	**0.000**
Asia-Pacific Markets										
China	p-value	0.189	0.891	0.289	0.074	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**
Hong Kong	p-value	**0.000**	0.078	0.603	0.214	**0.025**	**0.000**	**0.005**	**0.000**	**0.000**
India	p-value	**0.001**	**0.001**	**0.004**	0.337	0.533	0.056	**0.002**	**0.000**	**0.000**
Indonesia	p-value	**0.000**	**0.016**	0.088	0.226	0.702	0.246	**0.008**	**0.005**	**0.000**
Malaysia	p-value	**0.000**	**0.001**	**0.025**	0.058	0.695	0.234	**0.003**	**0.000**	**0.000**
Japan	p-value	1.000	0.156	0.652	0.252	**0.024**	**0.015**	**0.001**	**0.025**	**0.000**
Singapore	p-value	**0.001**	**0.009**	0.062	0.263	0.427	0.394	**0.010**	**0.000**	**0.000**
Taiwan	p-value	**0.021**	0.303	0.905	0.430	0.185	0.290	0.451	0.333	0.086

Equation [2] $Q_{r_t}(\tau | F_{t-1}) = \beta_0(\tau) + \beta_1(\tau) r_{t-1} + \beta_2(\tau) r_{t-1} D(r_{t-1}) + \varepsilon_t$.

The p-values reported in Table 2 show that there is an investor asymmetric behavior regarding to bear markets and bull markets. These differences are significant for all markets except China and Japan in the lowest quantiles and Taiwan in upper quantiles. Therefore, the reaction of investors in the quantile depends on the type (bull or bear) return that occurred yesterday.

4. Conclusions

In this paper we attempted to verify the autoregressive dependence in stock markets in Latin America, Europe, USA and Asia-Pacific during the recent period of 2001 to 2012. We chose to assess quantile autoregressive coefficients comparing them with estimates for Ordinary Least Squares coefficients. After the initial comparison, we verified the presence of asymmetric behavior of the investor associated with dependence on returns from yesterday in the various quantiles. We work with daily log-returns, which presented characteristics that favor the application of quantile regression. The returns showed asymmetric, leptokurtic and heteroscedastic behavior. These characteristics are already well documented in the financial literature and somewhat reduce the robustness of the estimates obtained by AR-GARCH which are widely used in the autocorrelation analysis.

The initial analysis identified a large difference in the dependence between today with yesterday's returns obtained by AR-GARCH and quantile regression. The dependence obtained by AR-GARCH tends to be close to zero and has low significancy. On the other hand, the quantile autoregressive coefficients showed a decreasing pattern of behavior over the quantiles. In lowest quantiles autoregressive dependence is positive and highly significant, as it travels to the higher quantiles autoregressive coefficient gradually becomes positive and in some markets it is significant.

In a more specific analysis, where we tried to identify an investor asymmetric behavior, it is proved that the positive dependence for lowest quantiles occurs in bear markets and bull markets for upper quantiles. The negative dependence occurs exactly in the opposite way. It occurs in lowest quantiles for bull markets and upper quantiles for bear markets. Thus, the investor behavior is conditioned to the specified quantile and market situation. These results corroborate the results of previous studies and somehow partly contradict the ideas of Fama (1970).

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