

**Volume 32, Issue 3****Long-Run Debt Sustainability and Threshold Adjustments: Non-Linear Empirical Evidence from the GIIPS**

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We assess the sustainability of the public finances of Greece, Ireland, Italy, Portugal and Spain (GIIPS), allowing for possible non-linearities in the form of threshold behaviour of the fiscal authorities. We provide some evidence of fiscal sustainability when debt gets “too high” relative to a threshold which is not necessarily fixed but varies with the level of debt relative to its recent history and/or the occurrence of a financial crisis. However, the Greek and Italian debt-to-GDP threshold levels (over which adjustment takes place) exceed 87% and rise further in periods of financial crises. This arguably adds to international investors' concerns, and as a result, raises the yields demanded for holding Greek and Italian debt. As debt is rolled over at high interest rates, fiscal prospects worsen making default more likely and adding to contagion effects from one Eurozone country to another.

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## 1. Introduction

Following from the recent financial crisis and the remarkable fiscal stimulus to drive the world economy out of recession, world markets have increasingly drawn their attention to the excessive debt of Greece, Ireland, Italy, Portugal and Spain (GIIPS). Greece was bailed-out twice (for €110bn in May 2010 and then again for €109bn in July 2011). In October 2011 and then again in February 2012, Greece negotiated a new €130bn rescue package involving a voluntary “haircut” of some 53.5% on the face value of its bonds held by the private sector. Ireland was bailed-out once (for €85bn in November 2010) and Portugal was also bailed once (for €78bn in May 2011) by the European Union, the European Central Bank and the International Monetary Fund. Despite the bail-outs, international markets remain extremely volatile and worried that the debt levels of all GIIPS are unsustainable (this is reflected, for instance, on Spanish and Italian government yields that are on the rise) posing a risk to the whole Euro zone<sup>1</sup>. These worries appear justifiable as the GIIPS, which account for around 35% of Euro zone’s GDP, currently run debt-to-GDP ratios well above the 60% threshold set by the Maastricht (1993) Treaty

Sustainability of the government’s intertemporal budget constraint (IBC), within a dynamically efficient economy, requires fiscal policies to satisfy the present value borrowing constraint, i.e. that the present value of outlays (current and future) equals the present value of revenues (current and future). This is equivalent to the imposition of a no-Ponzi game condition on the debt dynamics, preventing the government to pay interest on old debt by issuing new debt. Empirical tests on the IBC sustainability have generally been based on public debt unit root tests, cointegration tests between government revenues and expenditures (see, e.g. Quintos, 1995) and estimation of fiscal reaction functions (Bohn, 2007). Focusing our attentions on existing estimates for the GIIPS, we note that Afonso (2005) uses revenue-expenditure cointegration tests to report that most EU countries are at risk of unsustainability. However, Bohn (2007) warns against interpreting failure of stationarity and cointegration as evidence of unsustainable fiscal policy. Greiner et al. (2007), based on the fiscal reaction function, conclude that both Portuguese and Italian public finances are sustainable. All above tests, nevertheless, are implicitly based on a linear model of continuous fiscal adjustment. However, Bertola and Drazen (1993) argue that, due to difficulties in reaching necessary consensus for fiscal retrenchments, fiscal authorities initiate a corrective action only when the disequilibria reach a given trigger point, for instance when spending reaches levels high enough to be deemed critical. Only in this latter case, the necessary agreement can be reached and adjustment takes place. This suggests the opportunity of allowing for threshold behaviour of fiscal authorities, reacting only when fiscal variables exceed an endogenously estimated threshold. Applied to our fiscal policy set up, traditional linear tests might mistakenly suggest that given countries are on an unsustainable fiscal policy pact, when in fact, their IBC holds. Existing non-linear sustainability tests include (amongst others) Sarno (2001) who provides evidence of threshold behaviour for the US debt-to-GDP. Arghyrou and Luintel (2007) estimate

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<sup>1</sup> In July 2012, Euro-zone finance ministers approved a bank bailout of up to €100bn for Spain’s banks. In September 2012, the European Central Bank approved a plan paving the way for the ECB to make unlimited purchases in struggling euro members' bond markets (such as Italy and Spain) with the aim of lowering their government yields. The plan was conditional on struggling governments to sign on to a euro-zone program of budgetary discipline.

threshold revenue-expenditure models and report that Greece, Italy, Ireland and the Netherlands are fiscally sustainable. Chortareas et al. (2008) apply a non-linear unit root test to selected Latin American and Caribbean country debt series. Fincke and Greiner (2011) use a model of time-varying coefficients (on the grounds that any nonlinear model is approximated by a linear model with time-varying coefficients; see Granger, 2008), to infer that among EU countries, Greece and possibly Italy are fiscally unsustainable. Legrenzi and Milas (2012) provide evidence of non-linear adjustment for the Italian public finances.

Using long historical data on the debt-to-GDP ratios of the GIIPS, we extend previous literature by evaluating debt sustainability based on a number of non-linear models with fixed and time-varying thresholds. We provide evidence that fiscal sustainability occurs when debt gets “too high” relative to a threshold which is not necessarily fixed but varies with the level of debt relative to its recent history and/or the occurrence of a financial crisis.

The paper proceeds as follows. Section 2 discusses datasets and reports estimates of our empirical models for the GIIPS. Section 3 summarises our findings and concludes.

## 2. Data sets and empirical models

Long historical data for the GIIPS (reported in Figure 1) are available from Carmen Reinhart’s website at the Peterson Institute of International Economics (<http://www.carmenreinhart.com/data/>). For Greece and Italy, debt-to-GDP ratios refer to general government debt. For Portugal, Spain and Ireland, debt data refer to central government debt (general government debt data are only available from the early 1970s for Portugal and Spain and from 1980 for Ireland; their correlation with the ones used here are 0.72 for Portugal, 0.96 for Spain and 0.93 for Ireland, respectively). Figure 2 plots a composite measure of financial turmoil/crisis (which draws heavily on Reinhart and Rogoff, 2009). This is a world financial crisis measure which takes into account banking, currency, stock market, debt, and inflation incidences in the world. The index pools together world’s 20 largest economies with country specific weights given by their relative GDP share of the total GDP (based on Purchasing Power Parity). We also tried country-specific indices but empirical results were very poor and for this reason not reported.

Linear unit root tests (not reported for space considerations but full details available on request) suggest that all debt-to-GDP ratios are non-stationary. To examine this issue further, we proceed by considering the non-linear model of the form

$$\Delta x_t = \beta_0 + \beta_1 x_{t-1} \theta_{t-1} + \beta_2 x_{t-1} (1 - \theta_{t-1}) + \beta_l(L) \Delta x_{t-1} + u_t, \quad (1)$$

where  $x_t$  is the debt-to-GDP ratio,  $\beta_l(L)$  is a polynomial in the lag operator  $L$ ,  $u_t$  is a stochastic error term  $u_t \sim i.i.d.(0, \sigma_u^2)$  and

$$\theta_{t-1} = 1 - \frac{1}{1 + e^{-\gamma (x_{t-1} - \tau) / \sigma(x_{t-1})}} \quad (2)$$

is the logistic transition function discussed in e.g. van Dijk et al (2002). According to (1)-(2), when  $x_{t-1}$  is below the threshold  $\tau$ , the mean reversion is given by  $\beta_1$ , but when  $x_{t-1}$  is above the threshold  $\tau$ , the mean reversion is given by  $\beta_2$ . The parameter  $\gamma$ ,  $\gamma > 0$  determines the smoothness of the transition regimes. We make  $\gamma$  dimension-free by dividing it by the standard deviation of  $x_{t-1}$  (Granger and Teräsvirta, 1993).

However, there might be good reasons in favour of a time-varying threshold. If, for instance, countries are already running excessive debt-GDP ratios, the risk of triggering a recession might deter policymakers from bringing debt quickly back to a particular threshold. Instead, corrective (and smoother) action might be taken towards a time-varying threshold of the form:

$$\tau_t = \mu\tau + (1 - \mu)\left\{\frac{1}{n}\sum_{j=1}^n \text{debt\_GDP}_{t-j}\right\}, \quad (3)$$

Where  $\mu$  is the weight on a fixed threshold,  $\tau$  (such as the one estimated in (2)) and  $(1 - \mu)$  is the weight on past debt-GDP ratios. In what follows, we use  $n=4$  years; this (in general) corresponds to a particular government holding office and running its economic programme for a 4-year period (we also experimented with values of  $n$  up to 8 but results were less satisfactory in statistical terms).

We also allow for the possibility that corrective action depends on a measure of the state of the world financial crisis:

$$\tau_t = \tau_0 + \tau_1 \text{fin\_crisis}_t, \quad (4)$$

Where  $\tau_0$  is a fixed threshold and  $\tau_1 > 0$  ( $< 0$ ) implies that during a financial crisis, policymakers are willing to raise (lower) the debt ceiling above which corrective action is taken. For instance, the fear of a deep and lasting recession might lead to a higher debt ceiling ( $\tau_1 > 0$ ), or the fear of a debt downgrade by credit rating agencies (which will make debt servicing more difficult) might lead to a lower ceiling ( $\tau_1 < 0$ ).

Table 1A reports estimates of the non-linear models (1)-(2). The estimated and statistical significant thresholds are 88% for Greece, 50% for Ireland, 93% for Italy and 59% for Spain. Below these thresholds, corrective action/adjustment is insignificant (see estimates of  $\beta_1$ ). Above these thresholds, adjustment is significant (7% per annum for Greece, 6% for Ireland, 6.4% for Italy and 3.2% for Spain). The estimated thresholds for Greece and Italy are remarkably close to the 90% threshold that Reinhart and Rogoff (2010) deem to be of threat for the growth prospects of a particular country. The estimated thresholds for Spain and Ireland are not far from the 60% benchmark threshold of Maastricht's (1993) Treaty. Although not directly comparable (recall our use of central rather than general government debt for these two countries), we note that general government debt (as percentage of GDP) has exceeded, on average, central debt by 3 percentage points for Ireland and by 8 percentage points for Spain. Our model is unsuccessful for Portugal as all coefficients are statistically insignificant. This is also the case for the remaining models with time-varying coefficients in Tables 1B and 1C below (for this reason we drop further reference to Portugal). This might be because either Portugal's debt is unsustainable, or because this type of non-linear model is not able to explain the debt process (at the end of the day, there are infinite non-linear models).<sup>2</sup> Therefore, more research is needed to clarify this.

Table 1B reports estimates of the non-linear models (1)-(3). There is evidence of time-varying thresholds as a weighted average of threshold values very similar to those reported earlier and the debt-GDP values of the recent past. Compared with the remaining countries, Ireland "sticks" more with the estimated threshold ( $\tau = 47\%$ ) as

<sup>2</sup> As an alternative to (2), we tried for Portugal and the remaining countries an exponential function which allows for large versus small deviations from a threshold (see e.g. Sarno, 2001). However, we failed to get converging estimates.

it gives only 20% weight  $(1 - \mu)$  to past debt-GDP ratios; in this model, stronger correction takes place for Ireland below the estimated threshold. Table 1C reports estimates of the non-linear models (1)-(2) and (4). For Greece and Italy, the debt ceiling is raised during a financial crisis (i.e.  $\tau_1 > 0$ ), whereas the opposite is true for Ireland and Spain (i.e.  $\tau_1 < 0$ ). In terms of regression standard errors, models (1)-(3) provide a better fit for Greece, whereas models (1)-(2) and (4) provide a better fit for Ireland, Italy and Spain. All models pass parameter stability at the 10% level.

### 3. Summary and conclusions

Allowing for debt adjustment to depend on a threshold that varies with debt levels of the recent past and the impact of a financial crisis, we find evidence of fiscal sustainability for Greece, Ireland, Italy and Spain. However, the high Greek and Italian threshold levels over which adjustment takes place, rises further in periods of financial crises. This arguably adds to international investors' concerns, and as a result, raises the yields demanded for holding Greek and Italian debt. As debt is rolled over at high interest rates, fiscal prospects worsen making default more likely and adding to contagion effects from one Eurozone country to another. With this in mind, it would make sense to allow for potential cross-dependence amongst the European countries by estimating jointly debt-to-GDP equations as a non-linear panel (this model is in the spirit of a multi-sector smooth transition autoregressive model; see Fok et al, 2005). We intend to address this issue in future research.

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**Table 1A:** Estimates of non-linear models (1)-(2) (with fixed threshold  $\tau$ )

	Greece Sample 1850-2010	Ireland Sample 1924-2010	Italy Sample 1861-2010	Portugal Sample 1850-2010	Spain Sample 1850-2010
$\beta_0$	5.733 (3.27)	4.044 (1.91)	3.115 (2.64)	0.151 (0.09)	2.282 (1.86)
$\beta_1$	-0.040 (0.09)	-0.115 (0.08)	-0.019 (0.04)	-0.023 (0.03)	-0.051 (0.05)
$\beta_2$	-0.070 (0.02)	-0.060 (0.02)	-0.064 (0.02)	-0.040 (0.06)	-0.032 (0.01)
$\beta_t$	0.112 (0.08)	0.454 (0.10)	0.340 (0.07)	0.070 (0.06)	0.373 (0.07)
$\tau$	88.10 (31.12)	50.20 (19.38)	93.1 (30.1)	40.21 (64.1)	59.11 (24.21)
$\gamma$	10.11 (-)*	30.02 (-)	34.12 (-)	10.23 (-)*	9.10 (-)
Regression standard error	15.84	5.00	8.14	4.35	6.94
Parameter stability ( $p$ -value)	0.10	0.10	0.11	0.09	0.11

**Table 1B:** Estimates of non-linear models (1)-(3) with time-varying threshold

$$(\tau_t = \mu\tau + (1 - \mu)\{\frac{1}{n} \sum_{j=1}^n debt_{t-j} - GDP_{t-j}\})$$

	Greece Sample 1850-2010	Ireland Sample 1924-2010	Italy Sample 1861-2010	Spain Sample 1850-2010
$\beta_0$	8.001 (4.47)	5.921 (2.17)	2.891 (2.55)	2.601 (2.62)
$\beta_1$	-0.020 (0.07)	-0.198 (0.07)	-0.004 (0.03)	-0.010 (0.08)
$\beta_2$	-0.080 (0.02)	-0.080 (0.01)	-0.082 (0.03)	-0.021 (0.01)

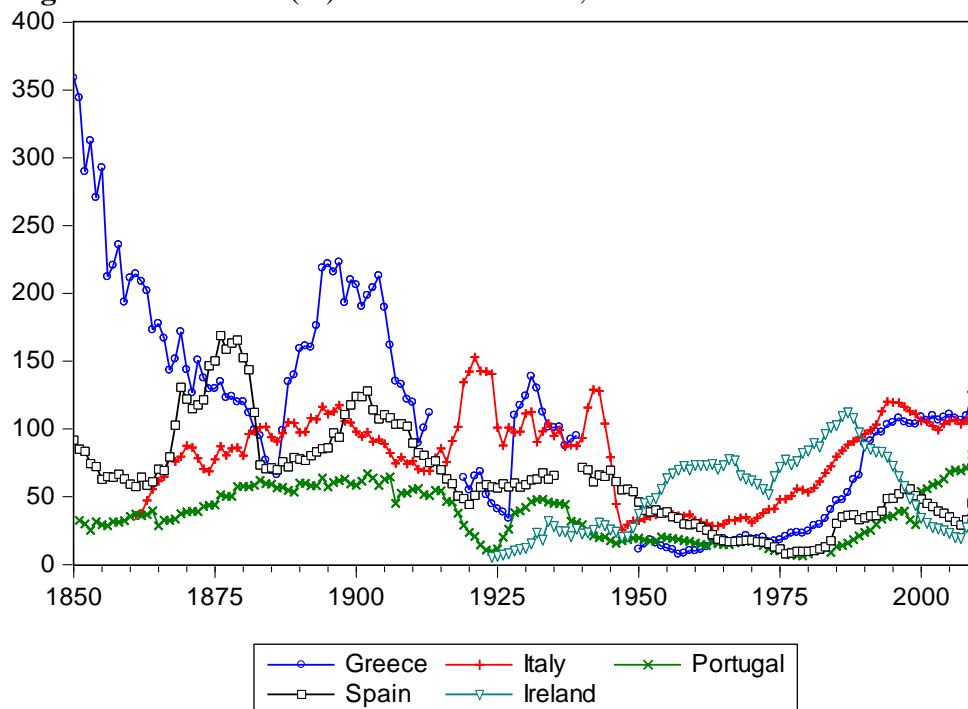
$\beta_1$	-0.040 (0.08)	0.416 (0.09)	0.380 (0.09)	0.396 (0.08)
$\tau$	87.20 (30.12)	47.14 (16.40)	94.92 (25.1)	59.01 (24.21)
$\gamma$	50.20 (-)*	33.26 (-)*	16.21 (-)*	22.20 (-)*
$\mu$	0.50 (-)*	0.80 (-)*	0.30 (-)*	0.58 (-)*
Regression standard error	14.50	4.86	7.90	6.95
Parameter stability ( <i>p</i> -value)	0.12	0.10	0.11	0.11

**Table 1C:** Estimates of non-linear models (1)-(2), (4) with time-varying threshold ( $\tau_t = \tau_0 + \tau_1 fin\_crisis_t$ )

	Greece Sample 1850-2010	Ireland Sample 1924-2010	Italy Sample 1861-2010	Spain Sample 1850-2010
$\beta_0$	4.030 (3.94)	4.764 (2.06)	2.727 (2.24)	2.666 (1.61)
$\beta_1$	-0.032 (0.08)	-0.152 (0.09)	-0.017 (0.030)	-0.070 (0.05)
$\beta_2$	-0.060 (0.02)	-0.064 (0.02)	-0.070 (0.03)	-0.040 (0.01)
$\beta_t$	-0.100 (0.08)	0.470 (0.10)	0.360 (0.08)	0.377 (0.07)
$\tau_0$	87.12 (31.12)	49.20 (10.11)	94.13 (30.20)	59.22 (25.22)
$\tau_1$	5.45 (1.24)	-4.13 (1.20)	13.13 (3.43)	-4.20 (1.23)
$\gamma$	5.32 (-)*	30.10 (-)*	54.12 (-)*	39.11 (-)*
Regression standard error	15.85	4.60	7.70	6.80
Parameter stability ( <i>p</i> -value)	0.11	0.11	0.11	0.11

Notes: Number in () are standard errors. \*Imposed value. van Dijk et al. (2002) argue that the likelihood function is very insensitive to  $\gamma$ , suggesting that precise estimation of this parameter is unlikely. For this reason, we run a grid search in the range [0.1, 250] and fix the  $\gamma$  parameter to the one that delivers the best fit of the estimated models. We set  $l=1$  and  $n=4$  above. In Table 1A, estimates of  $\mu$  are based on a grid search in the [0.1, 0.99] range. Parameter stability is an F test of parameter stability (see Lin and Teräsvirta, 1994).

**Figure 1: Debt/GDP (%) data for the GIIPS, 1850-2010**



**Figure 2: World composite measure of financial crises, 1850-2010**

