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Third-Party Budget Breakers and Side Contracting in Team Production

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Abstract

In a team production problem with unverifiable effort, budget breaking is essential to implementing efficient levels of effort. This short paper considers the use of a third party, who does not exert effort, in a setting with general contracts that can include message games, as a way to remove resources from the team. I show that if side contracting can influence behavior in a message game in the original contract, the addition of the third party is not helpful. My view of side contracting complements that of Baliga and Sjostrom (2009) in exploring the nature of side contracting that is needed in order for the third party to be useful for budget breaking.

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1. Introduction

In a team-production problem with unverifiable effort, budget breaking is essential to implementing efficient levels of effort. This budget breaking requires a third party who is to receive payments. However, budget breaking can be disrupted by side contracting between the third party and one of the team members. Baliga and Sjöström (2009) provide a novel contract that avoids the disruption of budget breaking when side contracting is possible. My analysis here is intended to complement Baliga and Sjöström's analysis in understanding the nature of side contracting that does and does not disrupt budget breaking and the implementation efficient levels of effort. I consider a version of Holmstrom's (1982) deterministic team production model that explicitly allows for a third-party budget breaker, side contracting between the third party and a team member, and message games. I show that if side contracting can influence behavior in a message game in the original contract, the addition of a third party is not helpful.

Consider the following standard team-production problem. Players $1, 2, \dots n$ choose costly unverifiable effort, which determines the value of production. Player i's effort is denoted by $a_i \in A_i$, and we let $A = \times_{i=1,\dots n} A_i$. The value of production, joint monetary output, is verifiable and given by $q : A \to \mathbf{R}$, and q is strictly increasing, concave, and differentiable with $q(0) = 0$. Player i's cost of effort is given by $v_i : A_i \to \mathbf{R}$, and v_i is strictly convex, differentiable, and increasing with $v_i(0) = 0$. An externally enforced contract is a function s from output to shares, satisfying that the sum of the shares does not exceed output. Player i's payoff is given by $s_i(x) - v_i(a_i)$, where $s_i(x)$ denotes the share of $x = q(a)$ that player i receives.

Holmstrom's Theorem 1 shows that when budget balance is required, $\sum_{i=1,\dots n} s_i(x) = x$, it is not possible to implement the optimal level of effort a^*. The optimal effort levels a^* solve

$$\max_{a \in A} [q(a) - v_1(a_1) - v_2(a_2) - \cdots - v_n(a_n)].$$

However, when the budget does not have to be balanced, Holmstrom's Theorem 2 shows that a^* can be implemented. Thus, if all of the output can be given to a third party, a^* can be implemented.[1] Holmstrom noted that if collusion between one of the players and the third party was possible, the usefulness of the third party as a budget breaker may be negated. Eswaran and Kotwal (1984) show this to be the case. However, neither of these studies allows for general contracts with message games. I allow for more general contracts and formally model the scope for side contracting with a third party who is used as a budget breaker.

My main result shows that if side contracting can influence behavior in a message game in the original contract, the addition of the third party is not helpful for breaking the budget. My view of side contracting is presented in Section 2. In Section 3 I discuss the relationship

[1]The example in his proof specifies

$$s_i(x) = \begin{cases} b_i & \text{if } x \geq q(a^*) \\ 0 & \text{if } x < q(a^*) \end{cases} .$$

The b_i's are chosen so that $\sum_{i=1,\dots,n} b_i = q(a^*)$ and $b_i > v_i(a_i^*) > 0$.

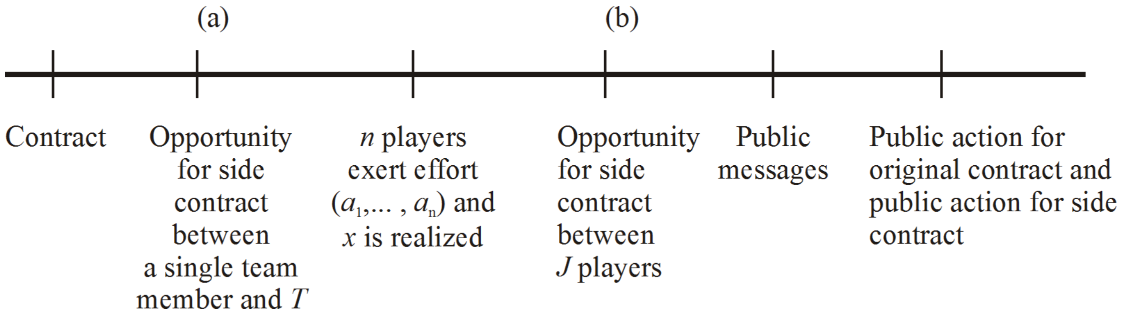

Figure 1: Timing of events.

of my model of side contracting with that of Baliga and Sjöström (2009), who also allow for general contracts and side contracting, and show, under their view of side contracting, the third party is helpful as budget breaker. Again, my results complement Baliga and Sjöström in understanding the nature of side contracting that is needed in order for the third party to be useful for budget breaking. Section 4 briefly concludes.

2. Incorporating Messages and Side-Contracting

I allow for a message game in both the original contract and the side contract. As in the Holmstrom setting, the team members' unverifiable effort choices deterministically lead to verifiable output x. I assume that a is commonly observed by the players.[2]

Prior to choosing effort, the team members and the third party contract. The timing of the players' interaction is illustrated in Figure 1. The contract specifies an allocation of the output x between all of the agents, which also represents transfers because negative allocations are allowed, as a function of verifiable items. The verifiable items include both the realized output x and messages that the players send following the realization of x. The players select messages from a grand message space $M \equiv \times_{i \in N} M_i$, where M_i is the set of feasible messages for player i and $N \equiv \{1, \ldots n, T\}$. A contract is $F : M \times \mathbf{R} \to \mathbf{R}^{n+1}$, where $F(m, x)$ is the allocation imposed given message profile m and output x. Player i's allocation, given a message profile and realized output is denoted by $F_i(m, x)$. I require, due to the possibility of renegotiation, that the players' allocations are balanced.

I now turn to opportunities for side contracting. Side contracting is not observed by the other players, and a side contract may contain a message game. A side contract is $f : M \times \mathbf{R} \to \mathbf{R}^{n+1}$, with $\sum_{j \in N} f_j = 0$ and $f_k = 0$ for all k not involved in the side contract. Note that the message profile that the side contract conditions on is the same one on which the original contract conditions. One interpretation is that both the original contract and the side contract condition on the same court proceedings and the information that comes out of those proceedings. Note that for a sufficiently rich grand message space, it could be as though there are messages for the original contract and messages for the side contract, and that both contracts can condition on the messages of the other. I do not allow the original

[2]It is straight forward to extend my results to the case where effort choices are private information.

contract to condition on whether there is a side contract, and neither contract can condition on the public action imposed by the other contract.

I allow for two opportunities for side contracting; these are labeled (a) and (b) in Figure 1. Recognizing that side contracting at either of these opportunities will disrupt using the third party to break the budget, I require that the contract the players agree to be *impervious to side contracting* at either opportunity. I follow Bull and Watson's (2004) treatment of coalitions for settings with externally-enforced side contracts.

Before describing the equilibrium/solution concept, which requires that contracts are impervious to side contracting, it is helpful to first examine what side contracts can achieve, given fixed F and behavior. This allows us to consider the potential for a side contract between one of the team members and the third party, as described at (a) in Figure 1. Fix F and behavior of the $-i$ team members at a_i, and assume that the messages of the $-i$ team members are functions of x and a. I denote player j's message choice by $\mu_j(a, x)$, and focus on pure strategies. It is possible for player i and the third party to use the side contract as a "forcing" contract to induce any effort level by player i, a_i, and any messages by player i and the third party as a function of x, $\mu_i(x)$, and $\mu_T(x)$. Here I consider the players side contracting over messages as a function of output, and not effort, because output is verifiable and effort is not.

Lemma 1: *Consider a side-contract between player i and the third party T, given an original contract and holding fixed the associated a^*_{-i} that would be induced without a side-contract. Any a_i, $\mu_i(x)$, and $\mu_T(x)$ can be implemented with a forcing contract between i and T.*

Proof: Hold fixed, the behavior of the players other than player i and the third party (the $-i$ players). Note that it is possible to induce any messages from i and T, as a function of x, with a side-contract. Given x the side-contract can specify an arbitrarily large transfer from a player who unilaterally deviates from $(\mu_i(x), \mu_T(x))$ to the non-deviating player. Given this, it is also possible to specify any attainable x, given the anticipated play of a^*_{-i}, with a forcing contract. This simply requires imposing a large transfer from player i to T if the level of x that corresponds to the desired a_i and anticipated a^*_{-i} is not observed. That is, if $q(a_i, a^*_{-i})$ is not realized, require that i pay an arbitrarily large transfer to T. *Q.E.D*

Thus, a team member i and the third party T can effectively act as single player.

I also consider side deals, between two or more players, concerning only messages and assume that these can be formed after the realization of output x but before the disclosure of messages. This opportunity is denoted by (b) in Figure 1. These side deals are also private and are not observed by the other players until after messages have been presented. Thus, the point of writing a side contract is to induce members of the coalition to change their behavior in the original-contract message game in a way that benefits the coalition. To evaluate whether this is possible, define $F_J(m) \equiv \sum_{j \in J} F_j(m)$, where $J \subset N$ is the coalition, and let $f_J(m)$ be defined analogously. Suppose that following effort profile a, and the corresponding output x, the players would coordinate on disclosure of messages $\mu(a)$ in the absence of side contracting. Further suppose that, by side contracting, a coalition J can induce its members to disclose messages $\hat{m}_J$. Then the coalition strictly gains from the side deal if and only if

$$F_J(\mu_{-J}(a), \hat{m}_J) + f_J(\mu_{-J}(a), \hat{m}_J) > F_J(\mu(a)) + f_J(\mu(a)),$$

which is equivalent to $F_J(\mu_{-J}(a), \hat{m}_J) > F_J(\mu(a))$ because $f_J = 0$. Here, "$-J$" denotes the complement of J.

In a manner similar to that used to induce shirking and then coordination on messages, the parties to such a side deal can force disclosure of specific messages. Specifically, a coalition J can specify f so that any player $j \in J$ who does not disclose a specified message must pay an amount y to each of the other players in the coalition.[3] Thus, after the realization of x and before the disclosure of messages, coalitions can effectively spot contract on which messages to disclose. In practical terms, a side contract may amount to joint disclosure of messages. Furthermore, side contracts can be used by a coalition to rearrange transfers internally.

I require that the original contract is impervious to side contracting at both opportunities, (a) and (b), for side contracting. Following Bull and Watson (2004), I use the term impervious to side contracting instead of *coalition-proof Nash equilibrium* since I'm dealing with externally-enforced contracts instead of self-enforced contracts (Nash equilibria).[4] I examine F and behavioral rule a, $\mu : A \to M$, and require that it passes the impervious to side contracting test.

Definition 1: *Given an externally-enforced contract F, a behavior rule a and μ is called **impervious to side contracting (ISC)** if*

(i) $F_{i,T}(\mu(a), q(a)) \geq F_{i,T}(\mu(a'_i, a_{-i}), q(a))$, for each i and a'_i, and

(ii) $F_J(\mu(a'), q(a')) \geq F_J(m', q(a'))$ for each a', each $q(a')$, each coalition $J \subset N$, and each $m' \in M$ that is a J-deviation from $\mu(a')$ at a' and $q(a')$.

Note that the ISC condition includes deviations by a single player, so every ISC disclosure rule is also an equilibrium disclosure rule.

The combination of the ISC condition and the constant-sum aspect of externally-enforced transfers prevents the existence of *message game phenomena*. Thus, the externally-enforced transfers imposed following a given level of output x must be the same for any effort levels a that led to x.

Lemma 2: *Consider any a' and a'' such that $x = q(a') = q(a'')$. For any ISC message rule $\mu(a)$, it must be that $F(\mu(a'), x) = F(\mu(a''), x)$.*

Proof: Suppose not. Then it must be for some $a' \neq a''$ such that $x = q(a') = q(a'')$ that $F(\mu(a'), x) \neq F(\mu(a''), x)$ and for some $J \subset N$ $F_J(\mu(a'), x) < F_J(\mu(a''), x)$.

Consider the message profile $(\mu_J(a''), \mu_{-J}(a'))$. Note that coalition J can induce this as a J-deviation from $\mu(a')$ and coalition $-J$ can induce this as a $-J$-deviation from $\mu(a'')$. In

[3] Then any sub-coalition $K \subset J$ will lose at least y when one or more of its members deviates from the prescription of f. The number y can be set large enough so that this loss is greater than any gain the sub-coalition can get by way of the original contract F.

[4] That is, ISC is Bernheim, Peleg, and Whinston's (1987) definition applied to externally enforced contracts. Here, the issue of sub-coalitions is easily handled because forcing contracts can always be designed to stifle any further side dealing by sub-coalitions.

order for μ to be ISC, given F, it must be that such a J-deviation is deterred following a', which requires

$$F_J(\mu_J(a''), \mu_{-J}(a'), x) \leq F_J(\mu(a'), x).$$

Additionally, deterring the $-J$-deviation from $\mu(a'')$ requires

$$F_{-J}(\mu_J(a''), \mu_{-J}(a'), x) \leq F_{-J}(\mu(a''), x),$$

which, because transfers must be balanced, is equivalent to

$$F_J(\mu_J(a''), \mu_{-J}(a'), x) \geq F_J(\mu(a''), x).$$

Combining yields

$$F_J(\mu(a'), x) \geq F_J(\mu_J(a''), \mu_{-J}(a'), x) \geq F_J(\mu(a''), x),$$

but we have assumed $F_J(\mu(a'), x) < F_J(\mu(a''), x)$. So this is a contradiction. *Q.E.D.*

Since any suboptimal value of output, $x < q(a^*)$, can be induced by any of the team members unilaterally deviating from a^* this prevents the contract from implementing the optimal effort level a^*.

Corollary 1: *Adding a third party to this team production problem does not allow efficient levels of effort a^* to be implemented.*

Adding the third party does not change the set of implementable effort profiles.

Theorem 1: *The set of implementable effort profiles a is the same with or without a third party.*

Proof: Without the third party, consider message game $F(\mu(a), x)$. In order for a to be implementable, there must be an $F(\mu(a), x)$ and an ISC message rule $\mu(a)$ such that selecting a is an equilibrium given F.

For $\mu(a)$ to be an ISC message rule, we need

$$F_J((\mu_J(a_J), \mu_{-J}(a_{-J}), x) \geq F_J((\mu'_J, \mu_{-J}(a_{-J}), x), \text{ for all } \mu'_J, a, x = q(a), J.$$

Given the ISC message rule, define $g(a) \equiv F(\mu(a), x)$. For a to be an equilibrium, given $g(a)$ requires

$$g(a_i, a_{-i}) - v_i(a_i) \geq g(a'_i, a_{-i}) - v_i(a'_i), \text{ for all } a_i, i.$$

Since we can view a colluding player and the third party as one player, these constraints do not change with the addition of a third party. *Q.E.D*

3. Relationship to Baliga-Sjöström

When it is not possible for a side contract to condition on the messages conditioned on by the original contract, there is scope for using a third party to break the budget.[5] Baliga and Sjöström (2009) consider a model in which the original contract and the side contract each have a private message game with private messages so a contract cannot condition on the messages of the other contract, and show the novel result of a third party can be beneficial for budget breaking. In this section, I follow their focus on teams comprised of two players. The timing in their model is the same as that of my model with the exception of the separate private message games for each contract. The message game for the original contract is played first, its allocation/transfer is imposed, and then the message game for the side contract is played. They assume, as I do, that the side contract cannot condition on the outcome of the original contract. Baliga and Sjöström's analysis demonstrates that the efficacy of third-party budget breaking relies on the side contract being unable to condition either on messages in the original contract or on the allocation imposed by enforcement of the original contract.[6]

Preventing side contracting, and the subsequent shirking, requires that the message game of the original contract can deter side contracting. For any $x < q(a^*)$ there are two unilateral deviations from a^* that can cause x: a unilateral deviation by player 1 and a unilateral deviation by player 2. In order to prevent side-contracting, there must be two equilibria following any such $x < q(a^*)$: one that "punishes" player 1 and one that "punishes" player 2. Consider player 1's potential side-contracting and deviation to $a_i^* - \varepsilon$. Player 1 must be required to pay a transfer to player 2 if we are to deter the side-contract. If instead the outcome of the message game required that player 1 pay a transfer only to the third party, this could be anticipated and accounted for in the side-contract between player 1 and the third party. So the transfer from player 1 to player 2 must be sufficiently large that it offsets any possible payment the third party would be willing to make to induce collusion. Thus, if the original contract specifies that the third party receive all of x when $x < q(a^*)$, the transfer from player 1 to player 2 must be at least x.

Lemma 3: *Any contract with $n = 2$, with messages games, that uses a third party to break the budget must induce two equilibria after any $x < q(a^*)$. The first of these must require that player 1 pay a positive transfer to player 2, and impose a lower overall transfer for player 1 than in the second equilibrium. The second equilibrium must require that player 2 pay a positive transfer to player 1, and impose a lower overall transfer for player 2 than in the first equilibrium.*

Baliga and Sjöström recognized this and cleverly constructed "whistle-blowing" equilibria, which rely on the third party naming, in the original contract message game, the team

[5] I feel that in most practical settings it's possible for the side contract to condition on those messages. So I study the model presented above.

[6] If the side contract can condition on the allocation imposed by enforcement of the original contract, the side-contracting players can effectively commit to behavior in the original contract even when the side contract cannot condition on messages in the original contract message game, which prevents using the third party to break the budget. The longer working paper version of my paper contains a formal treatment of this.

member with whom he colluded, when the realized output is not optimal.[7] In such a case, the team member, say team member i, is required to pay sufficiently large transfers to both the third party and the other team member j.

This equilibrium construction requires that the third party is indifferent as to which team member he names. To see this, suppose instead that the third party is to receive a larger transfer from team member i when he names team member i than he receives from j when he names j. Then team member j and the third party have the incentive to collude (so that j shirks) anticipating that the third party will name i—not j—as having colluded with him. Such an incentive would prevent the original contract message game from deterring side contracting, and prevent third-party budget breaking. So the "whistle-blowing" equilibria deter side contracting when the third party "rats" on his partner to the side deal. However, the third party is actually indifferent as to which team member he names.

Baliga and Sjöström's "whistle-blowing" equilibria are not the only way to prevent side contracting. Original-contract message games that contain versions of the familiar method of "Nash implementation with harsh punishments" will also deter collusion.[8] Under such a method, the team members are essentially forced to agree on which of them colluded or each must pay a large transfer to the third party. Of course, this method of deterring side contracting may be considered problematic because, as with the typical use of harsh punishment Nash implementation, there are multiple equilibria. Baliga and Sjöström do not consider this type of construction because they are concerned with strong implementation.

4. Conclusion

In this short paper, I have explored the use of message games to potentially allow a third party to be useful for breaking the budget in a team production problem. I have shown that when it is possible for the side contract to condition on the same messages as the original contract, the third party is of no use for breaking the budget and does not help in attaining efficient effort. Due the difference in assumptions about the effectiveness of side deals, my result differs from the novel result of Baliga and Sjöström (2009).

5. References

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[7] They require that, when team member i is named by the third party, an allocation such that i receives $-2Z$, j receives Z, and the third party receives $x + Z$. They show (their Theorem 3) there is $Z > 0$ will make this construction work.

[8] The earlier working paper version of this paper contains more discussion of this.

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