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Multipliers: A brief note on spending efficiency

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Abstract

This note discusses expenditure multipliers and their use in analyzing spending efficiency. Conceptual models are developed to examine the marginal propensities; namely consumption and investment and their role in influencing the expenditure multiplier, aggregate spending cycles, consumption and investment efficiencies, and aggregate income generation. Additional models are developed to examine the marginal propensities and their impact on the expenditure multiplier when the assumptions governing marginal propensity behavior are relaxed. It is suggested in this note that relaxing the assumptions governing the marginal propensities to consume and invest will alter the growth implications upon which multiplier analysis is based.

1. Introduction

This note discusses expenditure multipliers and their use in analyzing aggregate spending efficiency. Multipliers have a long history in the economics literature and have been used to examine a number of macroeconomic phenomena that are cumulative or cyclical in nature. Such phenomena include multiplier impacts on aggregate income and output resulting from capital investment (Clark 1935; and Keynes 1936), multipliers in combination with accelerators to evaluate economic fluctuations (Samuelson 1939), employment growth in urban and non-urban areas (Braschler and Kuehn 1976; and McDonald 1997), human capital inefficiencies (Lee 2011), impacts of expectations on spending cycles (Westerhoff 2006), and monetary and fiscal policy multipliers (Froyen 2009). As documented in the economics literature, multipliers can be used to measure more than just the impacts of investment alone (Samuelson and Nordhaus 2001).

In the spirit of gaining a better understanding of how changes in aggregate spending may impact an economy and aggregate income, this note attempts to extend the uses of basic multiplier analysis by utilizing the expenditure multiplier as a means of studying aggregate spending efficiency.

Conceptual models are developed in this note to examine the marginal propensities; namely consumption and investment and their role in influencing the expenditure multiplier, aggregate spending cycles, consumption and investment efficiencies, and aggregate income generation. Additional models of aggregate expenditure are also developed in this note to examine the marginal propensities and their impact on the expenditure multiplier when the assumptions governing marginal propensity behavior are relaxed. It is suggested in this note that relaxing the assumptions governing the marginal propensities to consume and invest will alter the growth implications upon which multiplier analysis is based.

2. Spending Cycles

As mentioned by Westerhoff (2006) and Dornbusch, Fisher, and Startz (2004), multipliers may be important in the generation of business and consumer spending cycles. The following model is an extension of the traditional Keynesian multiplier and attempts to capture the essence of business and consumer spending inter-linkages by developing a model that provides a sequence of linked multipliers in examining the dynamics of aggregate spending.

Given that Y is aggregate or national income, C is consumption, and I is net investment, following Samuelson (1971), an equation of national income can be specified as

$$Y = C(Y) + I(Y), \quad (1)$$

where consumption and investment are assumed to be induced. It is also assumed here that government spending, net exports, prices, and interest rates are constant. Additionally, it is assumed that the economy has excess production capacity or is expanding. Performing the necessary manipulations to the equation in (1) and specifying the resulting multipliers as a flow cycle allows a sequence of multipliers to be illustrated as

$$\Delta Y / \Delta I = 1 / 1 - \Delta C / \Delta Y \rightarrow \Delta Y / \Delta C = 1 / 1 - \Delta I / \Delta Y. \quad (2)$$

The multiplier on the left of the arrow is the investment multiplier. The multiplier on the right of the arrow is the consumption multiplier. The sequence of expenditures proceed from left to right such that $\Delta Y / \Delta I \rightarrow \Delta I / \Delta Y$. A change in investment leads to a change in aggregate income which leads to changes in consumption. These changes in consumption then lead to secondary changes in aggregate income that result in changes in investment. In effect then, an expenditure cycle emerges from the sequence. Please note that the second multiplier has a sequence that makes it similar to the accelerator.

Solving for ΔY on both sides of the arrow in (2) and writing the results as $\Delta Y = \Delta C + \Delta I$ yields

$$2 \Delta Y = [1 / (1 - \Delta I / \Delta Y)] \Delta C + [1 / (1 - \Delta C / \Delta Y)] \Delta I. \quad (3)$$

Please note that the notion of adding multipliers is borrowed from Froyen (2009). Subtracting the marginal propensities from one in the denominators in (3) allows (3) to be rewritten as

$$2 \Delta Y = [1 / (\Delta C / \Delta Y)] \Delta C + [1 / (\Delta I / \Delta Y)] \Delta I. \quad (4)$$

Rewriting the relationship in (4) gives us

$$2 \Delta Y = (\Delta Y / \Delta C) \Delta C + (\Delta Y / \Delta I) \Delta I. \quad (5)$$

Cancelling ΔC and ΔI allows (5) to be restated as

$$\Delta Y_C + \Delta Y_I = 2 \Delta Y, \quad (6)$$

where Y_C is the amount of aggregate income that results from consumption and Y_I is the amount of aggregate income that results from investment spending. Dividing (6) by 2 gives us

$$1/2 \Delta Y_C + 1/2 \Delta Y_I = \Delta Y. \quad (7)$$

For a greater degree of generality, if $\beta = 1/2$ then (7) can be rewritten as

$$\beta_1 \Delta Y_C + \beta_2 \Delta Y_I = \Delta Y. \quad (8)$$

Based on the equation in (4), the individual marginal propensities can be manipulated to serve as the individual spending efficiencies, $\beta = 1/2$, in the particular structure displayed in (8). Additionally, as can be seen from (2) and (3), the derived equation in (8) is cyclical or circular in nature. Therefore, theoretically speaking, once the marginal propensities or changes in investment and consumption efficiencies are converted into parameters, the right side of (8) will add to one. Also note that because β_1 and β_2 on the left side of (8) are identical in value, each component will contribute half to the growth in aggregate spending. This result may possibly have some policy implications. For instance, if we examine the U.S. economy where consumption spending represents roughly 70% of all aggregate spending, if there is a serious disruption in consumption spending the U.S. economy may experience undesirable declines. However, if the U.S. economy had a balanced spending structure, when one sector is affected with spending disruptions, other sectors may be able to somewhat offset those spending declines.

Suppose equation (8) had two additional spending sectors; government spending and net exports. If all the sectors, including the new ones, contributed equally to aggregate spending, then instead of each sector contributing half, as reported in equation (8), each sector would contribute twenty five percent. This type of spending balance is so much more efficient than an economy that depends on one sector contributing 70% to aggregate spending.

3. Expenditure Multipliers and the Marginal Propensity Assumptions

The next two sections of this note examine the impacts that the assumptions governing the marginal propensities will have on the values of the expenditure multiplier. The present section examines the assumptions of the marginal propensities as they currently exist. The next section examines the expenditure multiplier when the assumptions governing the marginal propensities are relaxed or altered.

The marginal propensity assumptions basically state that consumption, saving, and investment will increase with an increase in income but that the increase in consumption, saving, and investment will be less than the increase in income (Froyen 2009). Thus, the marginal propensity assumptions dictate that when increases in income are greater than increases in consumption, saving, and investment, the expenditure multipliers will take on positive values or

$$\text{if } \Delta C / \Delta Y < 1, \text{ then } \Delta Y / \Delta I > 1, \quad (9)$$

$$\text{if } \Delta I / \Delta Y < 1, \text{ then } \Delta Y / \Delta C > 1. \quad (10)$$

As can be seen from (9) and (10), because Y is increasing faster than C and I respectively, money will still be available for other purposes; namely investment and consumption. In effect, aggregate income will likely increase not only as a result of the initial change in consumption and investment, but will also increase as a result of the secondary increases in investment and consumption. Therefore, both multipliers in (9) and (10) will be positive. Also note that the accelerator is displayed in the relationship in (10).

4. Relaxing the Marginal Propensity Assumptions

This section examines the expenditure multipliers when the assumptions governing the marginal propensities are dropped. We will first examine the expenditure multipliers when the marginal propensities are greater than one, and then will examine the expenditure multipliers when the marginal propensities are equal to one.

Dropping the marginal propensity assumptions allows the economic system to be examined when it does not behave according to traditional economic theory. Essentially, when the marginal propensities are greater than one, consumption and investment expenditures will increase at greater rates than increases in income. When this occurs it is highly possible, and likely, that consumers will spend more than their income (dissave), and investors will commit more investment funds than they have (over-leverage). When consumers dissave and investors become over-leveraged, the marginal propensity assumptions have been violated.

Typically, as was stated in the previous section, the expenditure multipliers operate on the assumption that the marginal propensities are less than one or $\Delta C / \Delta Y < 1$ and $\Delta I / \Delta Y < 1$. When the marginal propensities are greater than one the multipliers will take on the following properties,

$$\text{if } \Delta C / \Delta Y > 1, \text{ then } \Delta Y / \Delta I < 1, \quad (11)$$

$$\text{if } \Delta I / \Delta Y > 1, \text{ then } \Delta Y / \Delta C < 1. \quad (12)$$

When the marginal propensities are equal to or approach one the multipliers will take on the following properties,

$$\text{if } \Delta C / \Delta Y = 1, \text{ then } \Delta Y / \Delta I = \infty, \quad (13)$$

$$\text{if } \Delta I / \Delta Y = 1, \text{ then } \Delta Y / \Delta C = \infty. \quad (14)$$

The results reported in (11) and (12) suggest that if the rates of consumption and investment increase faster than the rate of income, the expenditure multiplier will take on a negative value. In other words, for every unit increase in income, consumption and investment will increase by greater than one unit. If this occurs, multiplier theory suggests that negative exponential income growth will ensue. When C and I increase faster than Y , money will not be available for other purposes; namely investment and consumption. If money is not available for investment, then aggregate production capacity cannot be expanded to meet increases in demand. If money is not available for consumption, the increasing production capacity will just be wasted because of the lack of demand. Thus, when consumption and investment increase faster than increases in income, declines in aggregate income may be likely

The results reported in (13) and (14) are more difficult to explain. The problem is, how should infinity ∞ be interpreted? As $\Delta C / \Delta Y$ and $\Delta I / \Delta Y$ approach 1, the denominator in the multiplier approaches zero. Division by zero is not permitted. Therefore, because division by zero is not mathematically possible, when the marginal propensities equal one, ∞ can be thought of as some type of mathematical boundary. In other words, once the marginal propensities are equal to one, any further increase in the value of the expenditure multiplier is mathematically impossible. On the other hand, if the problem is interpreted as a limit problem, as the marginal propensities approach one, before the denominator hits absolute zero, it will take on an infinite number of extremely small values. These increasingly small values in the denominator will make the value of the expenditure multiplier increasingly large until zero is reached. The interpretation here is left open to the reader.

5. Some Final Comments on Spending Efficiency

The marginal propensity to consume and the marginal propensity to invest can be evaluated as forms of spending efficiency. For instance, if we examine the marginal propensity to consume $\Delta C / \Delta Y$, we can evaluate this ratio as the incremental change in consumption resulting

from an incremental change in income. In other words, this ratio represents the amount of change in spending resulting from a change in income. What we are interested in is how much spending or consumption will occur resulting from this increase in income. There are three overall possibilities for evaluating this: $\Delta C / \Delta Y < 1$, $\Delta C / \Delta Y > 1$, and $\Delta C / \Delta Y = 1$. When $\Delta C / \Delta Y < 1$, this represents spending inefficiency because for every dollar change in income spending increases by less than a dollar. The money that is left after consumption has taken place is equivalent to saving. When $\Delta C / \Delta Y > 1$, this represents spending efficiency because for every dollar change in income, spending will change by greater than one dollar. This type of spending occurs perhaps when consumers are extended credit or when consumers borrow against future income. Obviously, there are no savings available when this form of spending efficiency occurs. In fact, the consumer may actually find him/herself in debt. When $\Delta C / \Delta Y = 1$, this represents spending equilibrium because $\Delta C = \Delta Y$. Thus, every dollar increase in income will bring forth a one dollar increase in spending. Saving cannot take place here because all income increases have been exhausted.

In this note, we examined each of these spending efficiency possibilities in relationship to the expenditure multiplier. We saw that inefficient spending resulted in positive multiplier effects. We were also able to evaluate that efficient spending resulted in negative multiplier effects. Likewise, we also saw that the multiplier effects resulting from equilibrium spending were indefinite and the interpretation was less certain.

To better understand how changes in aggregate spending may impact an economy and aggregate income, this note attempted to extend the uses of multiplier analysis by utilizing the expenditure multiplier as a means of studying aggregate spending efficiency.

6. References

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