

Volume 32, Issue 4**Does noncausality help in forecasting economic time series?**

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Abstract

In this paper, we compare the forecasting performance of univariate noncausal and conventional causal autoregressive models for a comprehensive data set consisting of 170 monthly U.S. macroeconomic and financial time series. The noncausal models consistently outperform the causal models. For a collection of quarterly time series, the improvement in forecast accuracy due to allowing for noncausality is found even greater.

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1 Introduction

In this paper, we compare the forecasting performance of the linear causal autoregressive (AR) model with dependence only on the past with that of the noncausal AR model of Lanne and Saikkonen (2011), that explicitly incorporates dependence on the future. Noncausal models have hardly been applied to economic time series, and also the theoretical literature is scant, with Breidt et al. (1991) and Rosenblatt (2000) being the major early references. However, recently Lanne and Saikkonen (2011), and Lanne, Luoma and Luoto (2012) have found noncausality in U.S. inflation series, and Lanne, Luoto and Saikkonen (2012) provide evidence in favor of improvements in the accuracy of inflation forecasts. Inspired by these findings, we set out to assess whether more accurate forecasts in general result from allowing for noncausality in the predictive model.

In our forecast comparisons, we concentrate on the comprehensive data set of Marcellino, Stock and Watson (2006) consisting of 170 monthly U.S. macroeconomic and financial time series. As many important macroeconomic time series, such as the real GDP and its components, are measured only on a quarterly basis, we also consider a more limited collection of quarterly U.S. macroeconomic time series.

Overall, the results suggest that for most monthly time series, taking the presence of noncausality into account leads to improvements in forecast accuracy. The noncausal model consistently outperforms the causal model with few exceptions. For the quarterly time series even greater improvements due to allowing for noncausality are found.

The rest of the paper is organized as follows. Section 2 introduces the noncausal autoregressive model of Lanne and Saikkonen (2011), while Section 3 describes the simulation-based forecasting method of Lanne, Luoto and Saikkonen (2012). The forecasting results are reported in Section 4. Finally, Section 5 concludes.

2 Noncausal AR Model

The noncausal AR(r, s) model of Lanne and Saikkonen (2011) encompassing the causal AR model as a special case can be written as

$$\varphi(B^{-1})\phi(B)y_t = \varepsilon_t, \quad (1)$$

where $\varphi(B^{-1}) = 1 - \varphi_1 B^{-1} - \dots - \varphi_s B^{-s}$, $\phi(B) = 1 - \phi_1 B - \dots - \phi_r B^r$, and B is the usual backshift operator (i.e., $B^k y_t = y_{t-k}$). The polynomials $\phi(z)$ and $\varphi(z)$ are assumed to have their zeros outside the unit circle. Furthermore, ε_t is an independently and identically distributed (*i.i.d.*) non-Gaussian error term with mean zero and variance σ^2 . When $s = 0$ in (1) so that $\varphi(B^{-1}) = 1$, the model reduces to the conventional causal AR(r) model, i.e., y_t depends only on its past values. On the other hand, when $r = 0$, model (1) is the purely noncausal AR($0, s$) model with dependence of y_t only on its future values.

For forecasting purposes, it is useful to write model (1) in the following equivalent form

$$y_t = \phi_1 y_{t-1} + \dots + \phi_r y_{t-r} + v_t, \quad (2)$$

where

$$v_t = \varphi(B^{-1})^{-1} \varepsilon_t = \sum_{j=0}^{\infty} \beta_j \varepsilon_{t+j}. \quad (3)$$

This shows how the AR(r, s) model incorporates dependence on future error terms ε_{t+j} , $j \geq 0$.

As pointed out by Breidt et al. (1991), *inter alia*, distinguishing between causal and noncausal AR models requires a non-Gaussian error term. Lanne and Saikkonen (2011), and Lanne, Luoma and Luoto (2012) found Student's *t* distribution to fit U.S. inflation series well, and following their lead, we assume throughout that ε_t is *t*-distributed. With the exception of a number of financial time series, this distributional assumption turned out adequate in our data sets.

For selecting the correct orders, r and s , of the noncausal AR model, Lanne and Saikkonen (2011) proposed a two-step method that we also employ in Section 4. The first step involves finding an adequate causal Gaussian AR(p) model. To that end, we employ the Akaike (AIC) and Schwarz (BIC) information criteria with

the maximum number of lags equal to 8 and 12 for quarterly and monthly data, respectively. We also entertained the fixed lag length $p = 4$, which, however, turned out inferior. Hence, these results are not reported. In the second step, all noncausal $AR(r, s)$ models with the sum of r and s equal to p are estimated and the model yielding the greatest value of the log-likelihood function is used for forecasting.

3 Forecasting Method

To obtain forecasts of the noncausal $AR(r, s)$ model (1), we employ the simulation-based method recently proposed by Lanne, Luoto and Saikkonen (2012), whose main idea is briefly sketched here. The conditional expectation $E_T(\cdot)$ (conditional on the observed values y_1, \dots, y_T) of representation (2) of the $AR(r, s)$ model yields the mean-square sense optimal forecast of y_{T+h} , $h > 0$,

$$E_T(y_{T+h}) = \phi_1 E_T(y_{T+h-1}) + \dots + \phi_r E_T(y_{T+h-r}) + E_T(v_{T+h}). \quad (4)$$

Thus, provided a forecast of v_{T+h} is available, multiperiod forecasts at any forecast horizon h can be constructed recursively.

The forecast of v_{T+h} can be based on the approximation

$$v_{T+h} \approx \sum_{j=0}^{M-h} \beta_j \varepsilon_{T+h+j}, \quad (5)$$

where the integer M is assumed to be large enough to make the approximation error negligible. Therefore, an approximation to (4) is provided by

$$E_T(y_{T+h}) \approx \phi_1 E_T(y_{T+h-1}) + \dots + \phi_r E_T(y_{T+h-r}) + E_T\left(\sum_{j=0}^{M-h} \beta_j \varepsilon_{T+h+j}\right). \quad (6)$$

In computing forecasts, the idea is to approximate the conditional expectation of v_{T+h} by simulating N mutually independent realizations from the conditional distribution of $(\varepsilon_{T+1}, \dots, \varepsilon_{T+M})$. Lanne, Luoto and Saikkonen (2012) provide simulation evidence that even with relatively small values of the truncation parameter M and the number of simulation replications N , approximation (5) is quite accurate. Based on their simulation results, we set $M=50$ and $N=10\,000$, respectively.

4 Results

In this section, we report the forecasting results for a large number of monthly and quarterly time series. The forecasts are based on an expansive estimation window, and in forecasting, the model selection procedure described in Section 2 is repeated and the parameters are re-estimated at each date over the forecasting period. Forecast accuracy is assessed by the relative mean-squared forecast error (MSFE) criterion, with the causal Gaussian AR(p) model as the benchmark.¹

Our monthly data set consisting of 170 monthly U.S. macroeconomic and financial time series was originally compiled by Marcellino et al. (2006). For most series (except 29 series), the sample period ranges from 1959:1 to 2002:12. Following Marcellino et al. (2006), the first forecast date is 1979:1. The mean and median of the empirical distribution of the relative MSFEs at a number of forecast horizons are presented in the upper panel of Table 1. In addition, the fraction of the series with the relative MSFE below unity is reported in each case.

When the BIC is used in model selection, with the exception of the one-month forecast horizon, the noncausal model appears to forecast more accurately than the causal model, i.e., the mean and median of the relative MSFEs are less than unity, and for more than half of the series, the noncausal model produces more accurate forecasts. On the other hand, if the AIC is employed, the differences in the mean and median of the relative MSFE are minor, and in a few cases even slightly advantageous to the causal model. However, also in these cases, beyond the one-period horizon, the fractions always exceed 50%. The null hypothesis of no qualitative difference between the forecasts of noncausal and causal models can be rejected in most cases at least at the 10% significance level by the sign test of Diebold and Mariano (1995).

The t distribution turned out to be insufficient in capturing the excessive kurtosis of monthly interest rates and asset returns, and, therefore, we also report results for a data set excluding these 33 financial variables in the lower panel of Table 1. With the exception of the one-month forecasts, the superiority of the

¹ With the mean absolute forecast error, similar results (not reported) are obtained (for details, see the discussion paper version of this study, Lanne, Nyberg and Saarinen (2011).

noncausal model is evident. The means and medians of the relative MSFEs at all multiperiod forecast horizons are less than unity. The fractions often exceed 60% implying that qualitatively the noncausal AR model outperforms the causal model.

In Table 2, we break down the results of Table 1 to the five categories which are the same as examined by Marcellino et al. (2006). The results reveal that there are some differences in forecast performance between the different categories of variables. However, despite the limited number of variables in each category, the common finding appears to be that the noncausal model outperforms the causal model at all forecast horizons and all categories except for the one-period forecasts and the financial variables included in the category D.

Our quarterly data set comprises 18 U.S. macroeconomic time series including, e.g., the GDP and its components, price indices and (un)employment series.² For most series, the sample period ranges from the beginning of 1947 to the first or second quarter of 2010. Although our quarterly data set hence covers a somewhat longer time period than the monthly data set, for consistency, the first out-of-sample forecasts are made for the first quarter of the year 1979.

Table 3 reconfirms the overall superiority of the noncausal model found in the monthly data. In fact, the differences in the forecast performance between the noncausal and causal models appear to be even larger than in the monthly data set. In most cases, the qualitative differences are also statistically significant even at the 1% significance level. Furthermore, the lag length selection method seems to be less important than in the case of the monthly data set. It is only at the eight-month forecast horizon that the noncausal model performs clearly better when the AIC is employed compared to the other alternatives.

5 Conclusion

In this paper, we have compared the forecast performance of the noncausal AR model of Lanne and Saikkonen (2011) to that of the conventional causal AR model.

² A detailed description of the data set can be found in Lanne et al. (2011).

We have examined a comprehensive monthly data set consisting of macroeconomic and financial time series as well as a more limited collection of quarterly U.S. time series.

The noncausal model tends to yield superior multiperiod forecasts compared to the causal model. The improvement in forecast accuracy is not surprising given that noncausality was found for a vast majority of the time series considered. For the quarterly time series, the improvement in forecast accuracy due to allowing for noncausality is even greater than for the monthly series. With the exception of the interest rate and asset return series considered, the results also lend support to the adequacy of Student's *t* distribution for the errors. The question of whether the noncausal AR model is superior in forecasting these financial variables under a more suitable distributional assumption is left for future research.

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Table 1: Summary of relative MSFEs between the noncausal AR(r,s) model and the causal AR(p) model based on different model selection criteria.

Model Selection	Mean/Median/ Fraction	Forecast horizon (months)					
		1	3	6	9	12	24
All series							
BIC	Mean	1.0289	0.9972	0.9953	0.9957	0.9959	0.9963
	Median	1.0032	0.9979	0.9991	0.9996	0.9997	0.9998
	Fraction	0.4096	0.6145***	0.6506***	0.5783**	0.5361	0.5361
AIC	Mean	1.0882	1.0378	1.0987	1.0067	1.0052	0.9829
	Median	1.0119	0.9977	0.9982	0.9991	0.9998	0.9993
	Fraction	0.3735	0.5482*	0.6024***	0.5602*	0.5181	0.5663**
Excluding interest rates and asset prices (133 series remaining)							
BIC	Mean	1.0079	0.9909	0.9926	0.9936	0.9933	0.9927
	Median	1.0012	0.9977	0.9984	0.9993	0.9995	0.9997
	Fraction	0.4361	0.6165***	0.6917***	0.6090***	0.5940**	0.5564*
AIC	Mean	1.0489	0.9976	0.9913	0.9921	0.9911	0.9851
	Median	1.0084	0.9961	0.9959	0.9979	0.9995	0.9992
	Fraction	0.3985	0.5865**	0.6617***	0.6090***	0.5564*	0.5564*

Notes: Each entry is the indicated summary measure of the distribution of the ratio between the MSFE for the noncausal AR(r,s) model to the MSFE of the causal AR(p) model for the lag selection listed in the first column and the forecast horizon indicated in the column heading. Fraction is the percentage of variables with the relative MSFE below unity (i.e., the number of cases for which the AR(r,s) model yields a smaller MSFE than the AR(p) model). The statistical significances of the differences implied by the fractions are tested using the sign test statistic of Diebold and Mariano (1995). In the table, *, **, and *** denote the 10%, 5% and 1% significance levels, respectively. In the upper panel, four variables for which the estimation of the AR(r,s) model failed to converge are left out leaving 166 time series in total.

Table 2: Relative MSFEs in different categories of the monthly time series.

Model Selection	Mean/Median/ Fraction	Forecast horizon (months)					
		1	3	6	9	12	24
<i>(A) Income, output, sales, capacity utilization (38 series)</i>							
BIC	Mean	1.0031	0.9965	0.9907	0.9909	0.9917	0.9958
	Median	1.0011	0.9999	0.9997	0.9994	0.9995	1.0001
	Fraction	0.4737	0.5263	0.6842***	0.5789	0.6316**	0.3947
AIC	Mean	1.0176	0.9943	0.9902	0.9912	0.9881	0.9851
	Median	1.0073	0.9990	0.9987	0.9992	0.9992	0.9991
	Fraction	0.3947	0.5526	0.6579**	0.5263	0.6053*	0.6053*
<i>(B) Employment and unemployment (23 series)</i>							
BIC	Mean	1.0283	0.9876	0.9917	0.9934	0.9959	0.9967
	Median	0.9988	0.9848	0.9920	0.9969	0.9990	1.0000
	Frac	0.5217	0.6957**	0.6522**	0.6957**	0.6087	0.5217
AIC	Mean	1.1288	1.0024	0.9913	0.9946	0.9895	0.9852
	Median	1.0033	0.9937	0.9892	0.9971	0.9997	0.9990
	Fraction	0.4783	0.6087	0.6957**	0.6087	0.5652	0.5652
<i>(C) Construction, inventories and orders (37 series)</i>							
BIC	Mean	1.0013	0.9953	0.9949	0.9987	0.9962	0.9924
	Median	1.0009	0.9988	0.9991	0.9997	0.9999	0.9997
	Fraction	0.3784	0.5676	0.7297***	0.6216**	0.5405	0.6216**
AIC	Mean	1.0507	0.9895	0.9872	0.9915	0.9930	0.9846
	Median	1.0049	0.9904	0.9941	0.9991	0.9995	1.0004
	Fraction	0.4865	0.6757***	0.6486**	0.6486**	0.5135	0.4865
<i>(D) Interest rates and asset prices (33 series)</i>							
BIC	Mean	1.1137	1.0222	1.0063	1.0044	1.0066	1.0107
	Median	1.0161	0.9988	1.0001	1.0002	1.0005	1.0001
	Fraction	0.3030	0.6061*	0.4848	0.4545	0.3030	0.4545
AIC	Mean	1.2465	1.2001	1.5315	1.0660	1.0620	0.9740
	Median	1.0486	1.0332	1.1011	1.0183	1.0024	0.9995
	Fraction	0.2727	0.3939	0.3636	0.3636	0.3636	0.6061*
<i>(E) Nominal prices, wages and money (35 series)</i>							
BIC	Mean	1.0067	0.9825	0.9928	0.9912	0.9903	0.9870
	Median	1.0066	0.9927	0.9949	0.9980	0.9931	0.9987
	Fraction	0.4000	0.7143***	0.6857***	0.5714	0.6000*	0.6857***
AIC	Mean	1.0284	1.0065	0.9970	0.9919	0.9933	0.9856
	Median	1.0304	0.9996	0.9955	0.9939	0.9971	0.9961
	Fraction	0.2571	0.5143	0.6571**	0.6571**	0.5429	0.5714

Notes: In the table, the categories (A-E) are the same as in Marcellino *et al.* (2006). See also the notes to Table 1.

Table 3: Summary of relative MSFEs in quarterly macroeconomic time series.

Model Selection	Mean/Median/ Fraction	Forecast horizon (quarters)					
		1	2	3	4	5	8
All 18 series							
BIC	Mean	0.9823	0.9592	0.9690	0.9673	0.9710	0.9967
	Median	0.9836	0.9791	0.9790	0.9828	0.9888	1.0011
	Fraction	0.6667**	0.7778***	0.7778***	0.7778***	0.8333***	0.3889
AIC	Mean	0.9932	0.9598	0.9713	0.9642	0.9694	0.9854
	Median	0.9849	0.9625	0.9772	0.9673	0.9849	0.9935
	Fraction	0.6111	0.7778***	0.6667**	0.6667**	0.7222***	0.7778***

Notes: See the notes to Table 1.