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Time Preference and Long-Run Growth: the Role of Patience Capital

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Abstract

This paper studies the relation between long-term economic growth and time preference of households in the context of a simple model of endogenous growth. We assume that the rate of time preference depends on the level of household's patience (stock of patience capital). It is assumed that the patience capital is accumulated by household's future-oriented investment and decumulated by consumption activities. The paper focuses on how the level of patience capital is determined in the balanced-growth equilibrium.

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1 Introduction

Becker and Mulligan (1997) claim that patience is a kind of capital that can be accumulated by investment of the forward-looking household. On the other hand, Uzawa (1968) whose formulation has been widely employed in the literature assumes that the marginal impatience increases with the current level of consumption. We combine the ideas of Becker and Mulligan (1997) and Uzawa (1968) in the context of a simple model of endogenous growth. We assume that the patience capital is accumulated by the intentional investment of household but it is decumulated by consumption activities. In the balanced-growth equilibrium, the level of patience capital stays constant. We focus on how the level of patience capital is determined on the balanced-growth path.

Stern (2006), Nakamoto (2009), and Erol et al. (2011) introduce the Becker-Mulligan hypothesis into the neoclassical (exogenous) growth models. Stern (2006) and Nakamoto (2009) assume that the time preference depends on the future-oriented investment, while Erol et al. (2011) consider that the time discount rate is determined by the stock of physical capital. In analyzing endogenous growth models, Palivos et al. (1997) use the Uzawa formulation and Strulik (2012) assumes that the time discount rate is a decreasing function of physical capital. Both studies assume that the rate of time discount converges to some constant levels when the economy continues growing. Therefore, in their studies the balanced-growth characterization is the same as that of the standard setting with a fixed time discount rate. Meng (2006) examines an AK growth model in which the time discount rate depends on the social level of consumption-income ratio. Although the long-run time discount rate is endogenously determined in Meng (2006), his model does not follow the Becker-Mulligan hypothesis because it is assumed that external effects exclusively determine the rate of time preference.

2 The Model

Consider a representative agent economy where there is a continuum of identical households with a unit mass. The production technology is given by a simple AK production function such that $Y = AK$, where Y is output and K denotes aggregate capital. The optimization problem of the representative household is given by the following:

$$\max \int_0^{\infty} e^{-z} \frac{C^{1-\sigma}}{1-\sigma} dt, \quad 0 < \sigma < 1,$$

subject to

$$\dot{K} = Y - C - S - \delta K, \quad 0 < \delta < 1, \tag{1}$$

$$\dot{H} = \phi\left(\frac{S}{C}\right), \quad \phi'\left(\frac{S}{C}\right) > 0, \quad \phi''\left(\frac{S}{C}\right) < 0, \quad \phi(\theta) = 0, \quad \theta > 0, \tag{2}$$

$$\dot{z} = \rho(H), \quad \rho'(H) < 0, \quad \rho''(H) > 0, \quad \lim_{H\to\infty} \rho(H) = \rho_0 > 0, \quad \lim_{H\to\infty} \rho'(H) = 0, \tag{3}$$

as well as to the given initial conditions, $K(0) = K_0 > 0$, $H(0) = H_0 > 0$ and $z = 0$ when $t = 0$. Here, C is consumption, H denotes the level of patience (stock of patience capital) and S is the level of future-oriented investment. Note that to keep the utility level positive, it is assumed that the elasticity of intertemporal substitution $(1/\sigma)$ is larger than one.

Equation (1) is the flow budget constraint. Equation (2) describes the behavior of patience capital. We assume that investment for patience accelerates accumulation of patience capital under a given level of consumption. A rise in the current level of consumption decreases the effect of future-oriented investment. We also assume that to keep the patience capital constant in the steady state, the investment-consumption ratio, S/C, should be kept constant at a given level of θ. A smaller θ means that future-oriented investment is more efficient. As shown in (3), the rate of time preference is assumed to be a decreasing and convex function of H. This assumption follows Becker and Mulligan (1997).

The Hamiltonian function for the household's optimization problem is given by

$$\mathcal{H} = e^{-z} \frac{C^{1-\sigma}}{1-\sigma} + \hat{p}\left[AK - C - S - \delta K\right] + \hat{q}\phi\left(\frac{S}{C}\right) - \hat{\lambda}\rho(H),$$

where $\hat{p} > 0$, $\hat{q} > 0$, and $\hat{\lambda} > 0$ are costate variables. The necessary conditions for an optimum are:

$$\max_C H \Rightarrow e^{-z} C^{-\sigma} - \hat{p} - \hat{q}\phi'\left(\frac{S}{C}\right)\frac{S}{C^2} = 0, \tag{4}$$

$$\max_S H \Rightarrow -\hat{p} + \hat{q}\phi'\left(\frac{S}{C}\right)\frac{1}{C} = 0, \tag{5}$$

$$\frac{d\hat{p}}{dt} = \hat{p}(\delta - A), \tag{6}$$

$$\frac{d\hat{q}}{dt} = \hat{\lambda}\rho'(H), \tag{7}$$

$$\frac{d\hat{\lambda}}{dt} = -e^{-z}\frac{C^{1-\sigma}}{1-\sigma}, \tag{8}$$

together with the transversality conditions:

$$\lim_{t\to\infty}\hat{p}K=0,\qquad \lim_{t\to\infty}\hat{q}H=0,\qquad \lim_{t\to\infty}\hat{\lambda}z=0. \tag{9}$$

Define $p\equiv e^{z}\hat{p}$, $q\equiv e^{z}\hat{q}$, and $\lambda\equiv e^{z}\hat{\lambda}$. Then the above conditions are respectively rewritten as follows:

$$C^{-\sigma}=p+q\phi'\left(\frac{S}{C}\right)\frac{S}{C^{2}}, \tag{4'}$$

$$p=q\phi'\left(\frac{S}{C}\right)\frac{1}{C}, \tag{5'}$$

$$\frac{\dot{p}}{p}=\rho\left(H\right)+\delta-A, \tag{6'}$$

$$\frac{\dot{q}}{q}=\rho\left(H\right)+\frac{\lambda}{q}\rho'\left(H\right), \tag{7'}$$

$$\frac{\dot{\lambda}}{\lambda}=\rho\left(H\right)-\frac{1}{\lambda}\frac{C^{1-\sigma}}{1-\sigma}. \tag{8'}$$

3 Dynamic System

Define $x\equiv\lambda/q$, $v\equiv S/C$, and $m\equiv C/K$. Then $(7')$ and $(8')$ give

$$\frac{\dot{x}}{x}=\frac{\dot{\lambda}}{\lambda}-\frac{\dot{q}}{q}=-\frac{1}{1-\sigma}\frac{1}{x}\phi'\left(v\right)\left(1+v\right)-x\rho'\left(H\right). \tag{10}$$

From $(4')$ and $(5')$ we obtain

$$C^{1-\sigma}=q\phi'\left(v\right)\left(1+v\right).$$

The abve equation presents

$$\left(1-\sigma\right)\frac{\dot{C}}{C}=\rho\left(H\right)+x\rho'\left(H\right)+\frac{\psi'\left(v\right)v}{\psi\left(v\right)}\frac{\dot{v}}{v}, \tag{11}$$

where $\psi\left(v\right)\equiv\phi'\left(v\right)\left(1+v\right)$. Similarly, by use of $(5')$, $(6')$ and $(7')$ we can derive

$$\frac{\dot{C}}{C}=x\rho'\left(H\right)+\frac{\phi''\left(v\right)v}{\phi'\left(v\right)}\frac{\dot{v}}{v}-\delta+A. \tag{12}$$

Solving (11) and (12) with respect to $\dot{v}/v$ yields

$$\frac{\dot{v}}{v}=\xi\left(v\right)\left\{\sigma[\rho\left(H\right)+x\rho'\left(H\right)]+\left(1-\sigma\right)\left[\rho\left(H\right)+\delta-A\right]\right\}, \tag{13}$$

where

$$\xi\left(v\right)=\left[\left(1-\sigma\right)\frac{\phi''\left(v\right)v}{\phi'\left(v\right)}-\frac{\psi'\left(v\right)v}{\psi\left(v\right)}\right]^{-1}.$$

Finally, combining (12) with $\dot{K}/K = A - \delta - (C/K)(1 + S/C)$, we obtain the dynamic equation of m :

$$\begin{aligned}\frac{\dot{m}}{m} &= \frac{\dot{C}}{C} - \frac{\dot{K}}{K} \\ &= x\rho'(H) + \frac{\phi''(v)\,v}{\phi'(v)}\frac{\dot{v}}{v} - \delta + A - [A - (1+v)\,m - \delta]\,. \end{aligned} \tag{14}$$

From (13) we see that the right-hand side of (14) is a function of v, x, m and H.

To sum up, dynamic equations (10), (13) and (14), together with (2), constitute a complete dynamic system with respect to $x\,(= \lambda/q)$, $v\,(= S/C)$, H and $m\,(= C/K)$.

4 Balanced-Growth Equilibrium

In the balanced-growth equilibrium the state variables of the dynamic system stay constant over time, and thus it holds that

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{S}}{S} = g, \qquad \frac{\dot{q}}{q} = \frac{\dot{\lambda}}{\lambda} = (1-\sigma)\,g, \qquad \frac{\dot{p}}{p} = -\sigma g,$$

where g denotes a common growth rate. In addition, H stays constant, so that $S = \theta C$. The balanced-growth rate g is given by

$$g = -\frac{1}{\sigma}\frac{\dot{p}}{p} = \frac{1}{\sigma}\left[A - \delta - \rho\left(H^*\right)\right], \tag{15}$$

where H^* is the level of H on the balanced-growth path.

Condition $\dot{x} = 0$ in (10) presents

$$-\frac{1}{1-\sigma}\frac{1}{x}\phi'(v)\,(1+v) - x\rho'(H) = 0. \tag{16}$$

Provided that $\xi(v) \neq 0$, condition $\dot{v} = 0$ in (13) yields

$$\sigma[\rho(H) + x\rho'(H)] + (1-\sigma)\,[\rho(H) + \delta - A] = 0. \tag{17}$$

Eliminating x from (16) and (17) leads to

$$\left\{\left(\frac{1}{\sigma} - 1\right)(A-\delta) - \frac{\rho(H)}{\sigma}\right\}^2 = -\frac{\phi'(\theta)\,(1+\theta)}{1-\sigma}\rho'(H)\,. \tag{18}$$

This equation determines the steady-state level of patience capital, H^*. Given our assumptions, we find that the right-hand side of (18) is monotonically decreasing in H and converges to zero as H goes to infinity. The left-hand side of (18) is a U-shaped function whose minimum value is zero when H satisfies

$$(1-\sigma)(A-\delta)=\rho\left(\hat{H}\right).$$

This means that without imposing further restrictions on functional forms of $\phi(v)$ and $\rho(H)$, there may exist dual steady-state levels of H satisfying (18). In what follows, we restrict our attention to the case of a unique balanced-growth equilibrium with a positive growth rate.

As for the stability of the balanced-growth path, we can show that under a set of mild restrictions, the balanced-growth path with a positive growth rate satisfies saddle-point stability and that there locally exists a unique converging path around the balanced-growth equilibrium.[1]

5 A Remark

Since our model assumes that the level of patience capital is affected by the tension between household's future-oriented investment and present consumption, every factor that may affect those two decision makings may alter the time discount rate and thus it changes the long-term growth rate of the economy. For example, assume that the TFP of the economy, A, rises. From (18) on the balanced-growth path with a positive growth rate it holds that

$$\frac{dH^*}{dA}=\frac{2\left(1-\frac{1}{\sigma}\right)\left[\left(\frac{1}{\sigma}-1\right)(A-\delta)-\frac{\rho}{\sigma}\right]}{2\left(-\frac{\rho'}{\sigma}\right)\left[\left(\frac{1}{\sigma}-1\right)(A-\delta)-\frac{\rho}{\sigma}\right]+\frac{\phi'(\theta)(1+\theta)}{1-\sigma}\rho''}<0.$$

Namely, a higher A accelerates households' consumption relative to the future-oriented investment, so that the steady-state level of patience capital will decline. The impact of a rise in TFP on the growth rate is shown by

$$\frac{dg}{dA}=\frac{1}{\sigma}\left\{\frac{\frac{\phi'(\theta)(1+\theta)}{1-\sigma}\rho''-2\rho'\left[\left(\frac{1}{\sigma}-1\right)(A-\delta)-\frac{\rho}{\sigma}\right]}{2\left(-\frac{\rho'}{\sigma}\right)\left[\left(\frac{1}{\sigma}-1\right)(A-\delta)-\frac{\rho}{\sigma}\right]+\frac{\phi'(\theta)(1+\theta)}{1-\sigma}\rho''}\right\}>0,$$

implying that a higher A increases the long-term growth rate. However, the negative impact of a higher productivity on patience of households partially offsets its positive impact on long-term of the economy. This exercise suggests that conventional policy effect obtained in the simple endogenous growth models should be reconsidered if we endogenize the long-run time preference of the households.

[1] The detail of the stability analysis is available upon request.

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