Economics Bulletin

Volume 32, Issue 4

Social optimum in an OLG model with paternalistic altruism

Marion Davin
Aix Marseille University (Aix Marseille School of Economics), CNRS, EHESS

Karine Gente
Aix Marseille University (Aix Marseille School of Economics), CNRS, EHESS

Carine Nourry
Aix Marseille University (Aix Marseille School of Economics), CNRS, EHESS and IUF

Abstract

There is no consensus yet on the correct way to write the social utility function in the presence of paternalistic altruism. This note shows that the specification of the central planner objective is crucial for optimal capital accumulation and growth. When paternalistic altruism enters the social welfare function, we depart from the standard Modified Golden Rule. Capital intensity is no more determined by the equality between optimal returns on human and physical capital, and the optimal growth rate is higher when we consider altruism enters in the social welfare function. We calibrate the model for several countries and emphasize large differences on optimal growth rate, according to the specification of the social welfare function.

We thank T. Seegmuller and two anonymous referees for useful comments and suggestions.
Citation: Marion Davin and Karine Gente and Carine Nourry, (2012) "Social optimum in an OLG model with paternalistic altruism", *Economics Bulletin*, Vol. 32 No. 4 pp. 3417-3424.
Contact: Marion Davin - marion.davin@gmail.com, Karine Gente - karine.gente@univ-amu.fr, Carine Nourry - carine.nourry@univ-amu.fr.
Submitted: April 13, 2012. **Published:** December 20, 2012.

1. Introduction

How does the inclusion of paternalistic altruistic feelings in the social planner's objective affect the optimal growth rate? According to a strict definition, and assuming that utility is cardinal and unit-comparable (but not level-comparable) between generations, the social utility is the discounted sum of individual utility functions, namely a Utilitarian social function. However, Harsanyi (1995) recommends to exclude all external preferences, as altruism, from the social utility function (Harsanyi social function). Thus there is no consensus yet on the correct way to write this social utility function. On one side, considering an overlapping generations one sector model with consumption separable utility function, Cremer and Pestieau (2006) underline that optimal policy depends on the specification of the social utility function. Nevertheless, they do not clearly examine the implications on the optimal balanced growth path. On the other side, de la Croix and Monfort (2000) do not include the "joy of giving" term in the welfare function. In this paper, we show that the way to write the central planner objective is crucial for the optimal growth path. In this purpose, we consider the example of a paternalistic altruism where agents are concerned by the level of human capital of their children. We also assume that human capital is a simple function of parents investment in their child's education.

Our contribution is to quantitatively investigate the consequences of omitting the altruistic term in the social utility function. As long as education is ignored in the social utility function, the only way to increase welfare is to maximize consumption. Conversely, when child's education provides direct welfare to parents, there is an arbitrage in the social utility function between consumption and education. This is the reason why the relationship between human capital and capital intensity depends on preference parameters with the Utilitarian social function. We show that the optimal growth rate is higher with the Utilitarian social function than in the Harsanyi social function. We calibrate the model to quantify the difference between Utilitarian and Harsanyi optimal paths.

2. Social Optimum and paternalistic altruism

Consider a perfectly competitive economy in which the final output is produced using physical capital K and human capital H. The production function of a representative firm is an homogeneous function of degree one: $F(K,H)$. We assume for simplicity a complete depreciation of the capital stock within one period. Denoting, for any $H \neq 0$, $k \equiv K/H$ the physical to human capital ratio, we define the production function in intensive form as $f(k)$.

Assumption 1 *$f(k)$ is defined over $\mathbb{R}_+$, $\mathbf{C}^r$ over $\mathbb{R}_{++}$ for r large enough, increasing ($f'(k) > 0$) and concave ($f''(k) < 0$). Moreover, $\lim_{k \to 0} f'(k) = +\infty$ and $\lim_{k \to +\infty} f'(k) = 0$.*

We can also compute the share of capital in total income:

$$s(k) = \frac{kf'(k)}{f(k)} \in (0,1) \tag{1}$$

As in Michel and Vidal (2000), we consider a three-period overlapping generations model. In their first period of life, individuals are reared by their parents. In the second period, they work and receive a wage, consume, save and rear their own children. In their last period of life, they are retired and consume their saving returns. Following Glomm and Ravikumar (1992), we consider a paternalistic altruism according to which parents value the quality of education received by their children. Thus, the preferences of an altruistic agent born in $t-1$ are represented by:

$$U_t = u(c_t, d_{t+1}) + v(h_{t+1}) \tag{2}$$

where c_t and d_{t+1} represent adult and old aggregate consumption, and h_{t+1} child's human capital.

Assumption 2.
i) $u(c,d)$ is $\mathbf{C}^2$, increasing with respect to each argument ($u_c(c,d) > 0$, $u_d(c,d) > 0$), concave and homogeneous of degree a, with $a \in]0,1[$. Moreover, for all $d > 0$, $\lim_{c\to 0} u_c(c,d) = +\infty$, and for all $c > 0$, $\lim_{d\to 0} u_d(c,d) = +\infty$.
ii) $v(h)$ is $\mathbf{C}^2$, increasing ($v'(h) > 0$), concave and homogeneous of degree a, with $a \in]0,1[$. Moreover $\lim_{h\to 0} v'(h) = +\infty$.

Parents devotes e_t to his child's education, so human capital in $t+1$ is given by:

$$h_{t+1} = G(e_t) \tag{3}$$

Assumption 3. *The human capital production function $G(e)$ is strictly increasing and linear with e.*

At the decentralized equilibrium grandparents' expenditures in education generate a positive intergenerational external effect in human capital accumulation. Indeed, parents decide for their child's education but do not consider the impact of this decision on their grand child's education. We assume that population is constant over time and is normalized to 1, *i.e.* $N_t = N = 1$. Moreover, clearing condition on the labor market gives $H_t = Nh_t = h_t$ and thus $K_t = h_t k_t$.

The social planner maximizes the discounted sum of the life-cycle utilities of all current and future generations under the resource constraint of the economy and the human capital accumulation equation.

$$\max_{c_t, d_t, K_{t+1}, H_{t+1}} \sum_{t=-1}^{\infty} \delta^t \left(u(c_t, d_{t+1}) + \epsilon v(h_{t+1})\right) \tag{4}$$

$$\tag{5}$$

with $\delta \in (0,1)$, and ϵ taking alternatively the extreme values 0 (Harsanyi social function) and 1 (Utilitarian social function).
The Lagrange function is:

$$\begin{aligned}\mathcal{L} = \; & \delta^{-1}[u(c_{-1}, d_0) + \epsilon v(h_0)] \\ & + \sum_{t=0}^{\infty} \delta^t \left(u(c_t, d_{t+1}) + \epsilon v(h_{t+1}) + \; q_t \left(h_t f(k_t) - c_t - d_t - G^{-1}(h_{t+1}) - h_{t+1}k_{t+1}\right)\right)\end{aligned} \tag{6}$$

where q_t the Lagrange multiplier associated with the constraint. First order conditions for all $t \geqslant 0$ are:

$$u_c(c_t, d_{t+1}) = q_t \tag{7}$$

$$u_d(c_{t-1}, d_t) = \delta q_t \tag{8}$$

$$\delta^{t+1} f'(k_{t+1}) q_{t+1} = \delta^t q_t \tag{9}$$

$$\delta^t \epsilon v'(h_{t+1}) + \delta^{t+1} q_{t+1}(f(k_{t+1}) - k_{t+1} f'(k_{t+1})) = \delta^t q_t {G^{-1}}'(h_{t+1}) \tag{10}$$

$$h_t f(k_t) - c_t - d_t - G^{-1}(h_{t+1}) - h_{t+1} k_{t+1} = 0 \tag{11}$$

$$\lim_{t\to\infty} \delta^t q_t K_{t+1} = 0, \ \lim_{t\to\infty} \delta^t q_t H_{t+1} = 0 \tag{12}$$

Where equation (10) is obtained by differentiating the Lagrangean with respect to h_{t+1} and making a simplifying substitution using equation (9).

For initial conditions c_{-1}, K_0 and H_0 and for all $t \geqslant 0$, optimal solutions satisfy equations (7) to (12).

From (7), (8) and (9) we can rewrite the condition that gives optimal physical capital accumulation:

$$f'(k_{t+1}) = \frac{u_c(c_t, d_{t+1})}{u_d(c_t, d_{t+1})} = \frac{q_t}{\delta q_{t+1}} \tag{13}$$

From (7), (9) and (10), we obtain the following expression that determines optimal human capital accumulation:

$$MRS_{e/c} = {G^{-1}}'(h_{t+1}) - k_{t+1}\left(\frac{1}{s(k_{t+1})} - 1\right) \tag{14}$$

with $MRS_{e/c} \equiv \frac{\epsilon v'(h_{t+1})}{u_c(c_t,d_{t+1})}$ the marginal rate of substitution between education and first period consumption.

Equation (14) displays the main difference between the two approaches. With Harsanyi social function ($\epsilon = 0$), equation (14) becomes:

$$f'(k_{Ht+1}) = (f(k_{Ht+1}) - k_{Ht+1} f'(k_{Ht+1}))G'(e_{Ht}) \tag{15}$$

with k_H and e_H respectively the capital intensity and education spending in the Harsanyi case. The optimal return on investment in human capital (through education) is equal to the return on physical capital since the central planner does not differentiate between physical and human capital accumulation. The welfare increases only with consumption. Thus, along the balanced growth path, defined by a constant physical to human capital ratio, the optimal k_H corresponds to the standard Modified Golden Rule.

Conversely, with the Utilitarian social function ($\epsilon = 1$), from equation (14) we get:

$$f'(k_{Ut+1}) > (f(k_{Ut+1}) - k_{Ut+1} f'(k_{Ut+1}))G'(e_{Ut}) \tag{16}$$

with k_U and e_U respectively the capital intensity and education spending in the Utilitarian case. The optimal return on investment in human capital (through education) is lower than the one on physical capital because human capital accumulation provides direct welfare.

There is a trade off between consumption and education. Then, we depart from the Modified Golden Rule through the MRS term. In this latter case, preferences (time preference, altruism) affect the relationship between human capital and capital intensity, whereas they do not in the Harsanyi social utility function.

3. Optimal Growth and capital accumulation

The previous section has shown that qualitatively the way we specify the social utility function matters for capital accumulation. We determine here precisely optimal path for capital accumulation k_t and h_t. We know from k_0 given and equations (7) and (9), that optimal physical capital intensity K/H is constant from $t = 1$. Thus, the social planner has to determine the initial stocks H_1 and K_1 which drive the economy along the optimal path. Along this optimal path, K and H will grow at a constant rate g.

Proposition 1 *If* $G(h) = bh$, $1 \geq b > 0$, *the optimal path is determined by* $g^* = [\delta f'(k_1)]^{\frac{1}{1-a}} - 1$ *with* k_1, q_0 *and* h_1 *solutions of*

$$h_1 k_1 = h_0 f(k_0) - c_0(k_1, q_0) - d_0(q_0) - \frac{h_1}{b} \tag{17}$$

$$\epsilon v'(h_1) + q_0 k_1 \left(\frac{1}{s(k_1)} - 1 \right) = \frac{q_0}{b} \tag{18}$$

$$f'(k_1) \left(\frac{1}{b} + k_1 \right) - f(k_1) = \epsilon \left(\frac{\Psi(k_1)}{1 + \Psi(k_1)} \right)^{1-a} \Theta(k_1) \tag{19}$$

with c_{-1}, h_0, k_0 *predetermined,* $d_0(q_0)$ *solution of* $q_0 = u_d(c_{-1}, d_0)/\delta$ *and* $c_0(k_1, q_0) \equiv \phi^{-1}(f'(k_1)) \left[\frac{u_c(\phi^{-1}(f'(k_1)),1)}{q_0} \right]^{\frac{1}{1-a}}$ *with* $\phi(x) = u_c(x,1)/u_d(x,1)$ [1], $\Psi(k_1) = (1 + g^*(k_1))\phi^{-1}((1 + g^*(k_1))^{1-a}/\delta)$ *and* $\Theta(k_1) = \frac{\left[f(k_1) - \left(k_1 + \frac{1}{b}\right)(1+g^*(k_1))\right]^{1-a} v'(1)}{\delta u_c\left(1, \frac{1}{\phi^{-1}(f'(k_1))}\right)}$.

Proof. As marginal utility of consumption is homogeneous of degree $(a - 1)$, from (7) and (9), we have

$$\delta f'(k_1) = (1 + g^*)^{1-a} \tag{20}$$

Equations (17) and (18) in proposition (1) comes from equations (10) and (11) at time $t = 0$. From homogeneity (Assumption 1) and equations (7) at time $t = 0$ and (8) at time $t = 1$, we get $c_0 = \phi^{-1}\left(\frac{q_0}{\delta q_1}\right) d_1$. Substituting in (7) gives $d_1^{a-1} u_c \left[\phi^{-1}\left(\frac{q_0}{\delta q_1}\right), 1\right] = q_0$. As from equation (9) at time $t = 0$ gives $f'(k_1) = \frac{q_0}{\delta q_1}$, we finally get the result for $c_0(k_1, q_0)$.

Equation (19) is obtained using the first order conditions along the balanced growth path. Using homogeneity of u, equations (7) and (8) and balanced growth path properties, according to which $\frac{c_{t-1}}{d_{t-1}} = \frac{c_t}{d_t}$ and $\frac{d_{t-1}}{d_t} = \frac{d_t}{d_{t+1}}$, we have $c_1 = (1 + g^*(k_1))\phi^{-1}((1+$

[1] Under assumption 2, function $\phi(.)$ is invertible.

$g^*(k_1))^{1-a}/\delta)d_1$. Adding equation (11) and homogeneity of v, we can define the following relationship, $\frac{c_1}{h_1} = \frac{\left[f(k_1)-\left(k_1+\frac{1}{b}\right)(1+g^*(k_1))\right](1+g^*(k_1))\phi^{-1}((1+g^*(k_1))^{1-a}/\delta)}{1+(1+g^*(k_1))\phi^{-1}((1+g^*(k_1))^{1-a}/\delta)}$. Finally, using equation (10), we obtain the last equation of the system, from which we get k_1.

□

Proposition (1) describes the general system whose resolution gives the optimal growth rate and capital accumulations. We wish to show that the results obtained in terms of optimal growth and stocks highly depends on the way the social utility function is specified. Indeed, with the Harsanyi function, the relationship between human capital and capital intensity do not depend on time preference and altruism whereas they do with the Utilitarian social function.

For simplicity let us consider the following assumption:

Assumption 4. *Utility is characterized by* $u(c,d) = c^a + \beta d^a$ *and* $v(h) = \gamma h^a$, $0 < a < 1, 0 < \gamma < 1$*and technologies are given by* $f(k) = k^\alpha$.

Proposition 2 *Under assumption 4, there exists a unique value* k_{1i}, $i = U, H$, *satisfying equation (19). Moreover,* $k_{1H} > k_{1U}$, *hence optimal growth rate is always higher in the Utilitarian case.*

Proof. Under assumption 4, equation (19) becomes

$$\Omega_1(k_1) = \Omega_2(k_1) \tag{21}$$

with:

$$\Omega_1(k_1) \equiv \frac{\alpha k_{1i}^{\alpha-1}}{b} - (1-\alpha)k_{1i}^{\alpha} \tag{22}$$

and

$$\Omega_2(k_1) \equiv \frac{\gamma\epsilon}{\delta}\left(\frac{k_{1i}^{\alpha} - \left(\delta\alpha k_{1i}^{\alpha-1}\right)^{\frac{1}{1-a}}(k_{1i}+1/b)}{1+(\beta/\delta)^{\frac{1}{1-a}}}\right)^{1-a} = \frac{\alpha k_{1i}^{\alpha-1}}{b} - (1-\alpha)k_{1i}^{\alpha} \tag{23}$$

We have $\lim_{k_1\to 0}\Omega_1(k_1) = +\infty$, $\lim_{k_1\to+\infty}\Omega_1(k_1) = -\infty$, $d\Omega_1(k_1)/dk_1 < 0$, and $\Omega_1(k_1) = 0$ for a unique value $k_1 = \bar{k}_1$ with $\bar{k}_1 \equiv \frac{\alpha}{(1-\alpha)b}$. Concerning $\Omega_2(k_2)$, $\lim_{k_1\to 0}\Omega_2(k_1) = -\infty$, $\lim_{k_1\to+\infty}\Omega_2(k_1) = +\infty$ and for $\Omega_2(k_1) > 0$, $d\Omega_2(k_1)/dk_1 > 0$. Thus $\Omega_2(k_1) = 0$ for a unique value $k_1 = \hat{k}_1$. Moreover the sign of $\Omega_2(\bar{k}_1)$ is given by the sign of the term $\left[1 - \left(\delta\alpha^{a\alpha}(b(1-\alpha))^{a(1-\alpha)}\right)^{\frac{1}{1-a}}\right]$, which is always positive, and then $\hat{k} < \bar{k}$.
We deduce finally that, when $\epsilon = 0$, $\Omega_2(k_1) = 0$, and the unique solution to equation (21) is $k_{1H} = \bar{k_1}$, and when $\epsilon = 1$, the unique solution to equation (21) is $k_{1U} \in [\hat{k_1}, k_{1H}]$.

□

From Proposition (2), there is a negative relationship between k_1 and ϵ, hence a positive relationship between the optimal growth rate and the weight that planner gives to altruistic feelings.

Quantitatively, the spread between Utilitarian and Harsanyi optimal paths may be large. We show this through a numerical example calibrated on five countries using proxies for time preference (β) based on Wang *et al.* (2011), for altruism (γ) based on Armellini and Basu (2010) (using data from European and World Value Survey four-wave data-file 1981-2004, 2006). For the social planner discount rate (δ), we use the average real interest rate for 1980-2010 (World Development Indicators, World Bank 2010) following Armellini and Basu (2010). The proxy for δ in country i is the ratio between the country i average real interest and the Russian real interest rate (which is the highest) multiplied by 0.95 which is lower than one to guarantee the convergence of the social objective.

Calibrations. Consider the specific example given in assumption 4 to calibrate our model. Table I collects the parameter values. We are interested in highlighting the spread between the optimal paths obtained with the Harsanyi and Utilitarian social functions. Table II compare optimal paths in the two cases

Table I: Parameter Values

Country	Time Preference (β)	Altruism Degree (γ)	Social discount Rate (δ)
Germany	0.8	0.587	0.91
Japan	0.87	0.435	0.37
Russian Federation	0.77	0.563	0.95
United States	0.84	0.758	0.64

Table II: Calibration results

Country	g_U^*/g_H^*
Germany	1.023
Japan	1.228
Russian Federation	1.017
United States	1.151

Table II shows that there are important differences between the two specifications of the social utility function. From Proposition (2), we know that the optimal growth rate is always higher with the Utilitarian social function. The way we specify the social utility function matters a lot for the determination of the optimal growth paths. For example, in the United States, the Utilitarian approach leads to an optimal growth which is 15.15% higher than the one emerging with the Harsanyi utility function. An optimal educative policy would reach a human capital with the Utilitarian approach which is higher than the one obtained with the Harsanyi utility function. Consequently, the way we write the social welfare function is crucial to determine optimal policy.

References

Armellini, M and P. Basu (2010), "Altruism, education subsidy and growth", MPRA working paper number 23653.

Cremer, H and P. Pestieau (2006): "Intergenerational Transfer of Human Capital and Optimal Education policy," *Journal of Public Economic Theory* **8**, 529-545.

De la Croix, D and P. Monfort (2000): " Education Funding and Regional Convergence," *Journal of Population Economics* **13**, 403-424.

Galor, O. (1992): "A Two-Sector Overlapping-Generations Model: a Global Characterization of the Dynamical System," *Econometrica* **60**, 1351-1386.

Glomm, G and B. Ravikumar (1992): "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy* **4**, 818-833.

Harsanyi, J.C. (1995): "A theory of prudential values and a rule utilitarian theory of morality," *Social Choice and Welfare* **12**, 319-333.

Michel, P and J.P. Vidal (2000): "Economic Integration and Growth under Intergenerational Financing of Human-Capital formation," *Journal of Economics* **3**, 275-294.

Wang, M, MO. Rieger and T. Hens (2011) "How time preferences differ: evidence from 45 countries", Discussion Paper Series Number 182011, Department of Finance and Management Science, Norwegian School of Economics.