Economics Bulletin

Volume 33, Issue 1

Are better vaccines really better? The case of a simple stochastic epidemic SIR model

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Abstract

We consider a model of vaccine market where the buyer is centralized and shows an endogenous demand function based on a simple stochastic SIR model. When the seller is a monopoly, we show that better vaccines (in the sense of greater efficiency or inducing less side-effects) do not imply greater total surplus, greater buyer surplus or even greater profits. Since we consider a centralized buyer, our results cannot be caused by the well-known epidemiological externality of vaccination.

I gratefully thank an anonymous referee. The usual disclaimer applies.
Citation: Nicolas Houy, (2013) "Are better vaccines really better? The case of a simple stochastic epidemic SIR model", *Economics Bulletin*, Vol. 33 No. 1 pp. 207-216.
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Submitted: January 11, 2013. **Published:** January 23, 2013.

Economics Bulletin, 2013, Vol. 33 No. 1 pp. 207-216

1 Introduction

It is implicitly believed by the general population or even by health authorities that, *ceteris paribus*, vaccines should be as effective as possible in the sense that they should protect against infections with a probability as high as the technology permits and show as little side-effects as feasible.[1] This intuition is based on the observation that a dose of vaccine that would not protect is a loss of resources or even that a vaccine that would imply side-effects would be costly to the society. However, this argument should be counter-balanced in the case of a monopolistic supplier. Indeed, a less effective vaccine will imply a modification of the demand function and possibly a decrease of the mark-up imposed by the supplier and maybe a decrease of the social loss due to the existence of a monopoly. The purpose of this paper is to show, in a very simple framework, that the latter effect can be of higher magnitude than the former and hence that more efficient vaccines can decrease the social welfare.

We study the case of a monopolistic vaccine seller facing the demand function of a buyer, purchasing vaccines for a community of individuals in the presence of an infectious disease. The buyer can be a household facing a spreading disease, the head of an hospital facing a nosocomial epidemic (*Clostridium Difficile* for instance) or an health authority purchasing vaccines for the population it protects. Both the seller and the buyer fully know the epidemiology of the infection and are fully rational and forward-looking.

Considering a centralized buyer allows us to avoid the usual and well-known epidemiological externality from vaccination leading to under-consumption of vaccines when the demand is decentralized.[2] Regarding this effect, Xu (1999) has already explicitly studied the role of vaccine effectiveness on the decentralized demand for vaccines: precisely, the direct effect of an increased demand for a better product is counter-balanced by a decrease of the incentive to vaccinate due to a better protection of other agents. In our case, we concentrate on the market structure and show that it can be profitable for both the buyer and the seller to have access to a vaccine of lower quality, *i.e.* a vaccine that is more often ineffective or even a vaccine that implies more side-effects.

There is a growing literature studying vaccines using mathematical epidemiology and economics. A few studies concentrate on the issue of the market structure and in particular on the implications of a monopolistic seller.[3] Almost all these studies deal with infinite populations. Most of them concentrate on steady-state outcomes. On the contrary, we deal with a finite population and then, not only on steady-state outcomes.

[1]The Public Health Service Act of the United States (1944) states as a governmental responsibility to "provide direction to develop the techniques needed to produce safe and effective vaccines". The market for vaccines is often a market with limited production capacities. What should be done in the trade-off between production capacities and effectiveness is not specified. For vaccines effectiveness, see Decker and Schaffner (1990) for general information, Monto *et al.* (2009) for influenza, Salmon *et al.* (1999) for measles...

[2]See Geoffard and Philipson (1997), Bauch *et al.* (2003), Kremer *et al.* (2008) for instance.

[3]Forslid (2006) studies the optimal timing of entering the market for a monopoly. Forslid and Herzing (2010) study the endogenous production capacity set by a monopolistic seller. Kessing and Nuscheler (2006) study income discrimination by a monopoly.

2 The SIR model

The epidemic model we use is a standard stochastic SIR model in discrete time.[4] We consider a population of N individuals facing an epidemics caused by a virus (or a given strain of a virus). Each individual can be of type S (Susceptible), I (Infected) or R (Recovered). At each period of time, each individual of type I becomes of type R with a probability $\nu > 0$. All individuals are in contact with all other individuals and the transmission probability from any individual of type I to any individual of type S is β.[5] Individuals of type R are removed from the dynamics and remain R for ever. Hence, we do not consider any vital dynamics (no death and no birth) and we consider that once infected, individuals gain life-long immunity. The first assumption is a simplification and considers that the disease dynamics is much faster than the vital one and both assumptions are realistic for almost all viruses, see Anderson and May (1991, part. 1).

A vaccine is available. At the start of each period, at most one dose of vaccine can be administered to any individual of type S.[6] Immediately, the vaccine can be counter-productive and the individual becomes of type I with a probability δ_1.[7] Alternatively, the vaccine can be ineffective and the individual remains of type S with a probability δ_2. The vaccine is effective with a probability $(1 - \delta_1 - \delta_2)$ in which case the individual becomes of type R. Individuals' responses to vaccination are independent both in time and across individuals.

3 The buyer's decision

The objective of the buyer is to minimize the intertemporal costs he faces. On the one hand, at each period, each individual of type I costs κ. On the other hand, the buyer can buy vaccines at the unit price of c in order to minimize the number of infected individuals. Obviously, individuals are anonymous, so we can denote by $V_c(r, i, s)$ with $(r, i, s) \in \{(r, i, s) \in \mathbb{N}^3, r + i + s = N\}$, the expected intertemporal cost faced by the buyer at the start of each period if r is the number of people of type R, i the number of people of type I, s the number of people of type S and c is the unit cost of the vaccine. Obviously, whenever $i = 0$, we have $V_c(r, i, s) = 0$. For $i > 0$, the function V must satisfy

[4] Our model is very close to Tuckwell and Williams (2007).

[5] It would be equivalent to consider a transmission probability of 1 and a probability to be in contact of β or any halfway situation.

[6] The vaccine has no effect on individuals of type I or R and then, we can consider that it is only administered to individuals of type S.

[7] Notice that we model side-effects by considering a probability to get infected. We think for instance of the vaccine against Chicken Pox. The Centers for Disease Control and Prevention enumerating side-effects of this vaccine states that it can lead to "Mild rash, up to a month after vaccination (1 person out of 25). It is possible for these people to infect other members of their household, but this is extremely rare". A fixed cost would not modify our results.

the following Bellman equations:[8]

$$V_c(r,i,s) = \min_{k \ s.t. \ 0\leq k\leq s} k.c+$$
$$\sum_{k_1=0}^{k}\sum_{k_2=0}^{k-k_1}\left(\begin{array}{c}k\\k_1,k_2\end{array}\right)\delta_1^{k_1}\delta_2^{k_2}(1-\delta_1-\delta_2)^{k-k1-k_2}V'_c(r+k-k_1-k_2, i+k_1, s-k+k_2) \tag{1}$$

where equation V' is the following:[9]

$$V'_c(r,i,s) = i\kappa+$$
$$\sum_{i'=0}^{i}\sum_{s'=0}^{s}\left(\begin{array}{c}i\\i'\end{array}\right)\left(\begin{array}{c}s\\s'\end{array}\right)\nu^{i'}(1-\nu)^{i-i'}\gamma^{s'}(1-\gamma)^{s-s'}V_c(r+i', i-i'+s', s-s') \tag{2}$$

with

$$\gamma = 1-(1-\beta)^i$$
$$\left(\begin{array}{c}n\\m\end{array}\right) = \frac{n!}{m!(n-m)!}$$
$$\left(\begin{array}{c}n\\m_1,m_2\end{array}\right) = \frac{n!}{m_1!m_2!(n-m_1-m_2)!}$$

Then, from a situation (r,i,s) and for a given number of individuals vaccinated k (inducing a cost $k.c$), k_1 individuals will get the disease and become of type I, k_2 individuals will remain of type S and $k-k_1-k_2$ individuals will become of type R. This occurs with probability $\sum_{k_1=0}^{k}\sum_{k_2=0}^{k-k_1}\left(\begin{array}{c}k\\k_1,k_2\end{array}\right)\delta_1^{k_1}\delta_2^{k_2}(1-\delta_1-\delta_2)^{k-k1-k_2}$. Once this "vaccination step" has passed, the epidemiological step is governed by the probabilities given in the previous section.

Notice that it is possible to rewrite any of Equations (1) in order to have $V_c(r,i,s)$ depend only on the values $V_c(r',i',s')$ with $r'\geq r$, $i'\geq i$ and ($r'>r$ or $i'>i$). This implies that function V_c is well defined by (1) whenever $\nu>0$. We denote by $v_c(r,i,s)$ the vaccination policy at (r,i,s) for a price c, *i.e.* the value of k chosen in Equations (1). This is the instantaneous demand function.

Then, for $i>0$, the intertemporal expected demand function $D_c(r,i,s)$ is given by:

$$D_c(r,i,s) = v_c(r,i,s)+$$
$$\sum_{k_1=0}^{v_c(r,i,s)}\sum_{k_2=0}^{v_c(r,i,s)-k_1}\left(\begin{array}{c}\left(\begin{array}{c}v_c(r,i,s)\\k_1,k_2\end{array}\right)\delta_1^{k_1}\delta_2^{k_2}(1-\delta_1-\delta_2)^{v_c(r,i,s)-k1-k_2}.\\D'_c(r+v(r,i,s)-k_1-k_2, i+k_1, s-v_c(r,i,s)+k_2)\end{array}\right)$$

where

$$D'_c(r,i,s) = \sum_{i'=0}^{i}\sum_{s'=0}^{s}\left(\begin{array}{c}i\\i'\end{array}\right)\left(\begin{array}{c}s\\s'\end{array}\right)\nu^{i'}(1-\nu)^{i-i'}\gamma^{s'}(1-\gamma)^{s-s'}D_c(r+i', i-i'+s', s-s')$$

[8]Notice that we consider no time discounting rate and only pure strategies, *i.e.* the domain of minimization is the set of integers.

[9]Notice that when solving Equations 1, (r,i,s) in Equations 1 and in Equations 2 don't necessarily have the same values.

Obviously, $D_c(r, i, s) = 0$ whenever $i = 0$.

As an illustration, Figure 1 shows $D_c(0, 1, 9)$ (in light grey) and $v_c(0, 1, 9)$ (in dark grey) as functions of c with $\beta = 0.05$, $\nu = 0.2$, $\kappa = 1000$, $\delta_1 = 0$ and $\delta_2 = 0$.

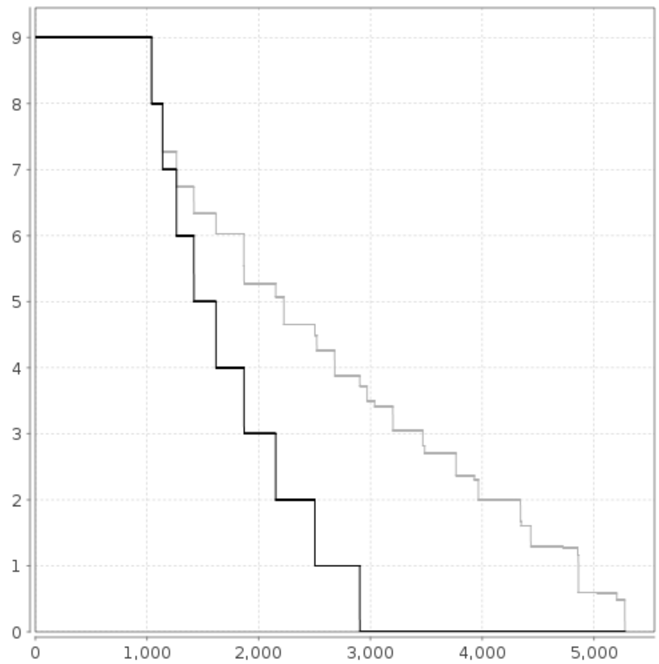

Figure 1: $D_c(0, 1, 9)$ (light grey) and $v_c(0, 1, 9)$ (dark grey) as functions of c.

First, notice that, in conformity with the intuition, $D_c(0, 1, 9)$ and $v_c(0, 1, 9)$ are decreasing with c. The higher the price of the vaccine, the lower the demand both instantaneous and intertemporal.

Second, notice that v_c is not necessarily an "all or nothing" vaccination strategy. Indeed, the incremental gains to vaccinate an individual depends on the number of individuals already vaccinated. Vaccinating an individual implies avoiding the direct costs of an infected. But it also implies avoiding further infections and this effect depends on the number of susceptible individuals left: the larger the number of susceptible individuals, the more important the indirect costs induced by an infection. Then, at each step, the marginal gain from vaccinating is decreasing with the number of vaccinations.

Third, notice that D_c and v_c differ. Since $\delta_1 = 0$ and $\delta_2 = 0$ in this case, it means that not vaccinating an individual the first day does not mean not vaccinating him later. Indeed, besides the effect given in the previous paragraph, the decision to vaccinate a susceptible depends on the probability that he gets infected one day. In Figure 1, there is only one infected (who will remain infected for $1/\nu$ periods on average). In this case, the probability to get infected for a susceptible can be considered too low to vaccinate for a price c. However, it is possible that later, the number of infected will be greater, making the probability for the susceptible individuals to be infected greater. But this case will not necessarily occur and there is a strictly positive option value to wait in this case. The possibility to vaccinate a susceptible individual in the future must be considered in the expected demand D_c even though he is not vaccinated in the first day and then does not count in v_c. Obviously this argument does not hold when everyone is vaccinated because there will be no susceptible individuals in the future (remember that $\delta_1 = 0$ and $\delta_2 = 0$). It is also not valid when only one individual is not vaccinated in the current period since the last susceptible will face at most one infectious individual in the future (all the others being recovered since vaccinated).

We denote by $V^-(r,i,s)$ the intertemporal expected cost at (r,i,s) if there was no vaccine or equivalently if the price of vaccines was infinite.

$$V^-(r,i,s) = V_\infty(r,i,s).$$

Then, the surplus from the existence of vaccines at price c earned by the buyer is:

$$SC_c(r,i,s) = V^-(r,i,s) - V_c(r,i,s)$$

4 The seller's decision

We consider a fully rational and perfectly forward-looking seller. Then, the seller can anticipate from any state how many vaccines will be demanded in expectation. Varying c, the seller knows the demand function he faces and from that, picks c^* such that it maximizes his profit. Notice that we implicitly assume that the unit price of the vaccines set by the seller is set before the first demand is expressed and cannot change afterward Then,

$$c^*(r,i,s) = \arg\max_c c.D_c(r,i,s),$$

and the profit earned by the monopolist seller is

$$\Pi(r,i,s) = \Pi_{c^*(r,i,s)}(r,i,s) = c^*(r,i,s).D_{c^*(r,i,s)}(r,i,s)$$

5 Surplus

The total surplus for the society is

$$S_c(r,i,s) = SC_c(r,i,s) + \Pi_c(r,i,s)$$

Because there is no cost to produce the vaccine, the optimal total surplus at (r,i,s), denoted $S^+(r,i,s)$ is

$$S^+(r,i,s) = SC_0(r,i,s)$$

It is also the total surplus that would be observed in a competitive environment, in which case profits would be null. Remember that contrary to the study by Xu (1999), we do not consider the demand as the result of decentralized decisions. Hence, there is no externality to make the outcome of a competitive environment and the social optimum different.

In the remainder, for computational reasons, we will consider a population of 10 individuals. Moreover, we will consider the case of a buyer facing the situation of an epidemic uprising with a single individual infected and all others susceptible, *i.e.* $(r,i,s) = (0,1,9)$. The parameters are set as follows: $\beta = 0.05$, $\nu = 0.2$, $\kappa = 1000$.

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Figure 2 shows the total surplus in a competitive environment, $S^+(0,1,9)$, as a function of δ_1 and δ_2. As can be seen, the more effective the vaccine (smaller δ_1 or δ_2), the larger the surplus. This is no surprise since a better vaccine means less infections and the buyer's objective is precisely only to limit the number of infections when $c = 0$. When $\delta_1 = 0$ and $\delta_2 = 0$ (the vaccine is fully effective), the social surplus is of about 24.5 days of disease avoided ($S^+(0,1,9)/\kappa \approx 24,475/1000$). When $1 - \delta_1 - \delta_2 = 0$, *i.e.* when vaccines are totally inefficient or imply side-effects, the social optimum is obtained when no vaccine is used (and hence, there is no case of infection avoided from the use of vaccines) and then, $S^+(0,1,9) = 0$.

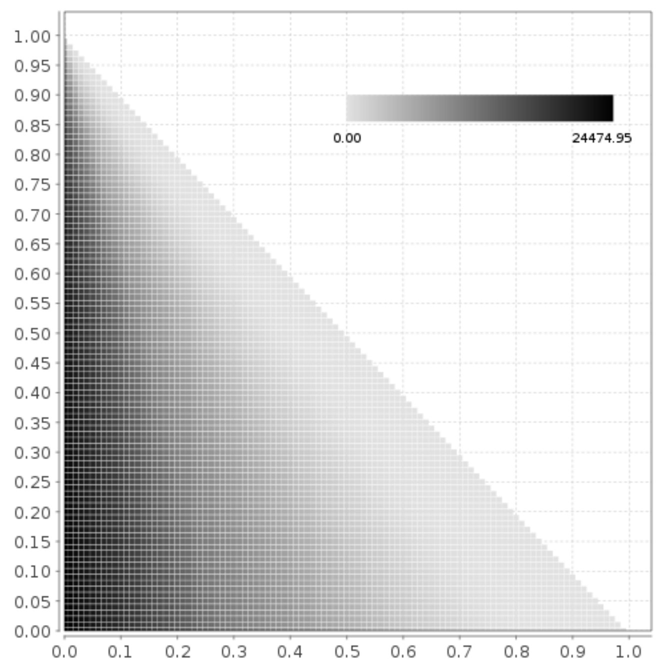
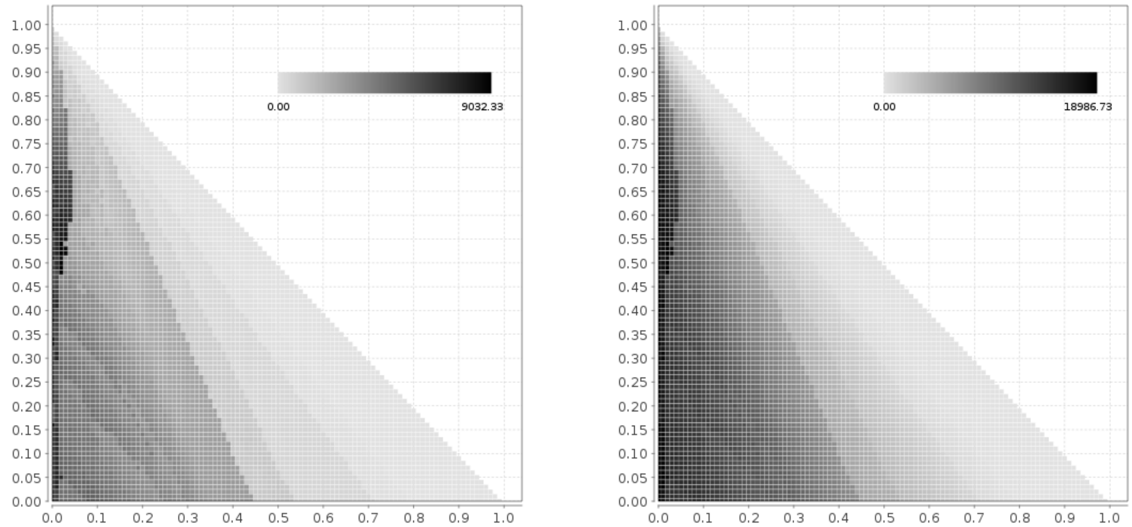


Figure 2: $S^+(0,1,9)$ as a function of δ_1 (abscissa) and δ_2 (ordinate).

Let us now see what happens when the market has a monopolistic structure. In this case, the fact that the social surplus or the buyer's surplus increases with the quality of the vaccines does not hold. In this case, the objective of the buyer is to limit his costs that depend on the number of infections but also on the price of the vaccine, set by the monopolistic seller beforehand. Figure 3a shows the buyer's surplus and Figure 3b shows the social surplus attained when the market structure is monopolistic as a function of δ_1 and δ_2. It can be easily seen that an increase in δ_2 (*i.e.* when the vaccine has no effect with a larger probability) does not imply a decrease in the social surplus. The same can be seen for an increase in δ_1 (*i.e.* when the vaccine has the side-effect of infecting with a larger probability).

Not only is it the case that a less effective vaccine can increase the social surplus, it can also increase the profit. This is shown in Figure 4 comparing the demand functions and profits for $\delta_1 = 0.11$ and $\delta_2 = 0$ or $\delta_2 = 0.06$.[10] The dashed curve represents the set

[10]Precisely, for $\delta_1 = 0.11$ and $\delta_2 = 0$, $c^*(0,1,9) = 2220.3$, $D_{c^*(0,1,9)}(0,1,9) = 4.354$ and $\Pi(0,1,9) = 9667.482$. For $\delta_1 = 0.11$ and $\delta_2 = 0.06$, $c^*(0,1,9) = 2168.4$, $D_{c^*(0,1,9)}(0,1,9) = 4.499$ and $\Pi(0,1,9) = 9755.708$.



(a) $SC_{c^*(0,1,9)}(0,1,9)$ as a function of δ_1 (abscissa) and δ_2 (ordinate).

(b) $S_{c^*(0,1,9)}(0,1,9)$ as a function of δ_1 (abscissa) and δ_2 (ordinate).

Figure 3: Buyer's and total surplus with a monopolistic seller.

of points such that $c.D_c(0,1,9) = 9755.708$. This curve is attained by the demand in the case $\delta_1 = 0.11$ and $\delta_2 = 0.06$ (in light grey) but not in the case $\delta_1 = 0.11$ and $\delta_2 = 0$ (in dark grey).

It seems natural to think that a less effective vaccine will be less demanded for the same fixed price. Actually, in some cases, the demand for vaccines can be higher for vaccines with a larger δ_2 (less effective vaccines). In Figure 4 (with 9 susceptible individuals and 1 infected), when the cost is low enough, the expected intertemporal demand for vaccine is 9 and takes place only in the first period. Indeed, in this case, the buyer's decision is to administer vaccines to all susceptible individuals. Then, the susceptible individuals become recovered or infected (remember that $\delta_2 = 0$), then there is no susceptible to vaccinate in the future. When δ_2 slightly increases (there is a possibility for the vaccine to be ineffective), the expected intertemporal demand can be higher since a portion of the vaccinated individuals for which the vaccine will be inefficient, will represent a future demand. Then, there can be an effect of profit cannibalization by the monopolistic seller[11] This increase in the demand for a less effective vaccine is the reason why the profit can increase when the vaccine supplied is of lower quality. Also notice that if the quality of the vaccine was endogenous and set by the monopolistic seller, the latter could choose to lower the quality of the vaccine he sells on the market and then avoid profit cannibalization by itself.

Notice that in the case shown in Figure 4, even though profit increases when the vaccine is of lower quality, the social surplus does not. Indeed, for $\delta_1 = 0.11$ and $\delta_2 = 0$: $SC_{c^*(0,1,9)}(0,1,9) = 4843.075$ and $S_{c^*(0,1,9)}(0,1,9) = 14510.558$ whereas for $\delta_1 = 0.11$ and $\delta_2 = 0.06$: $SC_{c^*(0,1,9)}(0,1,9) = 4286.878$ and $S_{c^*(0,1,9)}(0,1,9) = 14042.586$.

[11] This effect is also described in Kremer and Snyder (2003).

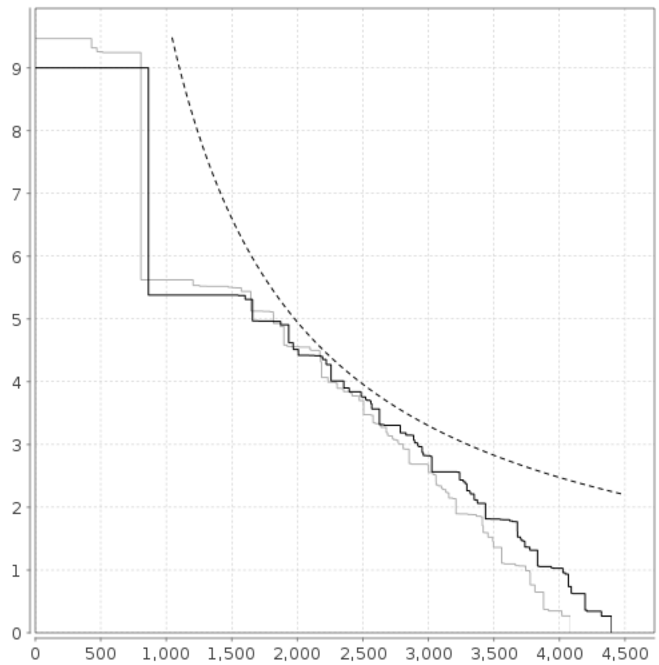


Figure 4: $D_c(0, 1, 9)$ as a function of c for $\delta_1 = 0.11$ and $\delta_2 = 0$ (dark grey) or $\delta_2 = 0.06$ (light grey). The dashed line is the set of points such that $c.D_c(0, 1, 9) = 9755.708$.

6 Conclusion

Then, with a simple model of vaccine market where the demand is endogenous and based on a basic SIR epidemiological model and where the supplier is a monopoly, we showed that more efficient vaccines do not imply greater total surplus, buyer surplus or even profits.

This result questions the general belief by the general population as well as by health authorities that vaccine quality should be as high as possible. It even questions the usage in many countries that two different departments evaluate the quality of a vaccine on the one hand and its economic impact on the other hand. In some cases, lower the quality of a vaccine in comparison with the highest quality achievable with the current techniques may induce higher economic impact.

Let us end our study with a couple of remarks about the market features we considered and that should be seen as a limitation to our results. First, we did not consider any cost of production. If there was a flat unit cost of production, our qualitative results would not change. However, the market for vaccines is often characterized by production capacity constraints or costs of production highly dependent on the demand level. Second, we considered that the buyer, even though it is a centralized one has no market power. The question of the robustness of our results with a buyer with some market power (think of a country-large health authority or a major employer) remains open. More generally, our model relies on a model with an monopolistic no-cost market. The generalization of our model to cases with markets closer to the actual vaccine market structure has not been addressed.

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