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### Inequality and Innovativeness

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#### Abstract

In this note, we construct two theoretical models that analyze the relationship between inequality of access and rates of innovation as well as correlative data that show a negative correlation between income inequality and levels of innovativeness. Our two models suggest that unequal access to problems slows innovation by reducing the level and variety of human capital applied to problems. More interestingly, both models show that the rate of innovation decline becomes much more pronounced as problems become more difficult. Thus, the costs of inequality may be increasing as the problems that societies face become more challenging.

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## Introduction

Evidence suggests that rising inequality can produce pernicious effects at the societal level including reductions in economic prosperity and lower average quality of life (Stiglitz 2012, Wilkenson and Pickett 2009). In this note, we consider how inequality might impact rates of innovation by constructing two models of problem solving and comparing levels of innovation as a function of equality of access. Both models assume that more equal societies enable more people to contribute to innovative processes and that less equal societies restrict innovative activities to a subpopulation. Thus, as would be expected, more equal societies prove more innovative. Both models also demonstrate that the reduction in innovation due to inequality increases markedly as problems become more difficult to solve. Thus, as the problems that societies face become more difficult, the costs of inequality may well increase substantially.

In our two models, we capture innovations as iterative improvements in the best current solution to a problem. These improvements arise as individuals apply their diverse cognitive tools to the problem (Hong and Page 2001). This approach differs from the common conception of innovation as a solitary genius shouting eureka and forever transforming how we see a problem. Though such singular events do occur, the bulk of empirical evidence demonstrates that most innovations result from small improvements in standard practices and designs which occur through the application of new and existing techniques as well as by recombining and borrowing existing ideas (Freeman 1985, Bessen and Maskin 2009, Carnabuci and Bruggeman 2009, Arthur 2009, Mokyr 2002). Hence, innovation is more accurately modeled as the sequential application of specialized, diverse talent as we have done here (Weitz 1998, Florida 2005, Quigley 1998, Glaeser 2011).

Our models enable us to explore the relationship between *socio-political means*, or access, and rates of innovation. The first model builds upon a problem solving framework introduced by Hong and Page (2004) that considers individuals as bundles of ways of looking at problems (perspectives) and tools that they apply to problems (heuristics). We add to that framework differential access to problems. The second model relies on a variant of a random search model. As mentioned above, in both models, we find that performance declines as means become restricted – inequality of access lowers rates of innovation and the extent of that decline increases as the problems confronting the collective become more difficult. This second result aligns with intuition that diversity matters more for hard problems (Page 2007).

## The Empirical Relationship Between Inequality and Innovativeness

We begin with a straightforward correlation exercise to explore the empirical relationship between income inequality and innovativeness. We measure country level innovation using estimates taken from the World Economic Forum's Global Competitiveness Report (Porter and Schwab 2009). Their innovation measure relies on patent data as well as extensive surveys that capture innovative capacity, scientific research quality, private R & D spending, collaboration between universities and industry, government spending on technology, and

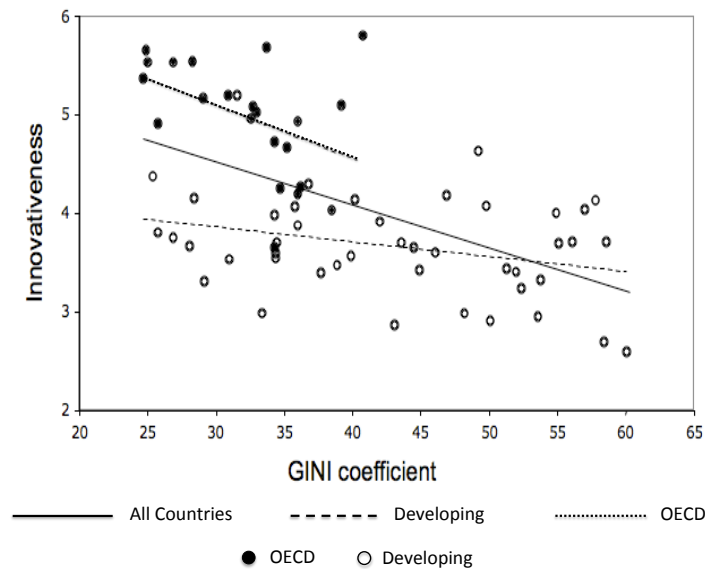


Figure 1: Decline in Innovativeness as a Function of Inequality (Empirical Data)

the number of scientists and engineers (Browne et al. 2009). To measure income equality we rely on GINI coefficients. We consider this to be a useful proxy for social inequality, particularly access. In Figure 3, we plot three lines, one for the entire sample and one each for developing and OECD countries. We split the data on the presumption that innovation and inequality may have a different relationship in more economically advanced countries.

In each case, we find a significant and negative relationship between inequality and innovativeness. For the entire sample, we obtain an estimated slope of  $-0.044$  (p-value 0.000002). For the developing countries, we obtain an estimated slope of  $-0.015$  (p-value 0.0456), and, finally, for the OECD countries, the estimated slope equals  $-0.05$  (p-value 0.04). The data show the effect of inequality on innovation to be larger for developed countries. Interestingly, the United States has both the highest innovativeness and highest inequality among OECD countries. This disconnect may well be explained by immigration. Hunt, and Gauthier-Loiselle (2009) estimate that a one percent increase in immigrant college graduates may produce a nine to eighteen percent increase in patents. Thus, the United States may be borrowing cognitive diversity from abroad to drive innovation (Page 2008).

Assuming developing countries face harder problems, then the difference in slopes aligns with our models' second prediction. However, a statistical test on differences of coefficients for the two models returns a Z-score of 1.51 which is in the right direction but not significant

at the 0.05 level.<sup>1</sup>

## A Computational Model of Innovation and Access

To gain insight into the relationship between inequality and innovativeness, we construct two models. We begin with a simple computational model that builds from the work of Hong and Page (2004). We assume a population of agents of variable means who attempt to find a solution that improves the value of some function  $V$ . An improvement in the value of  $V$  represents an *innovation*. We denote the set of possible solutions by the set  $X$  and assume that  $V$  maps  $X$  into the positive real numbers. The representation of solutions in an agent's internal language is called a *perspective*, while an agent's *heuristic* is a mapping  $A$  from solutions to subsets of solutions.

We also include a map,  $R$ , that represents an agent's socio-political means or capacity to incorporate her solutions into the set of proposed solutions under consideration. In an equal inclusive society,  $R$  equals the identity mapping. In an unequal, non inclusive society,  $R$  restricts the set of solutions put forward. In the most extreme case,  $R$  maps every set of proposed solutions to the empty set, i.e. the agent's ideas are ignored. Each agent can be represented by a triple: a perspective, a set of heuristics, and a level of socio-political means  $(M, A, R)$ .

In the model, we assume twenty agents, each represented by a heuristic set of three integers:  $h1$ ,  $h2$ , and  $h3$ . We assume a random status quo starting point. The agent moves the solution  $h1$  steps clockwise and compares that value to the status quo's. If an innovation occurs, that solution becomes the new status quo. The agent then moves  $h2$  steps clockwise and repeats the comparison, followed by a move of  $h3$  steps clockwise and another comparison. The agent then tries  $h1$  and continues trying all three heuristics until none improve the value. The second agent applies her  $hi$ 's in the same way. We loop through each of the twenty agents twice.

To capture changes in socio-political means on the value of the final solution, we assign an *inequality score* to each agent. Scores are uniformly drawn from  $[0, 1]$ . This represents the agent's social status, with higher values denoting higher status. We then compare status to a social equality threshold,  $G$ . Only if an agent's status exceeds the threshold will that agent's solutions be considered.

We apply this model to two sets of problems. Each solution set consists of 10,000 random values drawn from  $(0, 1)$  ordered on a circle. First, we considered problems in which the values were drawn from a uniform distribution. Second, to model harder problems, we distributed values of solutions as a power function  $f(x) = ax^b$ , where  $f$  denotes the probability density function. We used the parameter values  $a = 1$  and  $b = 30$ . All results are presented as a fraction of the best possible solution. A total of one hundred different heuristic sets were

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<sup>1</sup>We should be clear that we do not intend this correlation exercise as empirical validation or test of the models that follow. It merely provides context for thinking about the relationship between inequality and innovativeness

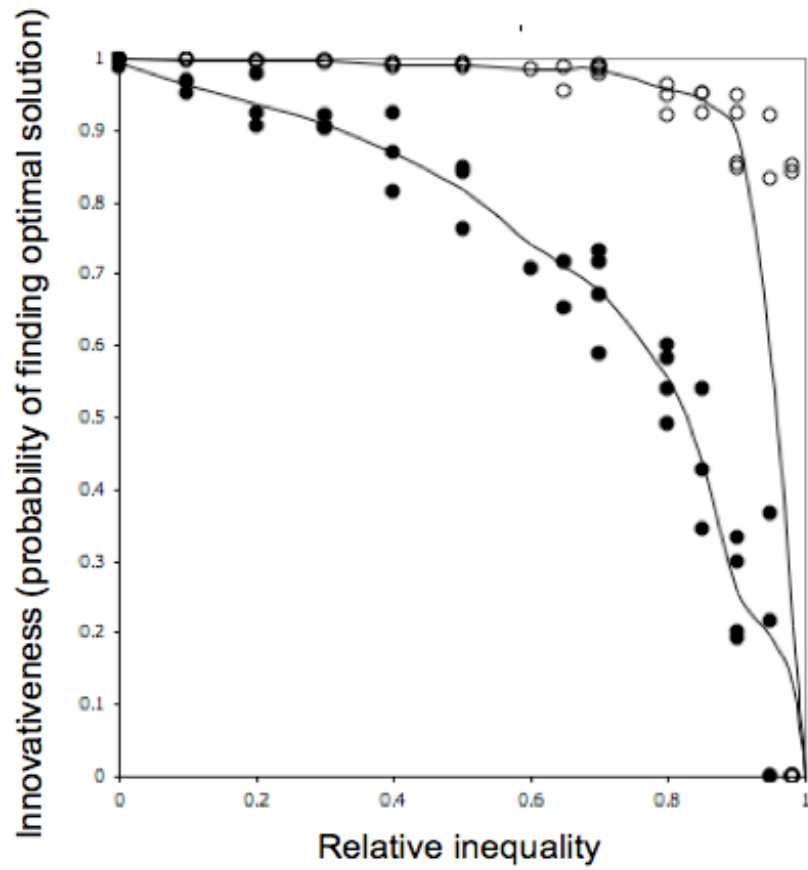


Figure 2: Innovativeness and Inequality (Computational) Open circle:easy closed circles:  
hard

evaluated for each solution set, and the average best solution found was recorded, for each  $G$ . The results are displayed in Figure 2.

By construction increasing  $G$  reduces the number of heuristics applied to problems, and therefore must, on average, reduce the number of innovations. However, as the figure shows, for simple problems the costs of inequality on innovation are relatively minor. For hard problems, the result becomes more pronounced. The model produces a steep decline in innovation as inequality increases. Thus, the computational model demonstrates an increasing cost to inequality as problem difficulty increases.

## An Analytic Model of Innovation as Search for Better Solutions

To gain analytic traction on why problem difficulty has such a large effect, we construct a second model of innovation as a random search model (Callendar 2011). This model differs from the previous model in two ways. First, in the random search model no spatial relationship exists between solutions. In the computational model, each solution had neighboring solutions producing a spatial structure. Here, no such structure exists. Subsequent solutions need bear no resemblance to earlier searches. Second, in this model problem difficulty becomes a parameter. We let  $x$  denote a proposed solution. We distribute the values of those solutions according to the density function  $f(x) = \frac{x^{-D}}{1-D}$ , where  $D$  is in the interval  $(0, 1)$ . The c.d.f for this distribution equals  $x^{(1-D)}$ . The parameter  $D$  serves as a proxy for the difficulty of the problem. As  $D$  approaches one, most of the draws have very low values, thus representing a hard problem. And, as  $D$  approaches 0, the distribution approaches a uniform probability distribution, representing an easy problem. Similar to the previous model, we capture *sociopolitical means* by a probability that a randomly chosen agent's solution will be considered. We denote this by  $p$ . We assume  $K$  agents. In the model, each of the  $K$  agents takes a draw from the distribution. With probability  $p$  that solution is considered. The best solution among those considered is then chosen. The expected value of the best solution given  $p$ ,  $D$ , and  $K$  are as follows:

$$\begin{aligned} \frac{\int_0^1 x(x^{1-D})^{pK-1} dx}{\int_0^1 (x^{1-D})^{pK-1} dx} &= \frac{\int_0^1 x^{pK-DpK+D} dx}{\int_0^1 x^{pK-DpK-1+D} dx} \\ &= \frac{pK - pKD + D}{pK - pKD + D + 1} \end{aligned}$$

In the special case  $(1-D) = \frac{1}{K}$ , the expected value of  $K$  agents equals  $\frac{p+D}{p+D+1}$ . The derivative with respect to  $p$  is as follows:

$$\frac{\partial}{\partial p} \left[ \frac{p+D}{p+D+1} \right] = \frac{1}{(p+D+1)^2}$$

For  $D$  near one, the derivative ranges from approximately  $\frac{1}{4}$  to approximately  $\frac{1}{9}$  as  $p$

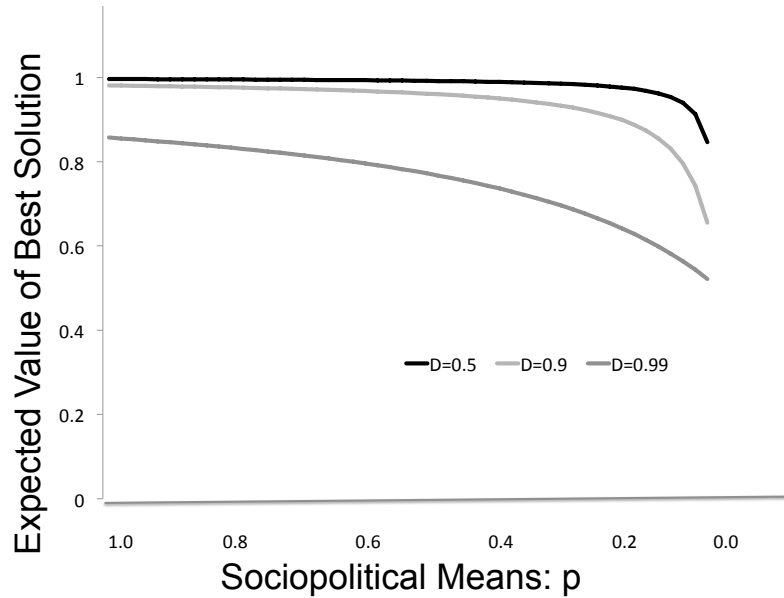


Figure 3: Decline in Innovativeness as a Function of Inequality (Mathematical Model)

ranges from 0 to 1. Thus, low socio-political means ( $p = 0$ ) has a large effect on the best solution for difficult problems. Figure 2 shows the expected values of the best solution for various value of  $p$  with  $K$  set at 500. We plot these graphs as function of  $G = (1 - p)$ . We consider values of  $D$  equal to 0.5, 0.9 and 0.99. Notice that as  $D$  increases – as the problem become more difficult – the costs of restricting sociopolitical means – increase. Relatedly, for relatively easy problems, restricting access has almost no effect until access is severely restricted ( $p$  near 0). This finding can be shown formally by taking the derivative with respect to  $D$  of the marginal value of changing  $p$ .

The analytic model reproduces both results of the computational model: innovativeness decreases in the level of inequality and the falloff becomes more pronounced as problems become more difficult. In each case, the interaction between our assumptions about the distribution of values with the statistical properties of order statistics produces the result.

## Discussion

In this note, we have constructed two models that relate the level of innovation to problem access. Both models show that decreasing access lowers performance and that the performance falloff becomes more pronounced for harder problems. The increased cost of inequality on innovation results from the fact that finding a good solution to a harder problem requires more searches. And, by construction, restricting access results in fewer searches. The costs of those lost searches become more pronounced when good solutions are more difficult to

find. Thus, although we have relied on a specific family of functional forms in this paper, any family of distributions in which good solutions become less probable would produce a qualitatively similar result.

The negative relationship between inequality and innovativeness that our models produce would seem to imply that one should also expect a negative relationship between inequality and growth rates (Stiglitz 2002, Persson and Tabellini 1994). While at first blush that intuition makes sense – more innovation should imply more growth – deeper thinking on the relationship between inequality and either productivity or growth reveals multiple causal forces of which innovation is only one. Our model highlights how inequality reduces access and opportunity, but increasing equality, particularly through transfers, could also reduce growth as transfers reduce the incentive to work (Okun 1975). More generally, whether productivity increases or decreases will depend on factors such as the complementaries produced by high income workers (Benabou 1996) or how inequality impacts public spending in areas such as education (Saint-Paul and Thierry 1993). Thus, causal arrows between inequality and growth point in both directions. And, not surprisingly, the empirical relationship inequality and growth proves muddled. Many papers demonstrate a negative relationship - where inequality stifles growth (Barro and Sala-i-Martin 1995, Berg and Ostry 2011) . Yet, several other careful well constructed analyses find a positive relationship (Forbes 2000) , where inequality increases growth. The fact that distinct specifications and data sets result in differing coefficients should come as no surprise given the number of control variables in these models (Achen 2005).

One way to make sense of that complexity is to look more deeply at the specific components – tounderstand theoretically and empirically how the many parts connect. Hence, our rather modest focus on on how inequality relates to innovativeness and specifically on how that relationship may depend on problem difficulty should be seen as contributing to the micro foundations of a deeper theory relating inequality and economic well being. That said, if future more sophisticated empirical work supports the correlative finding that innovation decreases in levels of inequality and if our theoretical finding that the falloff increases as problems become harder, then the future effects of inequality of growth may become more pronounced and negative. As the scientific, technological, and social challenges that lay before us become more difficult, the need for greater access to those problems, to have more sets of eyeballs looking for solutions, may become more acute.

In addition, one could argue that the costs of inequality of access may be even larger than our models suggest given we assume that those people restricted from participating in the innovative process are identical to those who do participate. In many societies, identity differences – differences in gender, race, religion, ethnicity, and physical ability – correlate with access. To the extent that these identity characteristics also correlate with cognitive diversity, the loss in innovation from restricted access would be greater than our models suggest. If in addition, individuals in more stratified societies lack the ability to work with and understand those from different ethnic or cultural groups (Gurin et al. 2013), then the ability to tap into those diverse ways of thinking may be limited even further.



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