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# Revealing Product Information to Bidders with Differentiated Preferences

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## Abstract

We study information disclosure in standard auctions where bidders preferences are horizontally differentiated, whose valuations depend on the matching between the product attribute and their preferences. The seller may reveal product information in the form of partition prior to the auction. Under a symmetric setting, we show in a close-form result that more precise information induces more dispersed distributions of bidders' posterior valuations, which, specifically, are ordered in terms of First Order Stochastic Dominance (FOSD). We also prove that optimal disclosure policy is extreme, in the sense that the seller will reveal either full or no information to the bidders, depending on the number of bidders.

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## 1 Introduction

Information disclosure in auctions is an important topic for research. In the early classic work of Milgrom and Weber (1982), they show that, in interdependent value auctions with affiliated signals, revealing information to bidders *on average* increases the expected auction revenues, and the best disclosure policy is full information disclosure. In their model, bidders' preferences are homogenous, and information affects bidders' valuations in the same manner.

The recent literature on information disclosure pays more attention to optimal information structures, and the central question is how much information a seller should reveal to consumers (Lewis and Sappington, 1994; Johnson and Myatt, 2006; Board, 2009; Ganuza and Penalva, 2010). And various concepts of information order are introduced, such as rotation order in Johnson and Myatt (2006) and precision order in Ganuza and Penalva (2010). Though different in definitions, they commonly emphasize the same intuition that more precise information will induce more dispersed posterior valuations. For example, revealing a specific product attribute may increase some consumers' willingness to pay, yet may reduce that of others. Therefore consumers' posterior valuations become more dispersed, and information disclosure can rotate the demand curve clockwise. In this case, the seller face the trade-off between a *niche market* strategy, by revealing precise information and charging higher prices, while only a portion of the consumers are served, and a *mass market* strategy, by revealing little information and charging lower prices, while most consumers are served. They show that, when information is costless, the seller should reveal either full or no information to consumers.

In those papers, although preference differentiation is pivotal for the result, it is not explicitly modelled. In fact, there are no explicit signals at all in those models. On the contrary, they just assume a monotonic relationship between signal precision and the degree of dispersion of posterior valuations, while remain silence on the underlying mechanism. The only exception is Ganuza (2004), where he models preference differentiation on a Hotelling circle, and investigates optimal disclosure with costly information. In his model, signal is in the form of Gaussian noise and bidders' utilities are quadratic. Yet with this setting, he is not able to provide clear characterization of posterior valuations, and therefore the underlying mechanism of demand rotation.

In this paper, we investigate a similar model as Ganuza (2004), yet with two major differences. First, bidder's utilities are in the form of absolute value. Second and more importantly, information structure is characterized by partitions of state space. Under this setting, we are able to derive close-form solutions and clearly reveal the mechanism on how information disclosure can rotate demand curves. Particularly, we show that, corresponding to increasingly precise information, the distributions of bidders' posterior valuations are ordered in terms of First Order Stochastic Dominance (FOSD). To our knowledge, it is the first close-form result in the literature.

We also show the similar result of extreme disclosure policy in our model of endogenous valuations, yet with some further qualifications. Let $n$ denote the number of bidders. We show that, when $n > 3$, it's optimal to reveal full information to the bidders; when $n < 3$, it's better for the seller to withhold all information; and when $n = 3$, the seller is indifferent between revealing full information or not. This result is similar to that of Board (2009), which is achieved in a reduced-form model. Here, in a model of endogenous valuations, we could not only provide new insights, but also show that the result is sensitive to the functional forms of utility functions.

The remaining parts of this paper are organized as follow: Section 2 is model setup; Section 3 we provide the main result of this paper on signal precision and the shape of the distribution of bidder's posterior valuations; Section 4 is about optimal disclosure policy; and Section 5 is a short conclusion.

## 2 The Model

A seller sells a product to $n$ *ex ante* homogenous bidders, indexed by $i = 1, 2, \cdots, n$, in a standard auction. The product is characterized by its vertical value, $V$, and horizontal attribute, $S$. The value of $V$ is commonly known, but $S$ is a random variable, whose realization, $s$, is observed only by the seller.

Bidder $i$'s ideal product attribute is $\theta_i$, which is also his type, and his valuation of the product depends on the matching between $\theta_i$ and the product attribute, as below

$$v_i(s; \theta_i) = V - \tau |s - \theta_i| \tag{1}$$

where $\tau$ is a coefficient measuring the degree of disutility of mismatching. The seller's valuation of the product is zero, and she is a revenue maximizer.

Both the product attributes $S$ and types of the bidders, $\theta_i$'s, are independent draws from a uniform distribution on a unit circle. Prior to the auction, the seller may reveal product information to the bidders, and the disclosure policy is characterized by a partition of the product attribute space.

A partition of degree $J$, denoted as $P_J$, is defined as a sequence of cutting points, $\{p_1, p_2, \cdots, p_J\}$, that divides the unit circle into $J$ sub-sections. And $\Delta_j = |p_j - p_{j-1}|$ is the length of the sub-section $[p_{j-1}, p_j]$. An equal partition of degree $J$, denoted as $P_J^E$, is a partition where all the sub-sections are of the same length.

**Definition 1** *A disclosure policy for a partition $P_J$ is a mapping from the attribute space to the signal space $M_J$, that is, $D(P_J) : S \longrightarrow M_J$. Specifically,*

$$D(P_J) = m_j \quad \textit{iff } s \in [p_j, p_{j+1}) \tag{2}$$

When receiving a signal $m_j$, a bidder $i$ will update his belief and forms the posterior estimate of product valuation, which is

$$v^i(m_j) = \mathbb{E}_{S|m_j} v_i(s;\theta_i) = \int_{s\in[p_j,p_{j+1})} v_i(s;\theta_i)dG(s|m_j) \tag{3}$$

where $G(\cdot)$ is the corresponding cdf function. The timing of this game is as below

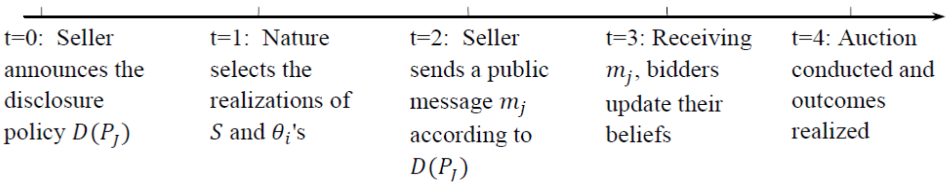


Figure 1: Timeline of the Game

It is worth attention that a disclosure policy of $D(P_J)$ implies $J$ possible signals, each inducing a distribution of bidders' posterior valuations. Therefore, an information partition $P_J$ generates a set of $J$ possible posteriors. For a signal $m_j$, we let $\mathbb{E}R(m_j)$ denote the corresponding *interim* expected auction revenue, and the seller's problem is to determine the optimal partition that maximizes the *ex ante* expected revenue.

$$\max_{P_J} \quad \mathbb{E}R = \sum_{j=1}^{J} \Pr(m_j)\,\mathbb{E}R(m_j)$$

The choice of information partition $P_J$ is twofold: one is the partition degree of $J$, and the other is locations of the cutting points, $(p_1, p_2, \cdots, p_J)$. With our symmetric setting of uniform distribution on a Hotelling circle, we focus on disclosure policies in the form of equal partitions, and therefore what is relevant here is just the degree of partition.

## 3 Disclosure and Distribution of Posterior Valuations

The focus of this section is to investigate how information disclosure affects the distribution of bidders' posterior valuations. First, we will define the precision of signals in an intuitive way. A signal $m_j$ is defined to be more precise than another one $m_{j'}$, if and only if $\Delta_j < \Delta_{j'}$. In our case of equal partitions, it is straightforward to define that an disclosure policy $D\left(P_J^E\right)$ reveals more precise information than another one, $D\left(P_{J'}^E\right)$, if and only if $J > J'$. Apparently, when $J > J'$, any signal under $D\left(P_J^E\right)$ is more precise than those from $D\left(P_{J'}^E\right)$ by definition.

In an equal information partition of $P_J^E$, we let the original point be 0 without loss of generality, and the cutting point is then $p_1 = \frac{1}{2J}$ and $p_J = -\frac{1}{2J}$. Therefore, for the message of $m_1$, corresponding to the sub-section of $\left[-\frac{1}{2J}, \frac{1}{2J}\right)$, and bidder $i$'s posterior valuation of the product is

$$v^i(m_1) = \int_{-\frac{1}{2J}}^{\frac{1}{2J}} (V - \tau|s-\theta_i|)\, dG(s|m_1) = \begin{cases} V + \tau\theta_i & \text{if } \theta_i \in \left(-\frac{1}{2}, -\frac{1}{2J}\right] \\ V - \frac{\tau}{4J} - \tau J\theta_i^2 & \text{if } \theta_i \in \left(-\frac{1}{2J}, \frac{1}{2J}\right) \\ V - \tau\theta_i & \text{if } \theta_i \in \left[\frac{1}{2J}, \frac{1}{2}\right] \end{cases} \tag{4}$$

As $\theta_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, the minimum and maximum of $v^i$, denoted as $\underline{v}$ and $\bar{v}(J)$ respectively, are $\underline{v} = V - \frac{\tau}{2}$ and $\bar{v}(J) = V - \frac{\tau}{4J}$. We notice that $\underline{v}$ does not depend on $J$. The value of $v^i$ at the inflection points of $\theta_i = \pm\frac{1}{2J}$ is $\hat{v}(J) = V - \frac{\tau}{2J}$. If $v \leq \hat{v}(J)$, then the cdf function of $v^i$ is $F(v) = \Pr(v^i(\theta_i) \leq v) = 1 - \frac{2V}{\tau} + \frac{2v}{\tau}$. If $v > \hat{v}(J)$, then from (4)

$$F(v) = \Pr\left(v^i(\theta_i) \leq v\right) = 1 - \Pr\left(v^i(\theta_i) > v\right) = 1 - 2\sqrt{\frac{V-v}{\tau J} - \frac{1}{4J^2}}$$

In summary, the cumulative distribution of $v^i$ is

$$F(v;J) = \begin{cases} 1 - \frac{2V}{\tau} + \frac{2v}{\tau} & \text{if } v \in [\underline{v}, \hat{v}(J)] \\ 1 - 2\sqrt{\frac{V-v}{\tau J} - \frac{1}{4J^2}} & \text{if } v \in (\hat{v}(J), \bar{v}(J)] \end{cases} \tag{5}$$

It is worth attention that $F(v;J)$ doesn't depend on the specific signal. In fact, under our symmetric setting, any signal $m_j$ in $P_J^E$ will induce the same distribution of posterior valuations. Then $F(v;J)$ is also the distribution of bidder's posterior valuations induced by the disclosure policy of $D\left(P_J^E\right)$ as a whole.

Figure 2 shows how $F(v;J)$ changes with $J$. It is interesting to observe that, when the seller reveals more precise information ($J$ increases), the induced distribution of bidders' posterior valuations becomes more dispersed. Specifically, they can be ordered in terms of first order stochastic dominance (FOSD). We provide the formal result as below:

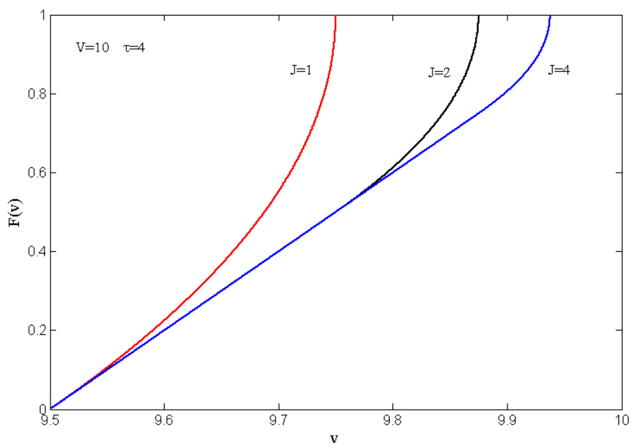


Figure 2: Distributions of Posterior Valuations

**Proposition 2** *For $J > J' \geq 1$,*

$$F(v;J) \leq F(v;J') \quad \textit{for } v \in [\underline{v}, V] \tag{6}$$

In (5) we provide a close-form solution to the distribution of bidders' posterior valuations. And Proposition 2 shows that, corresponding to increasingly more precise information disclosure, the distributions of bidders' posterior valuations are ordered in terms of FOSD. FOSD is a stronger concept that also implies the rotation order in Johnson and Myatt (2006), as well as the precision orders in Ganuza and Panalva (2010).

## 4 Optimal Information Disclosure

Upon receiving a public signal, the bidders are in a standard independent private value auction, where the one with the highest bid wins the auction, and the expected revenue is equal to the expected value of the second highest valuation. Therefore, the impacts of disclosure on expected auction revenues depend on how it affects the second highest valuation. The expected value of the highest valuation is obviously increasing in the precision of the signals, but this is not necessarily true for the second highest one, which depends on the number of bidders in the auction.

For a given signal of $m_j$, $v^i$'s are $n$ independent draws from the same distribution of $F(v;J)$. Let $v_n^{(1)}$, $v_n^{(2)}$, $\cdots$, $v_n^{(n)}$ be a rearrangement of these such that

$$v_n^{(1)} \geq v_n^{(2)} \geq \cdots \geq v_n^{(n)}$$

Then $v_n^{(k)}$, $k = 1, 2, \cdots, n$, are the *order statistics* of $v^i$'s when the number of bidders is $n$. Specifically, $v_n^{(k)}$ is the $k$th highest of the $v^i$'s, and we let $H_n^{(k)}(v;J)$ denote its cumulative distribution function.

For the highest valuation, $v_n^{(1)}$, apparently

$$H_n^{(1)}(v;J) = F^n(v;J) = \begin{cases} \left(1 - \frac{2V}{\tau} + \frac{2v}{\tau}\right)^n & \text{if } v \in [\underline{v}, \hat{v}] \\ \left(1 - 2\sqrt{\frac{V-v}{\tau J} - \frac{1}{4J^2}}\right)^n & \text{if } v \in (\hat{v}, \bar{v}] \end{cases} \tag{7}$$

from which we can derive the expected value of $v_n^{(1)}$, denoted as $\mathbb{E}v_n^{(1)}$. The result is shown in (10) in the Appendix. From the result on order statistics (Krishna, 2002, Appendix C) that

$$\mathbb{E}v_n^{(2)} = n\mathbb{E}v_{n-1}^{(1)} - (n-1)\,\mathbb{E}v_n^{(1)}$$

we get the expression of $\mathbb{E}v_n^{(2)}$ as below

$$\mathbb{E}v_n^{(2)} = \left(V - \frac{\tau}{4J}\right) - \frac{3\tau J}{2\,(n+1)\,(n+2)} + \frac{\tau\,(3J+n-1)}{2\,(n+1)\,(n+2)}\left(1 - \frac{1}{J}\right)^{n+1} \tag{8}$$

Furthermore, when the seller reveals no information ($J = 0$), a bidder's expected valuation of the product is, taking his ideal attribute as the origin

$$v^0 = V - 2\tau \int_0^{\frac{1}{2}} |s - 0|\, ds = V - \frac{\tau}{4}$$

which is the same for all the bidders. In this case, as in a second-price auction, the expected auction revenue is just $v^0$.

In summary, the expected auction revenue under an equal partition is

$$\mathbb{E}R\,(J, n) = \begin{cases} V - \frac{\tau}{4} & \text{if } J = 0 \\ V - \frac{\tau}{4J} - \frac{3\tau J}{2(n+1)(n+2)} + \frac{\tau(3J+n-1)}{2(n+1)(n+2)} \left(1 - \frac{1}{J}\right)^{n+1} & \text{if } J \geq 1 \end{cases} \tag{9}$$

We then get the following result of the seller's optimal disclosure policy.

**Proposition 3** *The optimal disclosure policy is extreme, that the seller will either reveal full or no information, depending on the number of the bidders. Specifically,*

*1) If $n = 2$, the seller reveals no information ($J = 0$);*

*2) If $n = 3$, she is indifferent between revealing full information or no information;*

*3) If $n \geq 4$, she reveals full information ($J = \infty$).*

Figure 3 below shows the value of $\mathbb{E}R$ as a function of $J$, corresponding to different number of bidders. The dashed line represents the expected revenue under no disclosure ($J = 0$). When $J \geq 1$ and the seller reveals informative information, it is clear that the expected revenues increase in $J$. Therefore, the more precise the signals, the higher expected revenue of the auction. When $n = 3$, the expected revenue under full disclosure is equal to that under no information disclosure, and therefore the seller is indifferent between revealing full or no information. However, this result of the critical number of $n = 3$ is sensitive to the specific form of the matching function[1]

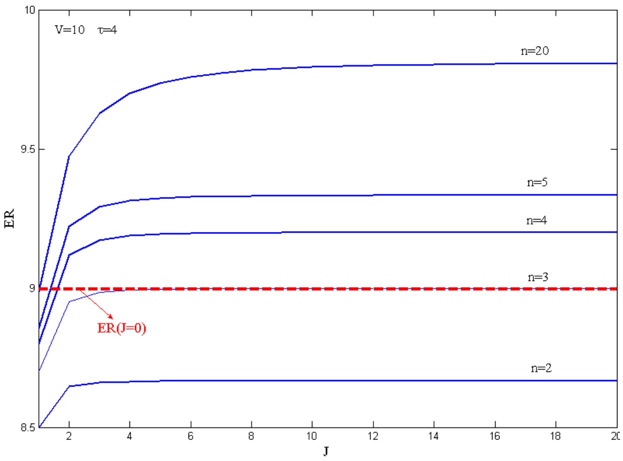


Figure 3: Expected Auction Revenues under Disclosure

[1]In this paper, bidder's payoff function is in the form of absolute value, as in (1). If it is quadratic, Jewitt and Li (2012) show that, when $n = 3$, the seller strictly prefers to reveal full information.

## 5 Conclusion

In this paper, we study information disclosure in auctions where bidders' preferences are horizontally differentiated. The information structure is characterized by partitions of state space, and the matching function in bidder's payoff functions is in the form of absolute value. With this setting, we provides a close-form result on how information disclosure can change the distributions of bidders' posterior valuations. Specifically, correspondingly to more precise information, those distributions are ordered in terms of First Order Stochastic Dominance in our model.

Secondly, we also show that the optimal disclosure policy is extreme in the sense that the seller will either reveal full or no information to the bidders, which depends on the number of bidders in the auction. The extreme disclosure policy result is similar to those in the literature. However, unlike those reduce-form models, we develop a model of endogenous valuations here and provide further qualification on that result.

## A Appendix: Omitted Proofs and Calculations

**Proof of Proposition 2**: If for any $J' \geq 1$, we show that $F(v;J'+1) \leq F(v;J')$ for $v \in [\underline{v}, V]$, then (6) is true for any $J > J'$. Let $J = J'+1$, and it is obvious that $\hat{v}(J) > \hat{v}(J')$ and $\bar{v}(J) > \bar{v}(J')$, as both $\hat{v}(\cdot)$ and $\bar{v}(\cdot)$ are strictly increasing in $J$. Furthermore, as $\bar{v}(J') - \hat{v}(J) = \frac{\tau}{4JJ'}(J'-1) \geq 0$, we then have

$$\hat{v}(J') < \hat{v}(J) \leq \bar{v}(J') < \bar{v}(J)$$

The interval of $[\underline{v}, \bar{v}(J)]$ is therefore divided into four subintervals, and we will show on each subinterval the above result is true. We define a new function $\phi(v) = \frac{1}{2}[F(v;J') - F(v;J)]$. (i) For $v \in [\underline{v}, \hat{v}(J'))$, $F(v;J) = F(v;J')$ from (5). (ii) For $v \in [\hat{v}(J'), \hat{v}(J))$, we have

$$\phi(v) = \frac{V-v}{\tau} - \sqrt{\frac{V-v}{\tau J'} - \frac{1}{4J'^2}}$$

In addition, $\phi'(v) = -\frac{1}{\tau} + \frac{1}{2\tau J'}\left(\frac{V-v}{\tau J'} - \frac{1}{4J'^2}\right)^{-\frac{1}{2}}$ and $\phi''(v) = \frac{1}{4(\tau J')^2}\left(\frac{V-v}{\tau J'} - \frac{1}{4J'^2}\right)^{-\frac{3}{2}} > 0$, where $\phi'(\hat{v}(J')) = 0$. So for $v \in (\hat{v}(J'), \hat{v}(J))$, $\phi'(v) > 0$ and $\phi(v)$ is strictly increasing. Given that $\phi(\hat{v}(J')) = 0$, we have$\phi(v) > 0$ for $v \in (\hat{v}(J'), \hat{v}(J))$. (iii) For $v \in [\hat{v}(J), \bar{v}(J'))$,

$$\phi(v) = \sqrt{\frac{V-v}{\tau J} - \frac{1}{4J^2}} - \sqrt{\frac{V-v}{\tau J'} - \frac{1}{4(J')^2}}$$

Let $\varphi(J) = \sqrt{\frac{V-v}{\tau J} - \frac{1}{4J^2}}$, and $\varphi'(J) = \frac{1}{2J^2}\left(\frac{V-v}{\tau J} - \frac{1}{4J^2}\right)^{-\frac{1}{2}}\left(\frac{V-v}{\tau} - \frac{1}{8J}\right) > 0$ for $v \in [\hat{v}(J), \bar{v}(J'))$. Given that $\phi(\hat{v}(J)) > 0$, then $\phi(v) > 0$ for $v \in [\hat{v}(J), \bar{v}(J'))$. (iv) For $v \in [\bar{v}(J'), \bar{v}(J)]$, $F(v;J') = 1 \geq F(v;J)$.

**Calculation of $Ev_n^{(1)}$ and $Ev_n^{(2)}$**: Given the distribution function of (7),

$$\begin{aligned}\mathbb{E}v_n^{(1)} &= \int_{\underline{v}}^{\hat{v}} v dH_n^{(1)}(v;J) + \int_{\hat{v}}^{\bar{v}} v dH_n^{(1)}(v;J) \\ &= vH_n^{(1)}(v;J)\Big|_{\underline{v}}^{\bar{v}} - \int_{\underline{v}}^{\hat{v}} H_n^{(1)}(v;J)\,dv - \int_{\hat{v}}^{\bar{v}} H_n^{(1)}(v;J)\,dv\end{aligned}$$

where

$$\begin{aligned}&\text{i) } \textstyle\int_{\underline{v}}^{\hat{v}} H_n^{(1)}(v;J)\,dv &&= \textstyle\int_{\underline{v}}^{\hat{v}} \left(1-\frac{2V}{\tau}+\frac{2v}{\tau}\right)^n dv = \frac{\tau}{2}\frac{1}{n+1}\left(1-\frac{1}{J}\right)^{n+1} \\ &\text{ii) } \textstyle\int_{\hat{v}}^{\bar{v}} H_n^{(1)}(v;J)\,dv &&= \textstyle\int_{\hat{v}}^{\bar{v}} \left(1-2\sqrt{\frac{V-v}{\tau J}-\frac{1}{4J^2}}\right)^n dv \\ & &&= \textstyle\frac{\tau J}{2}\left[\frac{1}{(n+1)(n+2)} - \frac{\left(1-\frac{1}{J}\right)^{n+1}}{n+1} + \frac{\left(1-\frac{1}{J}\right)^{n+2}}{n+2}\right]\end{aligned}$$

then we get the expression of $\mathbb{E}v_n^{(1)}$ as below

$$\mathbb{E}v_n^{(1)} = \bar{v} - \frac{\tau J}{2(n+1)(n+2)} + \frac{\tau(J-1)}{2(n+1)}\left(1-\frac{1}{J}\right)^{n+1} - \frac{\tau J}{2(n+2)}\left(1-\frac{1}{J}\right)^{n+2} \qquad (10)$$

We have the result that $\mathbb{E}v_n^{(2)} = n\mathbb{E}v_{n-1}^{(1)} - (n-1)\mathbb{E}v_n^{(1)}$ (Krishna, 2002, Appendix C), from which we get the expression of $\mathbb{E}v_n^{(2)}$ as in (8).

**Proof of Proposition 3**: For $J \geq 1$ and $n \geq 2$,

$$\frac{d\mathbb{E}R(J)}{\tau dJ} = \frac{1}{4J^2} + \frac{3}{2(n+1)(n+2)}\left[\left(1-\frac{1}{J}\right)^n\left(1+\frac{n}{J}+\frac{n^2-1}{3J^2}\right)-1\right] > 0$$

Therefore, if the seller decides to reveal information, she will fully reveal it ($J = \infty$). Then the expected revenue under full revelation is

$$\begin{aligned}&\lim_{J\longrightarrow\infty} \mathbb{E}R(J,n) \\ =\ & \lim_{J\longrightarrow\infty}\left\{V - \frac{\tau}{4J} - \frac{3\tau J}{2(n+1)(n+2)} + \frac{\tau(3J+n-1)}{2(n+1)(n+2)}\left(1-\frac{1}{J}\right)^{n+1}\right\} \\ =\ & V + \lim_{J\longrightarrow\infty}\left\{-\frac{3\tau}{2(n+1)(n+2)}\left[\sum_{i=1}^{n+1}\left(1-\frac{1}{J}\right)^{n+1-i}\right] + \frac{\tau(n-1)}{2(n+1)(n+2)}\left(1-\frac{1}{J}\right)^{n+1}\right\} \\ =\ & V - \frac{2n+4}{2(n+1)(n+2)}\tau\end{aligned}$$

Therefore, when $n = 2$, $\lim_{J\longrightarrow\infty}\mathbb{E}R(J,2) = V - \frac{\tau}{3} < v^0$, and the seller prefers no information disclosure; when $n = 3$, $\lim_{J\longrightarrow\infty}\mathbb{E}R(J,2) = V - \frac{\tau}{4} = v^0$, and the seller is indifferent between revealing full or no information; when $n \geq 4$, $\lim_{J\longrightarrow\infty}\mathbb{E}R(J,2) > v^0$, and it's optimal for the seller to reveal full information.

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