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Bias-Trigger Manipulation and Task-Form Understanding in Monty Hall

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Abstract

Monty Hall is a difficult task which triggers multiple biases. With sophisticated subjects and treatments that reverse and eliminate these triggers, non-rational choice is greatly reduced. Among task-familiar subjects, non-rational choice can can fall to background-error levels. But as our data also show, task-form recognition is necessary but not sufficient for rational choice when the task calls for conditional probability reasoning rather than simple rule-based behavior, as in e.g. 'Switch in Monty Hall.' Task-form understanding, a more stringent requirement, proves to be necessary and sufficient for rational choice in generalized Monty Hall conditional probability reasoning tasks.

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1 Introduction

What explains the robust disposition toward 'sticking' in the Monty Hall task?[1] A variety of plausible explanations have been posited: that humans are poor at probability reasoning in general and conditional probability reasoning in particular; that sticking is a manifestation of a default effect which defends against perceived strategic manipulation, or is triggered by the absence of clear decision-making cues; that by construction, the task induces an illusion of control effect; and that sticking reflects the preference for errors of omission over errors of commission.

Several extensions of the Monty Hall task are effective in reducing non-rational responses: repetition with feedback (Friedman, 1998), social learning (Palacios-Huerta, 2003), markets containing sophisticated traders (Kluger and Wyatt, 2004), institutions (competition and communication) (Slembeck and Tyran, 2004), learning through practice in valuing an engineered financial asset (Kluger and Friedman, 2010), and explicit guidance of intuition (Krauss and Wang, 2003). However, to date the literature does not include an experiment in which Monty Hall's bias triggers are tackled at source, resolving the problem of non-rational responses while preserving the one-shot nature of the task.

The experiment reported here is designed to fill this gap. It consists of a baseline treatment (T0), a treatment that reverses the directionality of the three above-mentioned task-triggered biases (T1), a treatment that eliminates the triggers for these biases (T2), and a treatment that reveals the error rate in subjects' unconditional probability reasoning responses (T3). Subjects are sampled from a pool of numerate students, who by self-selection and training have experience with probability reasoning problems and their solution. Accordingly, these data speak to the effects of (i) subject sophistication and (ii) task-embedded bias triggers.

In the standard one-shot Monty Hall task, rational response rates of 10%–13% are typical (Friedman, 1998; Krauss and Wang, 2003). In contrast, 79% of our full sample of sophisticated subjects ($N = 71$) choose rationally in the baseline treatment T0. And among the $n = 49$ subjects who recognize the Monty Hall task, 96% choose rationally in the baseline treatment T0. That is only 4% – less than the background error rate of 6% – shy of 100%. These data suggests that *task-form recognition* is even more discriminating in decision-theoretic experiments than *game-form recognition* is in game-theoretic experiments, where it serves as a powerful marker of immunity to behavioral anomalies (Chou et al., 2009; Ferraro and Vossler, 2010).

Treatment T1 shows, however, that task-form recognition is necessary but not sufficient for task-form understanding. It is the latter that is methodologically fundamental, and roughly corresponds to *teleological knowledge* as distinct from *procedural knowledge* (VanLehn, 1990). In treatment T1, the proportion of rational responses among subjects who recognize the Monty Hall task drops to 57%. By design, T1 requires understanding of conditional probability reasoning. The easily learned rule, 'It is rational to switch in Monty Hall', is not effective in treatment T1. The $96\% - 57\% = 39\%$ difference represents the proportion of subjects who (a) recognize the Monty Hall task form and choose correctly in T0, but (b) fail to apply conditional probability calculus successfully in T1. This 39% is a lower bound insofar as the bias triggered by T1 is in favor of the rational response. Inferences based on task-form recognition alone may be grossly misleading. In contrast a sufficient condition is *task-form understanding*, which excludes recognition-

[1]Readers unfamiliar with the Monty Hall task may refer to *Baseline treatment T0* on page 2.

triggered behavioral rules (procedural knowledge) similar in spirit to those that may be instilled through Skinnerian operant conditioning.

The treatments implemented in this experiment manipulate the triggers of three effects, all of which bias responses toward irrational sticking in the standard Monty Hall task: (i) the default effect (Margolis, 2007; Samuelson and Zeckhauser, 1988) (ii) the illusion of control effect (Granberg and Dorr, 1998) and (iii) the errors of omission vs. errors of commission effect (Gilovich, Medvec and Chen, 1995). In the baseline treatment T0, 41% of the $n = 22$ subjects who are unfamiliar with the Monty Hall task (MH-unfamiliar subjects) choose rationally. This is more than triple the 10%–13% rational response rate among non-sophisticated subjects. In treatment T1 – where biases reinforce the rational response option – 73% of MH-unfamiliar subjects choose rationally. This difference, which is in the direction predicted by the bias-trigger hypothesis, is statistically significant. In treatment T2 – where bias triggers are neutralized – the rational response rate is 55% among MH-unfamiliar subjects, which as predicted lies in-between the downward-biased 41% of T0 and the upward-biased 73% of T1.

Overall, the bias-trigger manipulations are successful. The fraction choosing rationally in the bias-trigger-neutralized treatment T2 – 66% overall (71% and 55% among MH-familiar and MH-unfamiliar subjects respectively) – surpasses that achieved in 'one shot' elsewhere. However neither approaches 100%, even adjusted for background error rates. Task-form understanding is less than ubiquitous even among sophisticated subjects who have received training in probability and statistics.

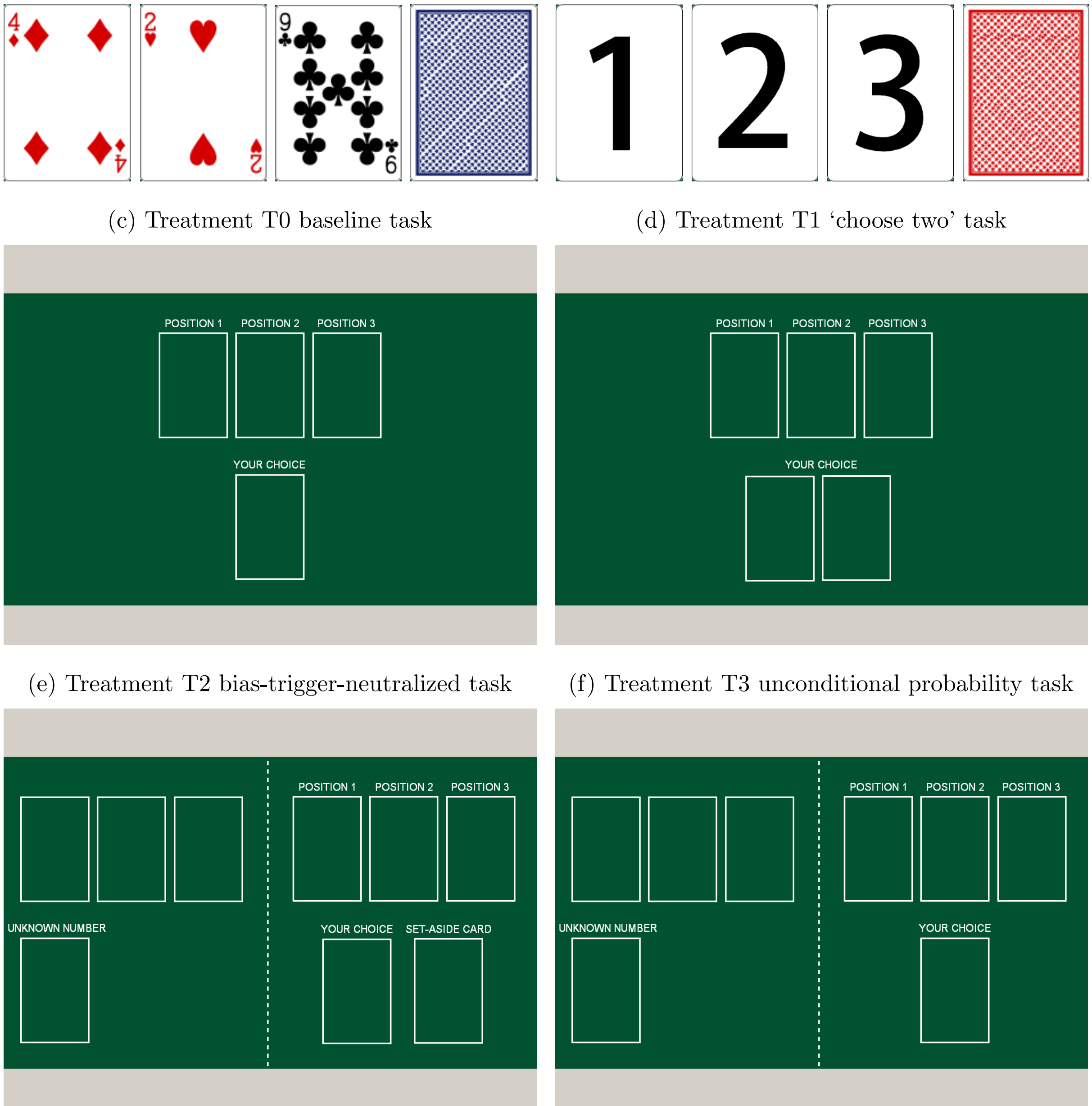
2 Design and procedures

This experiment combines three key elements: bias-trigger manipulation, direct elicitation of task-form recognition, and a numerate sophisticated subject pool. The experiment, including the Random Lottery Incentive (RLI) scheme, is implemented in z-Tree (Fischbacher, 2007). To enhance comparability with previous literature, Friedman's (1998) playing card presentation format is employed.

Baseline treatment T0. The subject is shown three playing cards, two of which are red and one of which is black as in Exhibit (1a). The subject stands to win €10 if she is left holding the black card at the end of the following procedure. The cards are shuffled and placed face down on a playing mat at positions 1–3 pictured in Exhibit (1c). The subject is asked to choose one of the three face-down playing cards. This card, still face down, is moved to the location marked YOUR CHOICE. From among the two cards remaining at their original locations, one is revealed to be a red, non-winning card. Now, just before revelation of whether the subject has won €10 or not, the subject is offered an opportunity to switch from the card at YOUR CHOICE to the remaining face-down card. The probability of obtaining the winning black card upon *switching* is 2/3.

'Choose two' treatment T1. Proceeds as in T0 except (i) that the playing mat is as pictured in Exhibit (1d), and (ii) that the subject is asked to choose two cards. These are moved to the two positions marked YOUR CHOICE. One of these two cards is revealed to be a non-winning red card. Now, just before revelation of whether the subject has won €10 or not, the subject is offered an opportunity to switch from her remaining face-down card at YOUR CHOICE to the non-chosen card. The probability of obtaining the winning black card upon *sticking* is 2/3.

Exhibit 1: Virtual cards and mats (not to scale).



(a) Playing cards used in all four treatments T0–T3 (b) Number cards used in treatments T2 and T3

(c) Treatment T0 baseline task (d) Treatment T1 'choose two' task

(e) Treatment T2 bias-trigger-neutralized task (f) Treatment T3 unconditional probability task

Bias-trigger-neutralized treatment T2. In this treatment, playing cards in Exhibit (1a) are shuffled and placed face-down at positions 1–3 on the right-hand-side of the playing mat pictured in Exhibit (1e). In addition, the number cards in Exhibit (1b) are shuffled and placed face-down at the three unmarked positions on the left-hand-side of this playing mat. Then one of these cards is allocated to the subject, and it is moved to the location marked UNKNOWN NUMBER. The value of this unknown number entitles the subject to the playing card at the correspondingly numbered position on the right-hand side of the playing mat. Now the playing card at POSITION 3 is identified as the set-aside card, and moved to the SET-ASIDE CARD location on the mat. Among the remaining playing cards at positions 1–2, a non-winning red card is revealed. Now, just before revelation of whether the subject has won €10 or not, the subject is offered the choice between (i) keeping the unknown position entitlement (possibly to the known non-winning red card at position 1 or 2), (ii) switching to the remaining face-down card at position 1 or 2, or (iii) switching to the set-aside card. The probability of obtaining the winning black card upon *switching* to the face-down POSITION 1 or 2 card is 2/3.

Unconditional probability treatment T3. Proceeds as in T2 except that the playing mat is as pictured in Exhibit (1f) and there is no set-aside card. A single playing card is revealed from among positions 1–3 to be a non-winning red card. Now, just before revelation of whether the subject has won €10 or not, the subject is offered the choice between (i) keeping the unknown position entitlement (possibly to the known non-winning red card at position 1, 2 or 3), or (ii) switching to one of the two face-down cards remaining at positions 1–3. The probability of obtaining the winning black card upon *switching* to a face-down POSITION 1 or 2 or 3 card is 1/2.

Subjects. 71 subjects from the Karlsruhe Institute of Technology (KIT) ORSEE pool were recruited in June/July of 2011. All subjects were enrolled in a KIT degree program requiring quantitative skills, and all reported having completed at least one course in mathematics or statistics. Less than 6% do not disclose their area of study. Among those who do, the distribution is as follows: 72% engineering or engineering consortial programs, 10% physics, 9% informatics or information technology, 6% mathematics or mathematics consortial programs, 3% geology or geology consortial programs. The average age is 22, and 19% are female.

Procedures. All 71 subjects faced four treatments in one of four randomly assigned sequences.[2] On average there were 9 subjects in each of eight 50-minute experimental sessions held in the Institute of Economic Theory and Statistics laboratory (KIT). Upon being seated at their stations, subjects were apprised of the rules of conduct and of the two-part structure of the session. The first part consisted of four choice scenarios (T0–T3). At the beginning of each scenario, subjects were directed to consult detailed scenario-specific written instructions, which set out the scenario procedure as clearly as possible. The second, non-incentivized part of the session, comprised of (i) a questionnaire eliciting the subject's probability beliefs across the four scenarios, and (ii) a screening question to determine whether the subject had recognized any of the four scenarios.

Each subject received a show-up fee of €5 plus a possible additional €10, depending on the playing-out of a randomly selected scenario from among T0–T3. Earnings were paid out in cash at the end of the session.

3 Results

The choice data from Treatments T0–T3 are presented in Exhibit (2a) and illustrated in Exhibit (2c). Associated hypothesis tests are presented in Exhibit (2b). The one-sided p-value reported for the first-row hypothesis $H_0\colon \hat{\theta}_{\text{T0F}} = \hat{\theta}_{\text{T0U}}$ is obtained from Boschloo's unconditional test of homogeneity. The remaining one-sided p-values are from exact McNemar tests for repeated measures. The final column reports Benjamini-Hochberg adjusted p^{BH}-values that control the false discovery rate in multiple hypothesis tests. Elicited probability beliefs are summarized in Exhibit (2d).

The overall proportion of rational responses in the benchmark task T0 (79%) is unusually high (see Exhibit (2a)). We ascribe this to the sophisticated subject pool's (i)

[2] T0,T1,T2,T3; T1,T2,T3,T0; T2,T3,T1,T0; T3,T2,T1,T0

Exhibit 2: Monty Hall choice data, hypothesis tests, and probability beliefs.

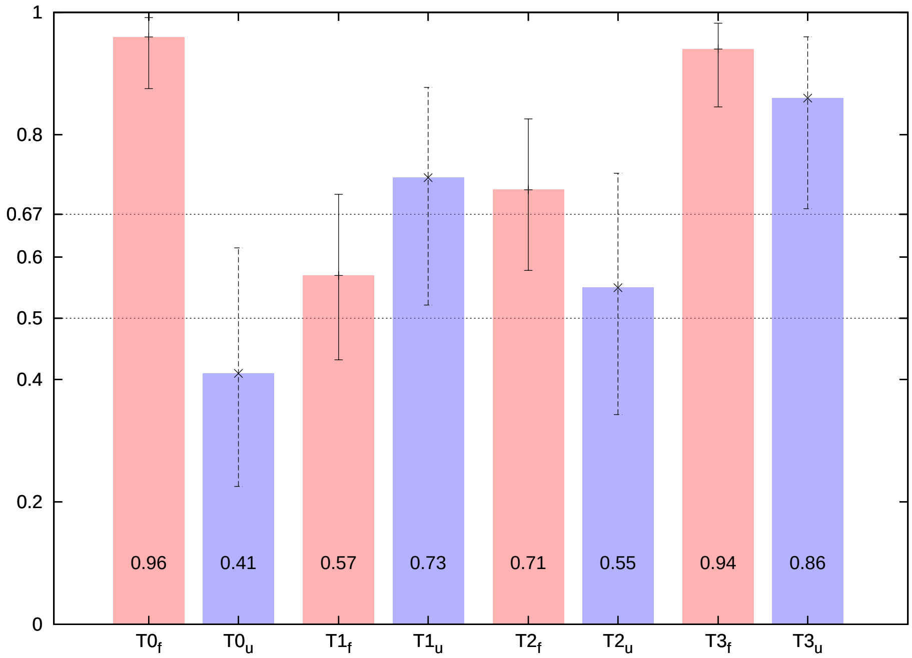
(a) Proportion choosing rationally, partitioned by familiarity with Monty Hall.

	Familiar	**Unfamiliar**	**All**
	$n = 49$	$n = 22$	$N = 71$
T0	96%	41%	79%
T1	57%	73%	62%
T2	71%	55%	66%
T3	94%	86%	92%

(b) Null hypotheses, one-sided p-values, and Benjamini-Hochberg adjusted p^{BH}-values.

H_0	p-value	p^{BH}-value
$\hat{\theta}_{\text{T0F}} = \hat{\theta}_{\text{T0U}}$	0.000	0.000
$\hat{\theta}_{\text{T0F}} = \hat{\theta}_{\text{T1F}}$	0.000	0.000
$\hat{\theta}_{\text{T1F}} = \hat{\theta}_{\text{T2F}}$	0.059	0.059
$\hat{\theta}_{\text{T0F}} = \hat{\theta}_{\text{T2F}}$	0.002	0.003
$\hat{\theta}_{\text{T0U}} = \hat{\theta}_{\text{T1U}}$	0.046	0.058

(c) Proportion choosing rationally among MH-familiar ($n = 49$, red) and MH-unfamiliar ($n = 22$, blue) subjects; Jeffreys method 95% CI error bars.



(d) Probability beliefs, separately for MH-Unfamiliar and MH-Familiar subjects: (i) correct probability beliefs, (ii) relative probability beliefs consistent with the rational choice, (iii) conditional probability calculation failure, (iv) number of observations.

		T0		**T1**		**T2**		**T3**	
		F	U	F	U	F	U	F	U
(i)	Correct	65%	15%	44%	14%	41%	11%	73%	60%
(ii)	Consistent	94%	30%	50%	19%	64%	11%	100%	90%
(iii)	50%–50%	2%	55%	6%	52%	21%	56%	na	na
(iv)	n	48	20	48	21	44	18	45	20

training in probability calculus and (ii) the 49/71 preponderance (69%) of subjects who are Familiar with the Monty Hall task. The rational response rate among MH-Familiar is 96%, while among the MH-Unfamiliar it is still 41%. Boschloo's unconditional test of homogeneity rejects the hypothesis that these proportions are equal (row 1, Exhibit (2b)).

The remaining rows in Exhibit (2b) report exact McNemar tests for repeated measures, comparing rational response rates within the MH-Familiar (MH-Unfamiliar) group across different treatments. Among the MH-Familiar, the 96% in T0 is statistically significantly different from the 57% in T1 and the 71% in T2. Among the MH-Unfamiliar, the 41% in T0 is statistically significantly different from the 73% in T1, but stops being so following the Benjamini-Hochberg adjustment that controls the false discovery rate across the multiple comparisons in Exhibit (2b). Since T2 is the treatment that *a priori* neutralizes the bias triggers present in T0 and T1, it is predicted that the T2 rational response rate falls in-between those of T0 and T1. Indeed this turns out to be the case, and it places bounds on the maximal effect size in comparisons involving T2, e.g. $|\hat{\theta}_{\mathrm{T0U}} - \hat{\theta}_{\mathrm{T2U}}| \leq \hat{\theta}_{\mathrm{T1U}} - \hat{\theta}_{\mathrm{T0U}}$ and $|\hat{\theta}_{\mathrm{T1U}} - \hat{\theta}_{\mathrm{T2U}}| \leq \hat{\theta}_{\mathrm{T1U}} - \hat{\theta}_{\mathrm{T0U}}$. This bound on effect size, combined with the MH-Unfamiliar sub-sample size ($n = 22$), restricts the power of McNemar tests on the MH-Unfamiliar.

Exhibit 3: GEE model parameter estimates

	Pooled (N=71)		Familiar (n=49)		Unfamiliar (n=22)	
	e^{β}	p-value	e^{β}	p-value	e^{β}	p-value
T0	5.540	0.000	23.50	0.000	0.692	0.396
T1	2.311	0.008	1.333	0.319	2.667	0.040
T2	2.764	0.001	2.500	0.004	1.200	0.670
Unfamiliar	0.360	0.001				
Familiar	1	na				

In order to explicitly incorporate and account for the effect of within-subject correlation, we estimate Generalized Estimating Equation (GEE) models, specifying the link function to be logit, the working correlation matrix to be unstructured, and the covariance matrix estimator to be robust. The first GEE model is estimated on the pooled (N=71) data, with T0–T3 and (Un-)Familiar entered as main effects without an intercept term. The second and third GEE models are estimated on the MH-Familiar (n=49) and MH-Unfamiliar (n=22) subsets separately. Table 3 reports the exponentiated parameter estimates, i.e. the population-average odds, for each main-effect category, as well as the associated two-sided Wald Chi-square statistic p-values. In the pooled model, all main effects are significant: treatments T0, T1 and T2, as well as the familiarity factor. Population-average probabilities may be recovered from the estimated odds (e^{β}) as $e^{\beta}/(1+e^{\beta})$, where the odds themselves are multiplicative between treatment categories and (Un-)Familiar. Notice that with the GEE population-average approach, the model-implied probabilities match the raw data proportions when the GEE model is estimated on the relevant restricted subset, e.g. $23.5/24.5 = 0.96$ for T0 in the second (MH-Familiar) model.

The probability belief data presented in Exhibit (2d) complements the choice data examined above. Row (i) shows that 41%–44% of MH-Familiar subjects form correct conditional probability beliefs in T1 and T2. In other words, the probability data confirms that a fraction of the MH-Familiar subjects are not going through the process of calculating conditional probability. Row (i) also shows that only 11%–15% of MH-Unfamiliar subjects correctly form conditional probability beliefs in the MH tasks. This is comparable to the 10%–13% rational response rates in standard one-shot MH tasks (Friedman, 1998; Krauss and Wang, 2003). Most MH-Unfamiliar subjects are poor conditional probability reasoners, despite being drawn from a 'sophisticated' pool and having had training in probability calculus. This is also borne out in row (iii), which shows that 52%–56% of MH-Unfamiliar subjects' probability beliefs – i.e. their modal beliefs – are of the 'equally likely' type across tasks T0–T2. Also evident in row (iii) is the relative paucity of MH-Familiar subjects displaying 'equally likely' type beliefs across T0–T2, although the format of T2 is ostensibly more difficult for these subjects than T0–T1. Finally, row (ii) reports the fraction of subjects whose probability beliefs – whether correct or erroneous – are consistent with the rational response. Among the MH-Familiar, the row (ii) rational-response-consistent beliefs (94%,50%,64%) closely match the Exhibit (2a) rational choice proportions (96%,57%,71%). However among the MH-Unfamiliar there is not this close correspondence: the vector of differences between rational-response-consistent beliefs and rational choice proportions is (11%, 54%, 44%). According to McNemar's exact test, the proportions are statistically significantly different on tasks T1 (p=0.002) and T2 (p=0.004). It is evident that among the MH-Unfamiliar, probability beliefs do not account for all of the variation in choice across MH tasks.

4 Conclusion

This experiment offers fresh perspective on the MH Paradox and its robustness. Its resilience is not attributable to a single factor, but to the co-incidence of (i) widespread prevalence of (erroneous) 50%–50% probability beliefs, in which the perceived probability of success is equally balanced between sticking and switching in the MH task, and (ii) multiple, mutually reinforcing biases triggered by the standard MH task. A decision-making bias will be particularly effective in swaying the choice of an individual whose probability beliefs leave her balanced on a knife edge between sticking and switching. And the structure of the MH task triggers three, mutually reinforcing biases: the default effect, the illusion of control effect, and the preference for errors omission over errors of commission. Repetition with feedback, especially in combination with social or market learning, eventually changes probability beliefs and thereby increases the rational response rate. But without changing the structure of the MH task, this learning process is hindered by the multiple, mutually reinforcing biases triggered by the standard MH task. By subtly altering the structure of the MH task, one may address the task-triggered biases, but this leaves the underlying erroneous probability beliefs unchanged. Hence conditional probability reasoning skills as well as bias triggers must be addressed in concert.

Heterogeneity in probability reasoning sophistication masks the effects of bias-trigger manipulation in raw sample averages. These data show that there are four subclasses: those who are poor conditional probability reasoners; those who do not recognize the MH task, but correctly calculate conditional probabilities (a small minority); those who recognize the MH task and have procedural knowledge that they associate with this recognition (i.e. 'Switch in Monty Hall'); and those who not only recognize the MH task, but have understanding – i.e. teleological knowledge – of the conditional probability reasoning underpinning rationality in the MH task. Among subjects who are familiar with the MH task, the T0–T2 data show that it is necessary to distinguish between task-form recognition (procedural knowledge) and task-form understanding (teleological knowledge). The importance of the former has been established in game-theoretic experimental economics

(Chou et al., 2009; Ferraro and Vossler, 2010). Yet the present experiment shows that task-form recognition is necessary but not sufficient for rational choice in MH-type conditional probability reasoning tasks. Instead, task-form understanding – a concept that is new to the field of experimental economics, but well known in the field of mathematics education as 'teleological knowledge' (VanLehn, 1990) – constitutes the necessary and sufficient form knowledge. Finally, among subjects who are not familiar with the MH task, the choice data confirm that treatment T0 (the standard MH task) triggers bias against the rational response option, treatment T1 triggers bias in favor of the rational response option, and treatment T2 neutralizes the bias triggers.

Overall, the data vindicate the paper's objective of tackling the MH task's bias triggers at source, preserving its one-shot nature. Rational choice may be enhanced or suppressed through bias-trigger manipulation. Indeed the MH task may also be modified to eliminate the bias triggers. Probability beliefs – whether reflecting failure of conditional probability reasoning, consistency with rational choice due to task-form recognition, or consistency with rational choice due to task-form understanding – become reflected in rational-choice proportions differently depending on the bias-trigger manipulation embodied in the particular MH task under investigation.

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