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Impact of financial constraint on incentive compensation and product market behavior

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Abstract

This paper introduces financing constraint in a model of incentive compensation and product market and develops key insights about the interactions of product market behavior, financial constraint and incentive compensation. A financially constrained firm faces higher cost of capital which results in lower output. The model suggests that a financially constrained firm offers higher incentive compensation to its manager if the degree of product differentiation is sufficiently low. This higher incentive encourages the manager to put more effort and produce more output in order to compensate for the loss in output due to financial constraint. Our paper generates the testable hypothesis that a financially constrained firm will offer higher incentive compensation to its manager which has not yet been tested empirically.

1. Introduction

Recent papers have reported how incentive contracts may affect product market behavior. Surprisingly, little work has been done to address how financing constraint may affect incentive contracts and product market behavior. We present a model to better our understanding of interactions of financial constraint, incentive compensation and product market behavior.

There is a theoretical literature on the topic of incentive contracts and competition (e.g. Schmidt (1997), Sklivas 1987, Fershtman and Judd 1987, Reitman 1993, Spagnolo 2000, Aubert 2009).

The main takeaway from these studies is that incentives have ambiguous effects on product market competition. On the one hand, in one-shot games with linear contracts, incentive pay seems to encourage non cooperative strategies that help increase sales at the expense of other firms. On the other hand, when contracts are nonlinear, as is the case with stock options, or in repeated games, incentive pay may actually enforce collusive behavior.

Our model introduces financial constraint in a framework of incentive compensation and product market behavior. It consists of a Cournot duopoly, with one firm more financially constrained than the other firm. The firm with higher cost of external financing is considered as more financially constrained than the other. Both the firms engage in a Cournot duopoly in order to maximize their respective firm values. In equilibrium, firm values depend on the output produced, which depends on the managerial efforts. A financially constrained firm faces higher cost of capital which reduces its output. The firms offer incentive compensations to the managers to encourage them to put more effort, which increases the equilibrium outputs and the firm values. Our model suggests that when the degree of product differentiation is sufficiently low, the financially constrained firm offers higher incentives to its manager which encourages her to put more effort and increase output and compensate for the decrease in production due to higher cost of capital.

This paper contributes to the literature of incentive compensation and product market behavior by introducing financial constraint and showing that incentive compensation will be higher for the manager of a financially constrained firm. This is the first paper in our knowledge which develops theoretical reasoning as to why a financially constrained firm should offer higher incentive compensation to its manager. Our paper generates the testable hypothesis that a financially constrained firm will offer higher incentive compensation to its manager which has not yet been tested empirically. Given the scarcity of research in addressing the relationship between financing constraint and incentive compensation, we believe that this area needs to be explored even further, both theoretically and empirically.

2. Theoretical Model

Without loss of generality, we assume that firm 2 is more financially constrained than firm 1.

2.1 Definition of Financial Constraint

A firm with higher cost of capital is more financially constrained. The cost of capital of firm 1(2) is $r(r+d)$, where d is the extra cost of capital the financially constrained firm faces. Higher is the degree of financial constraint, the greater is the value of d .¹

2.2 The Three Stage Game

The compensation contract offered by the equity holders of a firm to its manager is given by

$$w_i = \alpha_i + \beta_i V_i^{1/3}, i = 1, 2 \quad (1)$$

¹ In this model, we consider debt financing as the only source of external financing. Firm 1 pays a rate of return r on its debt, firm 2 pays a rate of return of $r+d$ on its debt obligation.

α_i and β_i are the compensation contract parameters which are exogenous to the manager's decision making process. V_i is the equity value of the firm.

2.2.1 The Set-Up

The two firms engage in a Cournot duopoly game to maximize their values. In the first stage, the equity holders of a firm chose the managerial compensation parameters in order to maximize the net equity value of the firm. In the second stage, the manager of the firm chooses her effort to maximize her utility. In the third stage, the manager of the firm engages in a Cournot duopoly with the other firm to maximize the equity value of her firm. Effort is unobservable to the equity holders. The equity holders of a firm design a compensation contract to ensure that the interest of the manager is aligned with that of the equity holders. Managerial compensation is composed of two parts. The first component is α_i , the fixed component of managerial compensation. The second component $\beta_i V_i^{1/3}$, the variable component of the compensation structure, is the incentive compensation of the manager. Several theoretical and empirical papers have reported that managerial compensation depends positively on the equity value of the firm. For example, Bebchuk and Grinstein (2005) report how equity based compensation has experienced substantial growth from 1993 to 2003. Smith and Stultz (1985) develops a model to show that equity based compensation incentivize risk averse CEOs to invest in risky projects. We draw upon the inferences of these papers to assume that managerial compensation is positively dependent on the equity value of the firm. The exact functional form of the dependence of managerial compensation on equity value of the firm is not standardized.² The parameter β_i is called the incentive parameter of the manager of firm i . Higher is the incentive parameter, greater is the incentive compensation of the manager.

In the second stage, the manager of firm i maximizes her utility by choosing her effort, denoted by e_i . Managerial utility is given by

$$\max_{e_i} U_i = w_i - \frac{e_i^2}{2}, i = 1, 2 \quad (2)$$

As a manager's compensation depends on the equity value of the firm, the manager has an incentive to maximize the equity value of the firm by putting more effort. But putting more effort is a disutility for the manager, given by the second term in the utility function³. There is an inverse market demand of the affine-linear form

$$p_i = \theta + e_i + z - q_i - \lambda q_j \quad (3)$$

where λ is the degree of product differentiation, $\theta - c$ is a positive constant and z is a random parameter, which represents the state of the nature. Higher is the value of λ , lower is the degree of product differentiation. We assume that z is uniformly distributed on a non-degenerate interval $[\underline{z}, \bar{z}]$ with the density function given by

² We use $V_i^{1/3}$ instead of V_i in equation 1 in order to facilitate easy algebraic calculations. As long as the compensation depends positively on a functional form of V_i , the basic intuitions of this model holds good.

³ We divide the second term of the utility function by 2 for simplifying the calculations in obtaining the optimal effort. The results of the paper remain unchanged if the second term of the utility function is not divided by 2. We

note that disutility from effort has been modelled by $\frac{e_i^2}{2}$ in several papers including Baggs and Bettignies (2007).

$$f(z) = \frac{1}{\bar{z} - \underline{z}} \quad (4)$$

In the third stage, the manager of a firm chooses output to maximize the equity value of the firm. We assume no fixed cost and the marginal cost of production is $c \geq 0$. Following Povel and Raith (2004), we assume that a firm i issues debt to finance its production cost so that its debt $D_i = cq_i$, where q_i is the level of production for firm i .

Switching state of nature \hat{z} is defined as that state of nature at which the revenue of a firm is exactly equal to its debt and interest on debt.

$$(1+r)D_i = R^i(q_i, q_{-i}, \hat{z})$$

where R^i is the revenue of firm i and r is the interest to be paid on debt D_i .

$$\text{For firm 1,} \quad (1+r)D_1 = R^1(q_1, q_2, \hat{z}_1) \quad (4a)$$

$$\text{For firm 2,} \quad (1+r+d)D_2 = R^2(q_1, q_2, \hat{z}_2) \quad (4b)$$

2.2.2 The Third Stage

This three stage game is solved by backward induction. In the third stage, the manager of a firm engages in a Cournot duopoly game with the other firm to maximize the value of her firm.

With limited liability, firm i 's manager maximizes

$$\max_{q_i} V_i = \max_{q_i} \int_{\hat{z}}^{\bar{z}} R^i(q_i, q_{-i}, z) f(z) dz \quad (5)$$

It can be shown that the maximized values of the firms are given by

$$V_i^* = \frac{(q_i^*)^3}{\bar{z}}, i = 1, 2 \quad (6)$$

$$\text{where} \quad q_1^* = \frac{\bar{z} + \theta + e_1 - (1+r)c - \frac{\lambda}{3} [\bar{z} + \theta + e_2 - (1+r+d)c]}{3 - \frac{\lambda^2}{3}} \quad (7a)$$

$$q_2^* = \frac{\bar{z} + \theta + e_2 - (1+r+d)c - \frac{\lambda}{3} [\bar{z} + \theta + e_1 - (1+r)c]}{3 - \frac{\lambda^2}{3}} \quad (7b)$$

2.2.3 The Second Stage

In the second stage, the manager of a firm chooses her effort simultaneously with the manager of the other firm to maximize her own utility.

Solving this maximization problem, the optimal efforts are given by⁴

$$e_i^* = \frac{\beta_i}{(\bar{z})^{\frac{1}{3}} (3 - \frac{\lambda^2}{3})} \quad (8)$$

⁴ We assume that $3 - \frac{\lambda^2}{3} > 0$. Given that the compensation parameter β_i is always positive, this assumption is needed to ensure that the optimal effort is positive.

Individual rationality constraint suggests that the utility of an individual manager must be greater than or equal to the reservation utility prevailing in the market.

$$U_i \geq \bar{U} \quad (9)$$

where \bar{U} is the prevailing reservation utility.

We assume that the labor market for managers is perfectly competitive which implies that a manager receives only the reservation utility. Using equations (1) and (8), the equilibrium managerial compensation contract and outputs are given by

$$w_i = \bar{U} + \frac{\beta_i^2}{2(\bar{z})^3(3-\frac{\lambda^2}{3})^2}, i=1,2 \quad (10)$$

$$q_1^* = \frac{[\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^3(3-\frac{\lambda^2}{3})} - (1+r)c] - \frac{\lambda}{3}[\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^3(3-\frac{\lambda^2}{3})} - (1+r+d)c]}{3-\frac{\lambda^2}{3}} \quad (11a)$$

$$q_2^* = \frac{[\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^3(3-\frac{\lambda^2}{3})} - (1+r+d)c] - \frac{\lambda}{3}[\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^3(3-\frac{\lambda^2}{3})} - (1+r)c]}{3-\frac{\lambda^2}{3}} \quad (11b)$$

2.2.4 The First Stage

The equity holders of a firm maximize the net equity value of the firm by choosing the compensation parameters. The maximization problem is given by the following equation.

$$\max_{\alpha_i, \beta_i} V_i^{net} = V_i - w_i \quad (12)$$

2.2.5 Proposition 1

The incentive compensation of a financially constrained firm increases with the degree of financial constraint, when the degree of product differentiation sufficiently low. The difference between the incentive compensation of a financially constrained firm and financially unconstrained firm increases with the degree of financial constraint, when the degree of product differentiation sufficiently low.

Proof: This is equivalent to showing that $\frac{d\beta_2^*}{dd} > 0$ and $\frac{d\beta_2^*}{dd} - \frac{d\beta_1^*}{dd} > 0$ for sufficiently high values of λ . Proof is in Appendix A.

Intuition behind this theorem is that a financially constrained firm (firm 2) has a higher cost of capital, resulting in higher cost of production. Higher cost of production reduces firm output which in turn decreases firm value. This is the first effect of financing constraint. When the degree of product differentiation is sufficiently low, the higher cost firm (firm 2) can compensate for this higher cost by offering higher incentive to its manager which encourages her increase her effort and produce more output. This is the second effect of financing constraint. Which effect dominates depends on the degree of product differentiation (represented by the parameter λ).

2.2.6 Proposition 2

When the degree of product differentiation is sufficiently low, a financially constrained firm is more aggressive in the product market.

Proof: This is equivalent to showing $\frac{dq_2^*}{dd} > 0$. See Appendix B.

When the degree of product differentiation is sufficiently low, the financially constrained firm offers incentive compensation to its manager which encourages her to put more effort and increase output in order to compensate for any loss in output due to financial constraint.

3. Conclusion

The relationship between financial constraint and incentive compensation has not been explored in the literature. This paper is the first attempt to address this relationship. In this paper, we introduce financial constraint in a model of incentive compensation and product market behavior. A financially constrained firm faces higher cost of capital which reduces its equilibrium output in a Cournot duopoly game. When the degree of product differentiation is sufficiently low, the model suggests that a financially constrained will offer its manager higher incentive compensation which incentivizes her to put more effort and produce more output. This paper proposes a new testable hypothesis that a financially constrained firm offers higher incentive compensation to its manager compared to a financially unconstrained firm.

Future work involves further exploring the relationship between financial constraint and incentive compensation both theoretically and empirically. The model can be extended with n firms and with the possibility of bankruptcy for the financially constrained firms.⁵ We admit the difficulties in setting up empirical tests in a duopoly setup. Empirical predictions are more easily testable in a model with n firms. We believe that our paper sheds light to an interesting area of research which is still relatively unexplored.

4. References

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⁵ We thank an anonymous referee for suggesting extending the model to n firms and with the probability of bankruptcy.

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5. Appendix

5.1 Appendix A

For firm 1, the optimization problem of the equity holders is

$$\max_{\alpha_1, \beta_1} V_1^{net} = \frac{[[\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r)c] - \frac{\lambda}{3}[\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r+d)c]]^3}{\bar{z}(3 - \frac{\lambda^2}{3})^3} - \frac{\beta_1^2}{2(\bar{z})^{\frac{2}{3}}(3 - \frac{\lambda^2}{3})^2} - \bar{U}$$

For firm 2, the optimization problem of the equity holders is

$$\max_{\alpha_2, \beta_2} V_2^{net} = \frac{[[\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r+d)c] - \frac{\lambda}{3}[\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r)c]]^3}{\bar{z}(3 - \frac{\lambda^2}{3})^3} - \frac{\beta_2^2}{2(\bar{z})^{\frac{2}{3}}(3 - \frac{\lambda^2}{3})^2} - \bar{U}$$

The first order condition with respect to β_1 is

$$3[[\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r)c] - \frac{\lambda}{3}[\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r+d)c]]^2 = (3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}} \beta_1 \quad (A1)$$

The first order condition with respect to β_2 is

$$3[[\bar{z} + \theta + \frac{\beta_2}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r+d)c] - \frac{\lambda}{3}[\bar{z} + \theta + \frac{\beta_1}{(\bar{z})^{\frac{1}{3}}(3 - \frac{\lambda^2}{3})} - (1+r)c]]^2 = (3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}} \beta_2 \quad (A2)$$

The equilibrium values of β_1 and β_2 satisfy equations (A.1) and (A.2) simultaneously. Differentiating equations (A.1) and (A.2) with respect to d and applying the first order conditions (A.1) and (A.2), we get,

$$\left[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_1^{*\frac{1}{2}}}\right] \frac{d\beta_1^*}{dd} - \frac{\lambda}{3} \frac{d\beta_2^*}{dd} = -\frac{\lambda}{3} c\bar{z}^{\frac{1}{3}} \left(3 - \frac{\lambda^2}{3}\right) \quad (\text{A3})$$

$$-\frac{\lambda}{3} \frac{d\beta_1^*}{dd} + \left[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_2^{*\frac{1}{2}}}\right] \frac{d\beta_2^*}{dd} = -\frac{\lambda}{3} c\bar{z}^{\frac{1}{3}} \left(3 - \frac{\lambda^2}{3}\right) \quad (\text{A4})$$

We solve for $\frac{d\beta_1^*}{dd}$ and $\frac{d\beta_2^*}{dd}$ which are given as below.

$$\frac{d\beta_1^*}{dd} = \frac{\frac{\lambda}{3} c\bar{z} \left(3 - \frac{\lambda^2}{3}\right)^3}{2\sqrt{3}\beta_2^{*\frac{1}{2}} D} \quad (\text{A5})$$

$$\frac{d\beta_2^*}{dd} = \frac{c\bar{z}^{\frac{1}{3}} \left(3 - \frac{\lambda^2}{3}\right) \left[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_1^{*\frac{1}{2}}} - \frac{\lambda^2}{9}\right]}{D} \quad (\text{A6})$$

$$\text{where } D = \left[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_1^{*\frac{1}{2}}}\right] \left[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_2^{*\frac{1}{2}}}\right] - \frac{\lambda^2}{9}$$

Using the FOC equations (A.1) and (A.2), the maximized value of V_i is $V_i^* = \frac{(\beta_i^*)^{\frac{3}{2}}}{3^{\frac{3}{2}}}$

$$\text{Maximized net value of the firm is } V_{i.net}^* = V_i^* - w_i = \frac{(\beta_i^*)^{\frac{3}{2}}}{3^{\frac{3}{2}}} - \frac{(\beta_i^*)^2}{2(\bar{z})^{\frac{2}{3}} \left(3 - \frac{\lambda^2}{3}\right)^2} - \bar{U}$$

\bar{U} is the reservation utility which is positive. Further, the maximized net value $V_{i.net}^*$ should be positive.

$$\text{This implies that } \frac{(\beta_i^*)^{\frac{3}{2}}}{3^{\frac{3}{2}}} > \frac{(\beta_i^*)^2}{2(\bar{z})^{\frac{2}{3}} \left(3 - \frac{\lambda^2}{3}\right)^2}$$

leading to $\frac{(\bar{z})^{\frac{2}{3}}(3-\frac{\lambda^2}{3})^2}{2\sqrt{3}(\beta_i^*)^{\frac{1}{2}}} > \frac{3}{4}$. $V_{i,net}^*$ is the maximized net value of firm i, maximized with

respect to β_i . Hence, $\frac{dV_{i,net}^*}{d\beta_i} < 0$ which leads to $\frac{(3-\frac{\lambda^2}{3})^2\bar{z}^{\frac{2}{3}}}{2\sqrt{3}(\beta_i^*)^{\frac{1}{2}}} < 1$. Hence, we get the following

upper and lower limits $\frac{3}{4} < \frac{(3-\frac{\lambda^2}{3})^2\bar{z}^{\frac{2}{3}}}{2\sqrt{3}(\beta_i^*)^{\frac{1}{2}}} < 1$. This means that we can have the following

inequality $-\frac{\lambda^2}{9} < [1 - \frac{(3-\frac{\lambda^2}{3})^2\bar{z}^{\frac{2}{3}}}{2\sqrt{3}(\beta_2^*)^{\frac{1}{2}}} - \frac{\lambda^2}{9}] < 1 - \frac{3}{4} - \frac{\lambda^2}{9}$. Sufficient condition for

$[1 - \frac{(3-\frac{\lambda^2}{3})^2\bar{z}^{\frac{2}{3}}}{2\sqrt{3}(\beta_2^*)^{\frac{1}{2}}} - \frac{\lambda^2}{9}] < 0$ is $\frac{3}{2} < \lambda$

$-\frac{\lambda^2}{9} < D = [1 - \frac{(3-\frac{\lambda^2}{3})^2\bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_1^{*\frac{1}{2}}}] [1 - \frac{(3-\frac{\lambda^2}{3})^2\bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_2^{*\frac{1}{2}}}] - \frac{\lambda^2}{9} < \frac{1}{16} - \frac{\lambda^2}{9}$

Sufficient condition for D to be negative is $\frac{3}{4} < \lambda$.

If $\frac{3}{2} < \lambda$, $\frac{d\beta_2^*}{dd} > 0$.⁶ Further when $\frac{3}{2} < \lambda$, $\frac{d\beta_1^*}{dd} < 0$. So as long as $\frac{3}{2} < \lambda$, $\frac{d\beta_2^*}{dd} - \frac{d\beta_1^*}{dd} > 0$

We note that $\frac{3}{2} < \lambda$ is a sufficient condition for these inequalities to hold, but not the necessary conditions. There can be other ranges of λ when these two inequalities may hold.

5.2 Appendix B

Using equations 8 and A.6, we get,

$$\frac{de_2^*}{dd} = \frac{d\beta_2^*}{dd} \frac{de_2^*}{d\beta_2} = \frac{c[[1 - \frac{(3-\frac{\lambda^2}{3})^{\frac{1}{2}}\bar{z}^{\frac{2}{3}}}{3}] - \frac{\lambda^2}{9}]}{2\sqrt{3}\beta_1^{*\frac{1}{2}} D} \tag{B1}$$

⁶ We consider only the positive values of λ

$$\frac{de_1^*}{dd} = \frac{d\beta_1^*}{dd} \frac{de_1^*}{d\beta_1} = \frac{\frac{\lambda}{3} c \bar{z}^{\frac{2}{3}} (3 - \frac{\lambda^2}{3})^2}{2\sqrt{3}\beta_2^{*\frac{1}{2}} D} \quad (\text{B2})$$

$$\frac{dq_2^*}{dd} = \frac{\frac{de_2^*}{dd} - c - \frac{\lambda}{3} \frac{de_1^*}{dd}}{(3 - \frac{\lambda^2}{3})} = \frac{c}{(3 - \frac{\lambda^2}{3}) D} \left[\frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_2^{*\frac{1}{2}}} \right] \left[1 - \frac{(3 - \frac{\lambda^2}{3})^{\frac{1}{2}} \bar{z}^{\frac{2}{3}}}{2\sqrt{3}\beta_1^{*\frac{1}{2}}} - \frac{\lambda^2}{9} \right] \quad (\text{B3})$$

If $\frac{3}{2} < \lambda$, $\left[1 - \frac{(3 - \frac{\lambda^2}{3})^2 \bar{z}^{\frac{2}{3}}}{2\sqrt{3}(\beta_2^*)^{\frac{1}{2}}} - \frac{\lambda^2}{9} \right] < 0$ and $D < 0$

The sufficient condition for $\frac{dq_2^*}{dd} > 0$ is that $\frac{3}{2} < \lambda$