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### Optimal nondiscriminatory auctions with favoritism

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### Abstract

In many auction settings, there is favoritism: the seller's welfare depends positively on the utility of a subset of potential bidders. However, laws or regulations may not allow the seller to discriminate among bidders. We find the optimal nondiscriminatory auction in a private value, single-unit model under favoritism. At the optimal auction there is a reserve price, or an entry fee, which is decreasing in the proportion of preferred bidders and in the intensity of the preference. Otherwise, the highest-valuation bidder wins. We show that, at least under some conditions, imposing a no-discrimination constraint raises expected seller revenue.

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# 1 Introduction

When auctions are used, the seller is frequently not indifferent as to which of the bidders will win. She runs an auction to enhance competition but, for a given selling price, she would prefer some of the bidders to win rather than others. This may occur when some of the bidders' welfare positively influences the seller's welfare. For example, in a government-run auction, domestic firms may generate more tax revenue than their foreign rivals. Alternatively, the seller and some of the bidders may be firms in the same conglomerate. We say that there is favoritism when the seller has such a preference for some bidders over others.

Favoritism usually motivates the design of discriminatory auctions.<sup>1</sup> For example, price preferences are sometimes introduced: to win, a non-preferred bidder may have to beat the highest bid made by a preferred bidder by at least some previously specified margin. Another usual way to discriminate, known as *right of first refusal*, awards one of the preferred bidders the right to match the highest bid that any of her rivals may submit.

However, in many situations discrimination is not possible. This happens quite often in public procurement, where laws and regulations sometimes forbid favoring some bidders over others to level the field and thus to foster competition. There may be higher-level regulations that explicitly prevent local authorities from favoring local firms.<sup>2</sup> In general, this constraint may be interpreted as one imposed by a principal on an agent who is in charge of the auction.<sup>3</sup>

Our aim here is to examine the auction design problem faced by a seller who places positive weight on some of the bidders' welfare,<sup>4</sup> but faces a no-discrimination constraint. We conclude that the seller will choose an auction where the highest-valuation bidder wins unless her valuation is too low, just as in the standard, revenue-maximizing auction. However, we find that the set of valuations excluded is smaller when there is favoritism, and it becomes even smaller when the weight attached to the utility of any favored bidder grows. Hence, favoritism raises the probability of selling the object. One possible justification for ruling out discriminatory auctions may be that doing so raises expected revenue. Indeed, we find that, in some cases, the optimal mechanism that the seller would select under favoritism when she *cannot* discriminate among bidders generates more revenue than the mechanism she would use if discrimination were allowed.

Our work contributes to the literature on favoritism in auctions. Laffont and Tirole (1991)

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<sup>1</sup>The justification for discrimination that we examine, which derives from the fact that the seller values some of the bidders' utilities, is not the only possible one. With a fixed number of bidders, biasing the auction against strong bidders raises revenue, as shown by optimal auction theory. With endogenous entry, discriminating against strong bidders may also be optimal for the seller, since it could encourage the entry of weak bidders. Those arguments require asymmetry among bidders, while our model is (but for the possibly unequal consideration of bidders' utilities by the seller) symmetric.

<sup>2</sup>For example, see Maasland et al. (2004) for a discussion on whether discriminating among bidders may be viewed as state aid -and thus be prohibited- under European Union rules or not.

<sup>3</sup>A more complete analysis would embed the auction design problem within a principal-agent model, but here we are simply concerned with the effects of a no-discrimination constraint on auction design under seller favoritism among bidders.

<sup>4</sup>In Section 3, we also analyze the case where the seller's welfare rises by a fixed value if certain bidders win.

and Vagstad (1995) study the case of multidimensional auctions, where favoritism may appear when the auctioneer assesses product quality. McAfee and McMillan (1989), Branco (1994), and Naegelen and Mougeot (1998) examine single-dimensional auctions, where price-preferences may be used. In all these papers, the seller is allowed to use discriminatory mechanisms. Arozamena and Weinschelbaum (2011) extend the analysis of the single-dimensional case to a situation where the number of bidders is endogenous, and conclude that the optimal auction in that setting is nondiscriminatory. Thus, not being allowed to discriminate is irrelevant when entry is endogenous. Here, though, we examine the case where the number of bidders is fixed.

In the following section we present the model and characterize the optimal mechanism for a seller that may discriminate among bidders. We derive the optimal mechanism under a no-discrimination constraint in section 3. In section 4, we examine the effect of that constraint on expected revenue. We conclude in section 5.

## 2 The model

The owner of a single, indivisible object is selling it through an auction.<sup>5</sup> For simplicity, we assume the seller attaches no value to the object. There are  $N \geq 2$  bidders whose valuations for the object are given by  $v_i$ ,  $i = 1, \dots, N$ . Each  $v_i$  is bidder  $i$ 's private information. These valuations are i.i.d. according to the c.d.f.  $F$  with support on an interval that is normalized to  $[\underline{v}, \bar{v}]$  and a density  $f$  that is positive and bounded on the whole support. The context is therefore one of symmetric independent private values. All parties to the auction are risk-neutral, and we assume that any bidder's virtual valuation,  $J(v) = v - \frac{1-F(v)}{f(v)}$ , is increasing in her actual valuation.<sup>6</sup>

We assume that the seller's objective function follows from adding to the seller's revenue each bidder's welfare, where bidder  $i$ 's welfare is weighted according to an exogenous parameter  $\alpha_i$ ,  $i = 1, \dots, N$ . Furthermore,  $\alpha_i \in [0, 1]$  for all  $i$ . That is, the seller attaches a weakly positive weight to each bidder's welfare, but cannot value the latter more than her own, "private" utility (i.e., her revenue). Note that if  $\alpha_i = 0$  for all  $i$ , we have a standard, revenue-maximizing seller. However, if  $\alpha_i = 1$  for all  $i$ , the seller will behave as she would when pursuing efficiency in the absence of favoritism.

In this specific context, as we mentioned above, we intend to characterize the optimal mechanism for a seller that cannot discriminate among bidders. However, the optimal mechanism for a seller who is allowed to treat different bidders in an asymmetric way will be useful to us as a reference point later on, so we describe it here first.

As presented so far, our problem is a slight modification of the standard optimal auction problem with independent private values.<sup>7</sup> Let  $H_i(v_1, \dots, v_N)$  ( $P_i(v_1, \dots, v_N)$ ) be the probability that bidder  $i$  gets the object (respectively, the expected price bidder  $i$  has to pay to the seller) if bidder valuations are given by  $(v_1, \dots, v_N)$ . Then, the seller has to choose

<sup>5</sup>All of our results, however, are applicable as well to the case of procurement auctions.

<sup>6</sup>This is what the literature calls the "regular case" after Myerson (1981).

<sup>7</sup>See Myerson (1981) and Riley and Samuelson (1981).

a mechanism  $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$  such that, for all  $(v_1, \dots, v_N)$ ,  $0 \leq H_i(v_1, \dots, v_N) \leq 1$  for all  $i$  and  $\sum_{i=1}^N H_i(v_1, \dots, v_N) \leq 1$ . In addition, let  $h_i(v_i)$  ( $p_i(v_i)$ ) be the expected probability that bidder  $i$  gets the object (respectively, the expected price she pays) when her valuation is  $v_i$ , and the valuations of all other bidders are unknown.

Bidder  $i$ 's expected utility when her valuation is  $v_i$  and she announces that it is  $v'_i$  is

$$\tilde{U}_i(v_i, v'_i) = h_i(v'_i)v_i - p_i(v'_i).$$

Additionally, let  $U_i(v_i) = \tilde{U}_i(v_i, v_i)$ . Then, the seller's problem is

$$\max_{\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N} \sum_{i=1}^N \left( \int_{\underline{v}}^{\bar{v}} p_i(v_i) f(v_i) dv_i + \alpha_i \int_{\underline{v}}^{\bar{v}} U_i(v_i) f(v_i) dv_i \right) \tag{1}$$

subject to the standard incentive compatibility and participation constraints

$$U_i(v_i) \geq \tilde{U}_i(v_i, v'_i) \quad \text{for all } i, \text{ for all } v_i, v'_i \tag{2}$$

$$U_i(v_i) \geq 0 \quad \text{for all } i, \text{ for all } v_i. \tag{3}$$

A slightly different version of this problem has been studied before, for instance, in Naegelen and Mougeot (1998).<sup>8</sup> We characterize the solution by following the usual steps in the literature. Let  $\tilde{v}_i(v_i)$  be the valuation that bidder  $i$  announces optimally when her true valuation is  $v_i$ . By incentive compatibility, it has to be true that  $\tilde{v}_i(v_i) = v_i$ . The envelope theorem then implies that

$$U'_i(v_i) = \frac{\partial}{\partial v_i} \tilde{U}_i(v_i, \tilde{v}_i(v_i)) = h_i(v_i).$$

Therefore,  $U_i(v_i) = \int_{\underline{v}}^{v_i} h_i(s) ds + U_i(\underline{v})$ . Noting that, in the solution to our problem,  $U_i(\underline{v}) = 0$  for all  $i$ ,<sup>9</sup> we have

$$p_i(v_i) = h_i(v_i)v_i - \int_{\underline{v}}^{v_i} h_i(s) ds$$

for all  $i$ . Substituting for  $p_i(v_i)$  and  $U_i(v_i)$  in the seller's objective function and integrating by parts, we have

$$\sum_{i=1}^N \int_{\underline{v}}^{\bar{v}} h_i(v_i) \left[ v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \right] f(v_i) dv_i.$$

<sup>8</sup>This problem can be thought of as an extension to the  $N$ -bidder context of a particular case of the analysis in Naegelen and Mougeot (1998), when there is no consumer surplus and the shadow cost of public funds is zero.

<sup>9</sup> $U_i(\underline{v})$  may be zero or positive for those bidders  $i$  with  $\alpha_i = 1$ . Given that we are adding the expected utilities of the seller and these bidders, how much they pay (as long as incentive compatibility holds) does not affect the seller's objective function. However, by no-discrimination,  $U_i(v_i) = U(v_i)$  for all  $i$ , so  $U(\underline{v}) > 0$  is only possible if  $\alpha_i = 1$  for all  $i$ . But even in this case there exists a solution where  $U_i(\underline{v}) = 0$  for all  $i$ .

Then, the seller solves

$$\max_{\{H_i(\cdot)\}_{i=1}^N} E_{v_1, \dots, v_N} \left[ \sum_{i=1}^N H_i(v_1, \dots, v_N) \left[ v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \right] \right]. \quad (4)$$

In the absence of favoritism, the seller would maximize the expected value of the winner's virtual valuation. Here, virtual valuations are adjusted to account for favoritism. Only a fraction  $(1 - \alpha_i)$  of bidder  $i$ 's rent is subtracted from that bidder's actual valuation in the seller's objective function.

The solution to this problem is straightforward, and is described in the following proposition.

**Proposition 1** *If discrimination is allowed, the optimal allocation rule for the seller is<sup>10</sup>*

$$H_i^D(v_1, \dots, v_N) = \begin{cases} 1 & \text{if } i = \arg \max_j v_j - (1 - \alpha_j) \frac{1 - F(v_j)}{f(v_j)} \\ & \text{and } v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

The seller thus defines individual minimum acceptable valuations  $r_i$  that solve

$$r_i - (1 - \alpha_i) \frac{1 - F(r_i)}{f(r_i)} = 0 \quad (5)$$

$i = 1, \dots, N$ . In order to be awarded the object, bidder  $i$ 's valuation must be larger than  $r_i$ , and her adjusted virtual valuation must be higher than all other bidders'.

### 3 No discrimination

As mentioned above, we are interested in the case where the seller *cannot* discriminate among bidders. Hence, we add a new constraint on the set of mechanisms  $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$  that the seller can select.

**No-discrimination constraint:** *The seller has to choose a mechanism  $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$  that, for any permutation  $\pi : \{1, \dots, N\} \rightarrow \{1, \dots, N\}$ , satisfies*

$$\begin{aligned} H_i(v_{\pi(1)}, \dots, v_{\pi(N)}) &= H_{\pi(i)}(v_1, \dots, v_N) \\ P_i(v_{\pi(1)}, \dots, v_{\pi(N)}) &= P_{\pi(i)}(v_1, \dots, v_N) \end{aligned}$$

for all  $i$ .<sup>11</sup>

<sup>10</sup>If there is a tie, the object may be allocated randomly among the bidders who have the highest adjusted virtual valuation.

<sup>11</sup>This condition has been used previously in Jehiel, Moldovanu and Stacchetti (1999).

**Remark 1** *The no-discrimination constraint may be viewed as one imposed on the specific indirect mechanism that the seller will use. Here, we present it as a constraint on the direct mechanisms that we are examining under the revelation principle. We may do that given that we are in a symmetric context, as described above: a nondiscriminatory auction with a symmetric equilibrium implements a nondiscriminatory direct revelation mechanism. If the distributions of valuations were not symmetric, though, clearly we would not necessarily be able to associate a nondiscriminatory auction with a symmetric direct mechanism.*

The seller's problem, then, is to maximize (1) subject to the incentive compatibility and participation constraints described in (2) and (3), and subject to the no-discrimination constraint. Following the steps described in the previous section, we can incorporate the incentive compatibility and participation constraints into the objective function. The seller's problem is that of maximizing (4) subject to the no-discrimination constraint.

The fact that the seller cannot discriminate among bidders, however, allows us to restate this problem in a more convenient way.

**Remark 2** *Let  $v_{(n)}$  be the  $n$ th order statistic associated with the vector of valuations  $(v_1, \dots, v_N)$ .<sup>12</sup> For any vector  $\mathbf{v} = (v_1, \dots, v_N)$ , we may define a function  $\gamma_{\mathbf{v}} : \{1, \dots, N\} \rightarrow \{1, \dots, N\}$  that assigns to each position  $1, \dots, N$  the identity of the bidder whose valuation ranks in that position. That is,  $\gamma_{\mathbf{v}}(n) = i$  if  $v_{(n)} = v_i$ .<sup>13</sup> The no-discrimination constraint implies that for any two vectors  $\mathbf{v} = (v_1, \dots, v_N)$ ,  $\mathbf{v}' = (v'_1, \dots, v'_N)$  such that  $(v_{(1)}, \dots, v_{(N)}) = (v'_{(1)}, \dots, v'_{(N)})$  we must have*

$$H_{\gamma_{\mathbf{v}}(n)}(v_1, \dots, v_N) = H_{\gamma_{\mathbf{v}'}(n)}(v'_1, \dots, v'_N), \quad n = 1, \dots, N.$$

To show this, assume  $H_{\gamma_{\mathbf{v}}(n^*)}(v_1, \dots, v_N) \neq H_{\gamma_{\mathbf{v}'}(n^*)}(v'_1, \dots, v'_N)$  for some  $n^*$ . Let  $\tilde{\pi} : \{1, \dots, N\} \rightarrow \{1, \dots, N\}$  be such that  $\tilde{\pi}(i) = \gamma_{\mathbf{v}}(\gamma_{\mathbf{v}'}^{-1}(i))$ . The functions  $\gamma_{\mathbf{v}}$  and  $\gamma_{\mathbf{v}'}$  are permutations, so  $\tilde{\pi}$  is a permutation as well. Note that  $(v'_1, \dots, v'_N) = (v_{\tilde{\pi}(1)}, \dots, v_{\tilde{\pi}(N)})$ . Then,

$$H_{\tilde{\pi}(i^*)}(v_1, \dots, v_N) \neq H_{i^*}(v_{\tilde{\pi}(1)}, \dots, v_{\tilde{\pi}(N)})$$

for  $i^* = \gamma_{\mathbf{v}'}(n^*)$ , which violates the no-discrimination constraint.

Thus, if for any two vectors of valuations the corresponding vectors of order statistics coincide, then the seller has to allocate the good with the same probability to those bidders that occupy each ordered position in the vectors of order statistics. In other words, the probability that any given bidder wins must depend only on the vector of order statistics and on her valuation's position in that vector. This, in turn, implies that the seller's problem can be expressed in terms of order statistics: she has to choose an allocation function

$$\{H_n(v_{(1)}, \dots, v_{(N)})\}_{n=1}^N.$$

<sup>12</sup>We follow the convention by which  $v_{(1)}$  is the highest value in the vector,  $v_{(2)}$  the second-highest, and so on.

<sup>13</sup>Since we are using continuous distributions, ties will occur with probability zero. Still, in the event of a tie, positions have to be allocated with equal probabilities among those bidders whose valuations coincide so that the no-discrimination constraint is satisfied.

We can focus only on which allocations the seller chooses when the vector of valuations is ordered. Allocations in all other cases follow from the no-discrimination constraint.

The seller, though, cares about bidder identities. Given a vector of order statistics  $(v_{(1)}, \dots, v_{(N)})$ , since valuations are independently drawn from the same distribution, the probability that bidder  $i$ 's valuation ranks in position  $n$  is the same for all bidders. Thus, the seller's problem can be expressed as follows.

$$\max_{\{H_n(v_{(1)}, \dots, v_{(N)})\}_{n=1}^N} E_{v_{(1)}, \dots, v_{(N)}} \left\{ \sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[ N v_{(n)} - \frac{1 - F(v_{(n)})}{f(v_{(n)})} \sum_{i=1}^N (1 - \alpha_i) \right] \right\}.$$

Since  $J(v) = v - \frac{1-F(v)}{f(v)}$  is increasing in  $v$ , it is easy to show that  $Nv - \frac{1-F(v)}{f(v)} \sum_{i=1}^N (1 - \alpha_i)$  is also increasing in  $v$ . Therefore, the solution to this problem is simple, and is described in the following proposition.

**Proposition 2** *The optimal allocation rule when discrimination is not allowed is<sup>14</sup>*

$$H_i^{ND}(v_1, \dots, v_N) = \begin{cases} 1 & \text{if } v_i > \max_{j \neq i} v_j \text{ and } Nv_i - \frac{1-F(v_i)}{f(v_i)} \sum_{j=1}^N (1 - \alpha_j) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

This direct mechanism can be implemented by any auction where the highest-valuation bidder wins, with an adequately chosen reserve price or entry fee. For example, the seller may choose a first-price or a second-price auction with reserve price  $r$  such that

$$Nr - \frac{1 - F(r)}{f(r)} \sum_{i=1}^N (1 - \alpha_i) = 0. \tag{6}$$

Since revenue (weakly) carries more weight in her objective function than bidders' welfare, the seller excludes some valuations. Discrimination is not allowed, though, so the corresponding reserve price is uniform: the seller just averages the weights  $(1 - \alpha_i)$  across bidders.<sup>16</sup> When the object is sold, revenue maximization and the fact that bidders have to be treated equally make it optimal to award the object to the highest-valuation bidder. Note that if  $\alpha_i = 0$  for all  $i$ , the optimal mechanism for the seller coincides with the standard, revenue-maximizing direct mechanism: the reserve price  $r$  satisfies  $r - \frac{1-F(r)}{f(r)} = 0$ . At the same time, if  $\alpha_i = 1$  for all  $i$ , then  $r = \underline{v}$  and no valuations are excluded. Therefore, for any vector of weights  $(\alpha_1, \dots, \alpha_N)$ , the seller chooses a mechanism that neither sells the good with probability one nor attains revenue maximization. She selects a mechanism that falls somewhere in between these two extreme cases.<sup>17</sup>

<sup>14</sup>In order to satisfy the no-discrimination constraint, as we mentioned above, if there is a tie all bidders with the highest valuation win with the same probability.

<sup>15</sup>As the left-hand side of this equation is increasing in  $r$ , there is a unique solution.

<sup>16</sup>Recall that  $(1 - \alpha_i)$  is the fraction of bidder  $i$ 's rent that is irrelevant to the seller's utility.

<sup>17</sup>When the number of bidders is endogenous, as Arozamena and Weinschelbaum (2011) show, even with favoritism it is optimal for the seller to use a nondiscriminatory mechanism which maximizes revenue and at the same time sells the good with probability one.

Note as well that, as long as  $\alpha_i > 0$  for some  $i$  (i.e., there is favoritism),  $r$ , the minimum acceptable valuation, depends on  $(1/N) \sum_{i=1}^N \alpha_i$ . Then, if a new bidder enters the auction with a weight in the seller's objective function that is higher (lower) than the average weight that the seller attaches to the existing bidders' utilities,  $r$  will fall (respectively, rise). This differs from the standard, revenue-maximizing reserve price, which is independent of  $N$ .

In addition, given  $N$ ,  $r$  is decreasing in  $\alpha_i$  for any  $i$ . If the seller places a larger weight on a given bidder's welfare, she can only enhance that bidder's welfare by reducing the reserve price or entry fee that she employs in any auction that implements the optimal mechanism. Doing so benefits all other bidders as well. Therefore, all bidders' expected utilities are increasing in any  $\alpha_i$ .

**Remark 3** *Our results also hold if we model favoritism in a different way. Assume, for example, that the seller attaches a fixed value  $w_i$  to bidder  $i$  winning the auction ( $i = 1, \dots, N$ ). Then, the seller's objective function is given by*

$$\sum_{i=1}^N \left( \int_{\underline{v}}^{\bar{v}} p_i(v_i) f(v_i) dv_i + w_i \int_{\underline{v}}^{\bar{v}} h_i(v_i) f(v_i) dv_i \right).$$

*In this case, it can be shown that, under a no-discrimination constraint, the bidder with the highest valuation wins and the optimal minimum acceptable valuation solves*

$$N \left( r - \frac{1 - F(r)}{f(r)} \right) + \sum_{i=1}^N w_i = 0.$$

*We could then find how that valuation changes with  $N$  and with each  $w_i$ . Our results would be analogous to the ones that follow when the seller cares about bidders' profits.*

## 4 Revenue

In our setup, one justification for imposing the use of nondiscriminatory mechanisms may be that allowing preferential treatment to some bidders could reduce revenue. In the two previous sections, we have characterized the optimal mechanisms for the seller both with and without discrimination. We examine in what follows how those mechanisms compare in terms of expected revenue. As is well known, expected revenue is given by

$$E_{v_1, \dots, v_N} \left[ \sum_{i=1}^N H_i(v_1, \dots, v_N) J(v_i) \right].$$

For any given weights  $(\alpha_1, \dots, \alpha_N)$ , let  $R^D(\alpha_1, \dots, \alpha_N)$  ( $R^{ND}(\alpha_1, \dots, \alpha_N)$ ) be the expected revenue generated by mechanism  $(H_1^D(\cdot), \dots, H_N^D(\cdot))$  (respectively,  $(H_1^{ND}(\cdot), \dots, H_N^{ND}(\cdot))$ ).

Figure 1 below helps visualize how discrimination influences revenue for the case where  $N = 2$ . Assume  $\alpha_1 > \alpha_2$ .

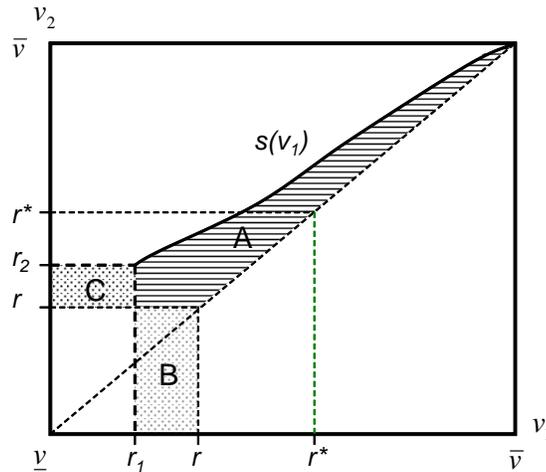


Figure 1

With discrimination, the seller sets individual minimum valuations  $r_1$ ,  $r_2$  as defined in (5). Figure 1 depicts a case where  $r_1 > \underline{v}$ . In addition,  $s(v_1)$  is the value of  $v_2$  such that the adjusted virtual valuations of both bidders coincide, i.e.,

$$s(v_1) - (1 - \alpha_2) \frac{1 - F(s(v_1))}{f(s(v_1))} = v_1 - (1 - \alpha_1) \frac{1 - F(v_1)}{f(v_1)},$$

as long as  $v_1 > r_1$ . If  $v_1 > r_1$  and  $v_2 > r_2$ , then she awards the object to bidder 1 if  $v_2 < s(v_1)$ , and to bidder 2 otherwise –clearly, if only one bidder’s valuation is above her own minimum acceptable level, that bidder is the winner. When discrimination is forbidden, the seller sets a uniform minimum valuation  $r$  defined in (6), and awards the object to the bidder with the highest valuation as long as it is larger than  $r$ . Finally,  $r^*$  is the minimum valuation that would be optimally selected in the absence of favoritism, defined in the usual way:  $J(r^*) = 0$ .

To examine the effect of the no-discrimination constraint on revenue, we focus on the pairs  $(v_1, v_2)$  where the allocation rule differs in the optimal discriminatory and nondiscriminatory mechanisms under favoritism. Those pairs are depicted in the figure in regions A, B and C. For  $(v_1, v_2)$  in region A, the object is awarded to a bidder under both mechanisms. With discrimination, bidder 1 wins, since her adjusted virtual valuation is higher. Without discrimination, bidder 2 wins, since her actual valuation is higher. Since  $J(v)$  grows with  $v$ , in region A revenue is larger when discrimination is not allowed. In region B, the seller awards the object to bidder 1 when she can discriminate. Once she cannot discriminate the seller keeps the object, since  $v_1, v_2 < r$ . Given that  $J(v_1) < 0$  in region B (as  $v_1 < r^*$ ), forbidding discrimination raises revenue. Finally, in region C the object remains unsold with discrimination, but is awarded to bidder 2 when the no-discrimination constraint applies. Since  $J(v_2) < 0$  in region C, revenue is larger with discrimination.

It is straightforward to see that these opposing effects apply as well when  $N > 2$ . Can we ascertain whether there is a positive or negative net effect on revenue? In some cases

we can.<sup>18</sup> If  $r_i = \underline{v}$  for some  $i$ , the seller never keeps the object when discrimination is allowed. In Figure 1, this means that region  $C$  vanishes, and so do the cases where the no-discrimination constraint could reduce revenue. This happens, for example, when  $\alpha_i = 1$  for some  $i$ . We then have the following proposition, that applies for any number of bidders and any distribution of valuations.

**Proposition 3** *Assume  $r_i = \underline{v}$  for some  $i$  (a sufficient condition for this is that  $\alpha_i = 1$ ). Then,  $R^{ND}(\alpha_1, \dots, \alpha_N) \geq R^D(\alpha_1, \dots, \alpha_N)$ , and the inequality is strict unless  $\alpha_1 = \alpha_2 = \dots = \alpha_N$ .*

When  $r_i > \underline{v}$  for all  $i$ , we can derive a result along the same lines for the two-bidder case and under some conditions on the distribution of valuations. These conditions are satisfied, for example, by power-function distributions,  $F(v) = v^k$ , with  $k \geq 1$ .

**Proposition 4** *Assume  $N = 2$  and the density  $f(\cdot)$  is differentiable. Then, if  $F(\cdot)$  has a monotone increasing hazard rate and  $J(\cdot)$  is convex, then  $R^{ND}(\alpha_1, \alpha_2) \geq R^D(\alpha_1, \alpha_2)$ , and the inequality is strict if  $\alpha_1 \neq \alpha_2$ .*

The proof is available in Arozamena, Shunda and Weinschelbaum (2012). In our setting, at least in the cases described in Propositions 3 and 4, there is a revenue-based justification for a principal to prevent an agent in charge of running an auction from discriminating among bidders when that agent's preferences exhibit favoritism.

## 5 Conclusion

We have examined the problem faced by a seller that cannot discriminate among bidders although her preferences exhibit favoritism. We have characterized the optimal mechanism, and concluded that, at least in some cases, it yields higher expected revenue than the optimal mechanism when discrimination is allowed.

Our analysis assumes a symmetric context. It would be interesting to study the effect of a no-discrimination constraint with bidder asymmetries, both with and without favoritism. That problem, though, is technically harder and we leave it for future research.

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<sup>18</sup>We have not been able to find an example where the no-discrimination constraint *reduces* expected revenue.

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