Economics Bulletin

Volume 34, Issue 2

A multiscale approach to emerging market pricing

Bruno Milani
Federal University of Santa Maria

Paulo Sérgio Ceretta
Federal University of Santa Maria

Abstract

Market risk measurement has a long tradition in finance and it has been drawing the attention of many academic studies since Markowitz (1952). But the CAPM model (and derived models) assumptions have been targets of much criticism, in the sense that beta estimation may be imprecise. The supposition of investor's homogenous expectations is one of its problems, knowing that investors have different profiles concerning risk exposure and time horizon. Thus, this article aims to verify the scale differences of emerging markets risk pricing based on the international CAPM model. To perform this analysis, it was used wavelet decomposition and panel regressions. The results confirm some literature trends regarding the beta tendency to increase at lower frequencies, as well as the best fit(R2). Additionally, we bring a unique contribution in relation to the long term leverage effect, showing that this form of risk affects only the long-term investors, causing a risk exposure not verified in the short term.

Citation: Bruno Milani and Paulo Sérgio Ceretta, (2014) "A multiscale approach to emerging market pricing", *Economics Bulletin*, Vol. 34 No. 2 pp. 784-792.
Contact: Bruno Milani - brunoprofess@gmail.com, Paulo Sérgio Ceretta - milani_bruno@yahoo.com.br.
Submitted: April 24, 2013. **Published:** April 23, 2014.

1 Introduction

Modern portfolio theory was introduced by the pioneering work of Markowitz (1952), which relies on a trade-off between expected return and risk. Hence, in his seminal paper, Markowitz (1952) implicitly provided a mathematical definition of risk, that is, the variance of returns. Thus, we can think about risk in terms of how spread-out the return distribution is.

Later on, the individual work of Lintner (1965), Treynor (1965), Sharpe (1966) and Jensen (1967) on performance measurement culminated in what is known today as the Capital Asset Pricing Model (CAPM). According to the CAPM, the systematic risk is the most relevant risk in portfolio analysis, since all the other risks can be minimized by portfolio diversification. The sensibility of the portfolio to the systematic risk is measured by the Beta coefficient, which is widely used.

Merton (1973) proposed the Intertemporal CAPM (ICAPM), which delineates a dynamic risk-return relationship, adjusted to the temporal shifts of the market sensibility of a given asset, supposing a stochastic variation in the set of investment opportunities. Besides the conditional aspect, the ICAPM also introduces a theoretical shift in claiming that the return of an asset can be priced by average market risk, which may also be due to changes in investment opportunities set due to decisions of local government. For Merton (1973), the interest rate is the simplest form to capture the specific risk of these policy changes. Thus, the model indicates that the global market risk is different from government policies risks.

Bekaert and Harvey (1995) use the ICAPM to analyze the risk-return dynamic in several emerging developed markets. The excess return of each market is priced by its covariance with the world market, whose proxy was the US market (NYSE), besides its own risk, i.e., its variance, which supplanted the interest rate proposed by Merton (1973) to capture the risk of government policies shifts. This variation of Merton’s (1973) model is known as international CAPM.

Many later studies discussed the difficulties in the CAPM model and derived models, due to their heavy assumptions that make Beta estimation difficult or non-representative of reality. For example, the dynamics of the relationship between stock returns and risk factors are likely to vary depending on the investor’s time horizon, but CAPM supposes homogenous expectations. So, the need of incorporating different time scales arises. A relatively new approach known as wavelet analysis takes may help to reduce these problems. Therefore, this paper aims to verify the differences in scale of the risk pricing in emerging markets, based on international CAPM model. In order to perform this analysis, wavelet decomposition and panel regressions were used.

2 Literature Review

Gençay et al. (2005) proposed the multiscale measurement of systematic risk, decomposing the traditional Beta into wavelets. The excess log-return of US, UK and Germany markets were individually analyzed with a different range of time for each one, but all of them with daily data. Their results showed that the higher the scale, the stronger the relationship between portfolio return and its beta, which means that the beta was higher at low frequencies (64-128 days dynamics).

Fernandez (2006) formulates a time-scale decomposition of an international version of CAPM that accounts for both market and exchange-rate risk, considering stock indexes of seven emerging countries of Latin America and Asia, for the sample period of 1990-2004. With daily data of the MSCI world index and the MSCI emerging markets index, two approaches are analyzed: the first consists in decomposing each index and recomposing its crystals by DWT and then estimate an OLS regression. The second approach is based on wavelet-variance analysis, which determines estimates for the slopes and the goodness of fit of the model (R^2) by the MODWT variance and covariance formulas. Both methods were used to estimate Beta. The results depended on which world index was used, although the emerging markets appear to depend more on the other emerging markets than the developed ones.

Cifter e Özün (2007) decomposed the variance and returns of 10 stocks of ISE-30 by the MODWT method, and then estimated a CAPM model to six scales. Their results showed that the return-risk maximization of the portfolio with these 10 stocks may be achieved at the scale of 32 days and the risk will be higher in the portfolios established at the scales different than 32 days. Rhaeim et al. (2007) estimated the systematic risk at different scales in the French stock market, with a sample composed of twenty-six actively traded stocks over 2002-2005 periods. Individual stocks and market returns were decomposed into 6 scales. Thus, Beta was estimated by OLS regression. The relationship between excess return and market portfolio becomes stronger at higher scales because beta increases as the scale increases.

Rua and Nunes (2012) illustrated the use of wavelets method assessing the risk of an investor in emerging markets over the last twenty years, using the monthly percentage returns of Morgan Stanley Capital International (MSCI), all country world index and the MSCI emerging markets index, expressed in US Dollars. Using the variance as a measure of total risk, the wavelet spectrum analysis shows that the volatility of monthly stock returns is concentrated at high frequencies, which means that short-term fluctuations dictate the variance of the series. In fact, frequencies associated with movements longer than one year are almost negligible in terms of contributions to total variance. They identify changes in variance across different time-scales in each country, which are clearly linked to well-documented crisis, although there is no evidence of an upward or downward trend in the volatility of emerging countries.

The overall beta of emerging countries is 1.17, seeming to be more stable over time at low frequencies and more time-varying at high frequencies. At high frequencies, one can identify regions in the time-frequency space where the beta is near 3. Given that, their conclusions oppose others like Gençay et al. (2005), Fernandez (2006) and Rhaeim et al. (2007). However, the periods where the beta is high include several crises, which mean that if the crises effects were controlled, these results could not hold.

Counterpointing results are also found by Masih et al. (2010), who estimates beta at different time scales in the context of the emerging Gulf Cooperation Council (GCC) equity markets by applying wavelet analysis, finding a multiscale tendency. They analyzed companies of the Saudi stock market (88), Muscat Securities Market (114), Kuwait stock exchange (189), Bahrain stock exchange (43), Doha securities market (38), Abu Dhabi securities market (61) and Dubai financial market (46), in different time ranges, comprising February 2007 to April 2008, with daily data. Each return series is separated into its components multiresolution (multihorizon) constituents using orthogonal Haar wavelet transformation. Then, an OLS estimation is ran to each stock and for each frequency, generating several multiscale Betas. They found that Beta and its variability increase between lowest and highest scale, which makes long-term investors more exposed to systematic risk than short-term investors. Also, R^2 decreases when moving to higher scales (longer interval), which means that market return is more able to explain individual stock return at higher frequencies, similarly as the study of Rua and Nunes (2012).

Additionally, Rua and Nunes (2012) also computed the wavelet of R^2 as a multiplication of the country's conditional Beta by the wavelet of market return divided by the country return, analogously to the traditional R^2. This is due to the importance of the systematic risk in explaining total risk, since the overall value of R^2 was near 0.5, but changing considerably over time and frequencies. In low frequencies, 80% of total variance is explained by the systematic risk, but in high frequencies, only 30%.

Deo and Shah (2012) applied the multiscale Beta estimation approach based on wavelet analysis to all stocks comprising BSE-Sensex, using the wavelet decomposition from the maximal overlap discrete wavelet transform (MODWT). With daily data from the BSE-30 (a representative index of the thirty biggest companies of the Indian stock market) from 5 January 2010 to 31st march 2012 (562 observations), they separate out each return series into its constituent multi-resolution (multi-horizon) components. The MODWT was chosen because giving up orthogonality, they gain attributes that are more desirable in economic applications, as the possibility to handle data of every length, not just powers of two; it is translation invariant – that is, a shift in the time series results in an equivalent shift in the transform; it has increased resolution at lower scales since it oversamples data; the choice of a particular wavelet filter is not so crucial; it is slightly affected by the arrival of new information.To each scale of stock return series, two equations are estimated by the OLS method, one with the conventional Beta and other with two coefficients analogous to Beta, one associated to a short periodicity series and the other to a long-periodicity series of market returns.The market index is also decomposed and the Beta coefficient estimated in each level. Beta coefficients were significantin all cases but, they observed that the R^2 is higher at lower scales, implying that major part of market portfolio influence on individual stocks is between medium to higher frequencies. If market risk is concentrated at the medium and higher frequencies, the model predictions would be more relevant at medium to long-run horizons as compared to short time horizons.

Conlon et al. (2008) explored multiscale analysis for Hedge Funds, due to their wide acceptance by institutional investors because their seemingly low correlation with traditional investments and attractive returns. The Hedge Funds correlation and market risk scaling properties are analyzed by the MODWT, with monthly data from April 1994 to October 2006, tracking over 4500 funds holding at least US$ 50 million under management. They found that both correlation and market risk level with respect to S&P500 varies greatly according to the strategy and time scale examined. The correlation between Convertible Arbitrage, Fixed Income and Multi-strategy, besides the S&P500 and the Hedge Fund Composite Index was found to increase as the time scale increases. But the correlation between Dedicated Shorted Bias, Equity Market Neutral, Global Macro and Managed Futures strategies correlation with S&P500 and the Hedge Fund Composite Index was found to decrease as the time scale increases. Also, the market risk level held by different Hedge Funds strategies varies according to the time horizon studied. The level of market risk of convertible Arbitrage, Emerging Markets, Event-Driven and Long/Short Equity was found to increase as the time scale increased. The market risk of Dedicated Short Bias, Global Macro and Managed Futures was found to decrease as the time scale increased.

Thus, in general, there is certain consensus among the studies, in the sense that betas are higher at low frequencies (large scales), pointing that an asset (local market) dependency on the market (global market) is stronger and easily verified in the long-run analysis. R^2 are also higher at low frequencies, showing that the market return is more able to explain a stock return in the long run, which may be due to a high degree of speculative behavior at the short-run.

3 About Wavelets

Wavelets are small "waves" that grow and decay in a limited time period. The wavelets transforms decomposes a time series in terms of some elementary functions, called the daughter wavelets or, simply, the wavelets ($\psi_{\tau,s}(t)$). These wavelets are new time series resulting from a mother wavelet $\psi(t)$ that can be expressed as a function of the time position τ (translation parameter) and the scale s (dilatation parameter), which is related to the frequency.

Wavelets are similar to sine and cosine functions because they oscillate around zero, but differ because they are localized both in the time and frequency domains. In contrast to Fourier analysis, wavelets are compactly supported, because all projections of a signal onto the wavelet space are essentially local, not global, and thus it doesn't need to be homogeneous over time. In fact, wavelet analysis can be seen as a refinement of Fourier analysis.

Wavelets are flexible in handling a variety of non-stationary signals, considering the non-stationarity as an intrinsic property of the data rather than a problem to be solved. Basic wavelets are characterized into father and mother wavelets. A father wavelet (scaling function) represents the smooth baseline trend, while the mother wavelets (wavelet function) are used to describe all deviations from trends. Formulations (1) and (2), respectively represents the father and mother wavelets.

$$\phi_{j,k}(x) = 2^{\frac{j}{2}}\phi(2^j x - k). \tag{1}$$

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^j x - k). \tag{2}$$

Where $j, k \in \mathbb{Z}$, for some coarse scale j_0, that will be taken as zero. j=1, in a j-level decomposition. The father wavelet integrates to one and reconstructs the trend component (longest time scale component) of the series. The mother wavelets integrate to zero and describe all deviations from the trend. In order to compute the decomposition, wavelet coefficients at all scales representing the projections of the time series onto the basis generated by the chosen family of wavelets need to be calculated first. They are $D_{j,k}$ (smooth; mother wavelet) and $S_{j,k}$ (detailed; father wavelet), as expressed by the formulation (3), that generates an orthonormal system. For any function f that belongs to this system we may write, uniquely:

$$f(x) = \sum_k S_{0,k}\,\phi_{0,k}(x) + \sum_{j\geq 0}\sum_k D_{j,k}\psi_{j,k}(x). \tag{3}$$

In (3), $S_{0,k} = \int f(x)\phi_{0,k}dx$ and $D_{j,k} = \int f(x)\psi_{j,k}\,dx$ are the Smooth and Detail component wavelet coefficients. We could also understand that $f(x)$ is reconstructed, containing the separate components of the original series at each frequency j. After we decompose the function $f(x)$ into j crystals, the crystals d_j are recomposed into a time domain. Formulation (3), thus, represents the entire function $f(x)$, where $\sum_k D_{j,k}\psi_{j,k}(x)$ is the recomposed series in the time domain from the crystal d_j and $\sum_k S_{0,k}\,\phi_{0,k}(x)$ is the recomposition of the residue. In this sense, $\sum_k D_{j,k}\psi_{j,k}(x)$ represents the contribution of frequency j to the original series.

Considering a time series $f(t)$ that we want to decompose into various wavelet scales. Given the father wavelet, such that its dilates and translates constitute an orthonormal basis for all subspaces that are scaled versions of the initial subspace, we can form a Multiresolution Analysis for $f(t)$. The wavelet function in formulation (3) depends on two parameters, scale and time: the scale or dilation factor j controls the length of the wavelet, while the translation or location parameter k refers to the location and indicates the non-zero portion of each wavelet basis vector.

The Discrete Wavelet Transform (DWT) is the usual approach for this multiresolution analysis, but it is restricted to sample sizes to a power of 2, i.e., for j levels we must have a sample of size 2^j. In order to overcome this difficulties, in this study we adopt the Overlap Discrete Wavelet Transform (MODWT), which can handle data of any length, not just powers of two; it is translation invariant, i.e., a shift in the time series results in an equivalent shift in the transform; it has also increased resolution at lower scales since it oversamples the data; the choice of a particular filter is not so crucial if MODWT is used and it isn't affected by the arrival of new information, except for the last few coefficients.

Differently from DWT, MODWT is a highly redundant linear filter that transforms a series into coefficients related to variations over a set of scales (Gençay et al. 2001). This way, giving up of orthogonality, MODWT gains attributes that are more desirable in economic applications.

4 Data and Methodology

We use daily returns of market indexes of six emerging countries such as Argentina, Brazil, Chile, Colombia, Mexico and Peru and a proxy for the world market, all obtained from the Morgan Stanley Capital International (MSCI). The sample period is from 11/07/2002 to 13/07/2012, with 2612 observations. We choose this period and this frequency due to its availability. Squared returns are used as a proxy for market volatility. Each market return and volatility, as well as the world return and volatility, was decomposed into wavelets by the MODWT transform, generating a multistage decomposition of the series at seven different scale crystals (*j*) as follows:D1 (2-4 days); D2 (4-8 days); D3 (8-16 days); D4 (16-32 days); D5 (32-64 days); D6 (64-128 days); D7 (128-256 days).

Considering the large data quantity and the deriving difficulty to analyze it, we chose to disregard the intermediates D2, D4 and D6 crystals, allowing us to focus on D1, D3, D5 and D7 crystals. This way, we were able to verify the scale differences on a larger frequency range, reducing the outputs and making it easier to interpret them. Once the time series were decomposed, they were rearranged in four panels, one for each regarded frequency. Each wavelet panel contained four variables: the emerging markets return wavelets and volatility wavelets, and the same for the world market. The model was estimated with fixed effects.

This article aims to estimate the multiscale pricing of each emerging market, through an ICAPM model, as described in Section 2. The model can be compared to the Bekaert and Harvey (1995) model, in the sense that it uses the variance as a proxy for country risk and analyzes an international dynamics. Using a panel regression, equation (4) was estimated for each of the four regarded scales.

$$R_{m,t}(\tau_j) = \alpha(\tau_j) + \beta_1 R_{w,t}(\tau_j) + \beta_2 \sigma^2_{m,t}(\tau_j) + \beta_3 \sigma^2_{w,t}(\tau_j) + \mu_t(\tau_j) \qquad (4)$$

Where $R_{m,t}(\tau_j)$ is the market proxy return wavelet *j* at time *t*; $\alpha(\tau_j)$ is the estimated linear coefficient for each *j*; $R_{w,t}$ is the world proxy return wavelet *j* at time *t*; $\sigma^2_{m,t}(\tau_j)$ is the market proxy volatility wavelet for each *j* at time *t*; $\sigma^2_{w,t}(\tau_j)$ is the world proxy volatility wavelet for each *j* at time *t*; $\mu_t(\tau_j)$ is the estimated residual for each *j*, at time *t*; β_1, β_2 and β_3 are parameters.

In addition, we ran a conventional estimation for the original data (without scale decomposition) for comparison purposes. Section 5 will present the obtained results.

5 Results and Discussion

Initially, we checked the series stationarity. The Im, Pesaran and Shin (1997), Levin, Lin and Chu (2002) and the Harris and Tzavalis (1999) unit root tests for panel data reject, at 5% significance level, the null hypotheses that the panels present unit root. After that, we calculated the descriptive statistics of emerging markets return panel and, also, of the world market return series, which are presented in Table 1.

Variable	$R_{m,t}$	$R_{w,t}$
Mean	0.0007	0.0001
Median	0.0011	0.0008
Minimum	-0.1835	-0.0733
Maximum	0.1662	0.0910
Std Deviation	0.0196	0.0115
V.C	277.655	850.855
Skewness	-0.4049	-0.3668
Ex. Kurtosis	857.885	761.966

Table 1: Descriptive statistics of daily emerging markets return panel and daily world market return series.

Table 1 shows that the six emerging markets average return is much higher than world market average return, but it appears to be riskier too, considering that the minimum-maximum amplitude, as well as the standard deviation, are higher. Although the maximum point is much higher, the minimum point is much lower. It is also evident that emerging markets are more left-skewed than the world market, confirming that they are riskier. Thereby, emerging markets appear to have higher expected returns and higher risk, a widely known effect. The risk pricing was estimated by a conventional ICAPM model and the coefficients are presented in Table 2:

Variable	Coefficient (β)	t test	p value	prob>F	R^2overall
α	0.1310	4.9700	0.0040	0.0000	0.3789
$R_{w,t}$	1.0432	9.1800	0.0000		
$\sigma^2_{m,t}$	0.0017	-0.2400	0.8200		
$\sigma^2_{w,t}$	-0.0998	-3.7400	0.0130		

Table 2: Panel regression coefficients for the conventional ICAPM model estimated with the original variables (non-decomposed).

The results show that all coefficients are significant, except for the emerging markets volatility($\sigma^2_{m,t}$), at 5% degree of significance. This means that the emerging markets return can be explained only by the world market return and its volatility. The presented goodness-of-fit of the regression model (R^2 overall) indicates that 37,89% of the emerging markets variance is explained by the model. Table 3 presents the estimating results of the ICAPM model for the four wavelet panels analyzed.

D1 (2-4 days)					
Variable	Coefficient (β)	t test	p value	prob>F	R^2
$\alpha(\tau_1)$	0.0000	-0.9400	0.3900	0.0400	0.3258
$R_{w,t}(\tau_1)$	0.9994	78.400	0.0010		
$\sigma^2_{m,t}(\tau_1)$	-0.3253	-17.800	0.1350		
$\sigma^2_{w,t}(\tau_1)$	0.0608	19.100	0.1140		
D3 (8-16 days)					
Variable	Coefficient (β)	t test	p value	prob>F	R^2
$\alpha(\tau_3)$	0.0000	-14.200	0.2160	0.0000	0.4366
$R_{w,t}(\tau_3)$	10.957	125.000	0.0000		
$\sigma^2_{m,t}(\tau_3)$	0.0366	0.6300	0.5540		
$\sigma^2_{w,t}(\tau_3)$	0.0315	0.3300	0.7540		
D5 (32-64 days)					
Variable	Coefficient (β)	t test	p value	prob>F	R^2
$\alpha(\tau_5)$	0.0000	-16.100	0.1680	0.0007	0.4479
$R_{w,t}(\tau_5)$	11.578	101.000	0.0000		
$\sigma^2_{m,t}(\tau_5)$	-0.2513	-0.5600	0.6020		
$\sigma^2_{w,t}(\tau_5)$	0.1151	0.8600	0.4290		

D7 (128-256 days)					
Variable	Coefficient (β)	t test	p value	prob>F	R^2
$\alpha(\tau_7)$	0.0000	-19.200	0.1130	0.0016	0.4389
$R_{w,t}(\tau_7)$	13.806	82.700	0.0000		
$\sigma^2_{m,t}(\tau_7)$	0.0608	14.300	0.2110		
$\sigma^2_{w,t}(\tau_7)$	-0.2181	-41.900	0.0090		

Table 3: Panel regression coefficients of the four wavelet panels analyzed. There is one regression for each of the four panels containing the world market/emerging markets return wavelets and volatility wavelets. The ICAPM model explains the emerging markets return as a function of their volatility and the world market return and volatility.

To assess the estimation results, we present Table 3, which shows that in the four panels there was only one significant coefficient: the world market return($R_{w,t}$), except for the D7 panel, whereupon the world market volatility ($\sigma^2_{w,t}$) was also significant, however negatively. As previous studies reported (Gençay et al. 2005, Rhaeim et al. 2007, Rua and Nunes 2012, Deo and Shah 2012) the market coefficient (Beta) is higher at low frequencies (large scales), showing that long-term investors are more exposed to systematic risk than short-term investors. It is noteworthy that the Betas of the three largest scales are higher than one. For example, D7 beta is 1,38, what makes the emerging markets very aggressive and susceptible to systematic risk, because the emerging markets proxy oscillates more than the world market proxy.

At large scales (long-term) emerging markets return exhibit a negative relationship with the world volatility, so that the return increases when the volatility decreases. This can be explained by the leverage effect, first discussed by Black (1976), who observed that the volatility of stocks tends to increase when the price falls. The negative relationship between return and volatility could be related to a market panic effect (Bouchaud et al. 2001), but since we are able to identify the time horizon where this happens, due to the wavelet technique, the traditional explanation of Black (1976) is more suitable for the long-run (low frequency), which gives support to an accounting interpretation.

Importantly, when comparing the panel regression of the non-decomposed panel (Table 2) with the panel regression of multiscale panels (Table 3), we can see that the world volatility coefficient that was significant in the first case, remains significant only at a specific frequency, what could have led to an inaccurate conclusion if the multiscale approach was not considered. Besides, another trend verified in the literature is confirmed too: the goodness-of-fit (R^2) is higher at low frequencies, showing that in large scales, returns are more predictable.

6 Conclusions

Although most studies assume that volatilities and covariances are constant, it has long been acknowledged that systematic risk and other factors may vary over time. This paper allowed better understanding of the scale differences between the short and long run dynamics when it comes to pricing of emerging markets return. This feature has fundamental implications because of profile differences among investors, in terms of risk and time horizon. Clearly, the short-term investor is more interested in the risk associated with higher frequencies, whereas the long-term investor focuses on lower frequencies.

We confirm some literature trends, with respect to the Beta tendency to increase at lower frequencies, as well as the model goodness-of-fit (R^2), like it was verified by Rua and Nunes (2012), who made a similar analysis of the emerging countries using monthly data. Our paper is consistent with their results in the sense that the emerging market dependency to the world market is higher at large scales, even though we used daily data. We extend their study including each market's volatility and world market volatility as independent variables. However, we found that the return of an emerging market is a function of the world market return, but local volatility and world volatility are not significant in a general way.

Additionally, we bring a contribution regarding the long run leverage effect, showing that this risk form can affect the long-term investors, causing a risk exposure not verified in the short-run. For further studies, we suggest the use of a more flexible kind of estimation, such as the copula regression.

7 References

Bekaert, G. and Harvey, C. Time-varying world market integration. Journal of Finance, v. 50, p. 403–444, 1995.

Bouchaud, J. P.; Matacz, A.; Potters, M. Leverage Effect in financial markets: The retarded volatility model. Physical Review Letters, v. 87, i. 22, 2001.

Black, Fischer (1976). The pricing of commodity contracts, Journal of Financial Economics, 3, 167-179.

Cifter, A. and Özün, A. Multiscale Systematic Risk: An application on ISE-30. MPRA Paper No. 4288, 2007.

Conlon, T., Crane, M. and Ruskin, H. Wavelet Multiscale Analysis for Hedge Funds: Scaling and Strategies. Physica A: Statistical Mechanics and Its Applications, v. 387, i. 21, p. 5197-5204, 2008.

Deo, M. Shah, A. Wavelet-Based Beta Estimation: Applications to Indian Stock Market. Kuwait Chapter of Arabian Journal of Business and Management Review, v. 1, n. 11, 2012.

Fernandez, V. The international CAPM and a wavelet-based decomposition of Value At Risk. Studies in Non-linear Dynamics & Econometrics, v. 9, i. 4, 2005.

Gençay, R., Selçuk, F. and Whitcher, B. Multiscale Systematic Risk.Journal of International Money and Finance v. 24, p. 55-70, 2005.

Jensen, M. C. The performance of mutual funds in the period of 1945-1964.Journal of Finance, v. 23, n. 2, p. 389-416, 1967.

Lintner, J. Security prices, risk and maximal gains from diversification.Journal of Finance, v. 20, i. 3, p. 587–615, 1965.

Markowitz, H. M. Portfolio selection.Journal of Finance, Oxford, v. 7, n. 1, p. 77-91, 1952.

Masih, M., Alzahrani, M. and Al-Titi, O. Systematic Risk and time scales: New evidence from an application of wavelet approach to the emerging Gulf stock markets. International Review of Financial Analysis, v. 19, p. 10-18, 2010.

Merton, R. C. An Intertemporal capital asset pricing model.Econometrica, n. 41, v. 5, p. 867-887, 1973.

Rhaeim, N., Ammou, S. B. and Mabrouk A. B. Wavelet estimation of systematic risk at Different Time Scales: Application to French stock market. The international Journal of Applied Economics and Finance v. 1, i. 2, p. 113-119, 2007.

Rua, A. and Nunes, L. C. A wavelet-based assessment of market-risk: The emerging markets case. The Quarterly Review of Economics and Finance, v. 52, p. 84-92, 2012.

Sharpe, W. F. Mutual Fund Performance. The Journal of business, v. 39, n. 1, p.119-138, 1966.

Treynor, J.How to Rate Management of Investment Funds. Harvard Business Review, v. 43, p. 63-75, 1965.