Economics Bulletin

Volume 34, Issue 2

On Ramsey's conjecture with AK technology

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Abstract

This paper analyzes growth and wealth distribution in a simple AK model in which households are heterogeneous not only in time-preference but in intertemporal substitution. Contrary to the result without long-run growth, the most patient household does not always own the entire capital in an economy with perpetual growth. In addition, it is shown that, if the most impatient household has high intertemporal substitutability, it can own the almost all (but not entire) capital of the economy in the long-run.

The author would like to thank an anonymous referee for helpful comments. He is also grateful to Professors Tetsugen Haruyama, Kentaro Iwatsubo, Daishin Yasui and seminar participants at the Kobe University for their valuable comments, but retains responsibility for any remaining errors. The financial support of Grants-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (grant no 22530180) is gratefully acknowledged.
Citation: Tamotsu Nakamura, (2014) "On Ramsey's conjecture with AK technology", *Economics Bulletin*, Vol. 34 No. 2 pp. 875-884.
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Submitted: April 04, 2014. **Published:** May 06, 2014.

1. Introduction

Intertemporal preferences of households are in general characterized by time-preference and intertemporal substitution. The two concepts seem similar but differ in essence: the former captures human nature to prefer today's consumption to tomorrow's while the latter reflects the human desire to smooth consumption over time. To be precise, the intertemporal elasticity of substitution measures the degree at which a household adapts to changes in his consumption over time. Focusing on the former concept, three seminal contributions have been made to the literature of long-run wealth distribution. Ramsey (1928) concludes his path-breaking paper with the remark, "equilibrium would be attained by a division of society into two classes, the thrifty enjoying bliss and the improvident at the substance level." (Ramsey, 1928, p.559). Becker (1980) confirms Ramsey's conjecture in a discrete time model by proving that the most patient household owns the entire capital of the economy while others consume their wage income in the long-run. Using a continuous version of Becker's model, Mitra and Sorger (2013) not only confirm the conjecture, but also show that the exclusive possession by the most patient household takes place after some finite time.[1]

Assuming a Stone-Geary type non-homothetic preference, Alvarez-Pelaez and Diaz (2005) and Obiols-Homs and Urrutia (2005) analyze the dynamics of wealth in a discrete-time model without long-run growth. Due to non-homothetic preference, intertemporal substitution changes over time on the equilibrium path. This fact produces various important and interesting implications for the dynamics of wealth distribution. In their models, however, deep parameters such as an intertemporal elasticity of substitution and a rate of time-preference charactering the preference are the same across households.

Time preference surely plays a crucial role in determining growth and the dynamics of wealth distribution in a growing economy. Shedding light on the effect of social conflicts on the

[1] They also develop an important argument about the fundamental issues arisen from the difference in the modeling between the continuous-time and the discrete-time.

individuals' preference, Borissov and Lambrecht (2009) endogenizes the time-preference rate to examine growth and income distribution in a discrete-time AK model. As a result, it is shown that the steady state equilibria are indeterminate and the set of equilibria is a continuum parameterized by an index of income inequality. In addition, under reasonable assumptions, the relationship between growth and inequality becomes hump-shaped.

Using a continuous-time AK model with perpetual growth, this paper shows that intertemporal substitution can be an important factor in determining not only the growth of the economy but also the long-run wealth distribution. As is explained above, without perpetual growth, only time preference plays a crucial role in determining the long-run wealth distribution. Even in a growing economy, the minimum patience is required for a household to own capital in the long-run. If, however, it has enough intertemporal substitutability over consumption, the most ***impatient*** household can own the almost all (but not entire) capital and determine the growth of the economy in the long-run.

The rest of the paper is organized as follows. Section 2 sets up a simple AK growth model with two types of heterogeneous households. Section 3 analyzes the growth and the dynamics and long-run distribution of the wealth of the economy. Section 4 concludes the paper.

2. The Model

Consider the competitive equilibrium of an infinite horizon production economy that consists of infinitely-lived heterogeneous households and identical firms that produce a homogenous good. The population of households is constant over time and normalized to unity.

2.1 Technology

To make the point clear, let us assume a simple AK technology. Output $Y(t)$ is produced by using only by capital $K(t)$ according to the following production function:

$$Y(t) = AK(t), \tag{1}$$

where A is a positive constant and stands for the net marginal product of capital. $K(t)$ can be interpreted as either physical capital or human capital. The rate of return on capital $r(t)$ is equal to the marginal product in the equilibrium, i.e.,

$$r(t) = A. \tag{2}$$

2-2. Preferences and intertemporal optimization

They are two types of households, H and L. The number of households H is constant over time at λ and hence that of households L is $1-\lambda$ $(0<\lambda<1)$. Preference of household i is characterized by

$$\int_0^\infty \frac{c_i(t)^{1-(1/\varepsilon_i)}-1}{1-(1/\varepsilon_i)} e^{-\rho_i t} \quad i = H, L, \tag{3}$$

where $c_i(t)$ is consumption, ε_i $(0<\varepsilon_i<1)$ is a constant intertemporal elasticity of substitution, and $\rho_i > 0$ is a constant rate of time-preference.

Each household maximizes the above lifetime utility subject to the following flow budget constraint:

$$\dot{k}_i(t) = r(t)k_i(t) - c_i(t) \text{ with } k_i(0) > 0, \qquad i = H, L, \tag{4}$$

where $k_i(t)$ is household i's capital stock, and the initial capital levels, $k_H(0)$ and $k_L(0)$, can be different. Solving the maximization problem, we obtain the following Keynes-Ramsey rule:

$$\dot{c}_i(t) = \varepsilon_i (r(t) - \rho_i) c_i(t), \qquad i = H, L, \tag{5}$$

The substitution of (2) into (5) gives

$$\dot{c}_i(t) = \varepsilon_i (A - \rho_i) c_i(t), \qquad i = H, L. \tag{6}$$

Also, the following transversality condition must hold:

$$\lim_{t\to\infty} e^{-\rho_i t} c_i(t)^{-1/\varepsilon_i} k_i(t) = 0, \qquad i = H, L. \tag{7}$$

To assure that both types of households accumulate capital and hence exist in the long-run, let us assume that the rate of return on capital is larger than their time-preference rates. In addition, we assume that household L is more patient than household H. Therefore, the following inequality holds:

$$A > \rho_H > \rho_L . \tag{8}$$

2-3. Household capital accumulation

Defining $x_i = c_i / k_i$ or the consumption-capital ratio, from (4) and (6) , we have

$$\frac{\dot{x}_i}{x_i} = \frac{\dot{c}_i}{c_i} - \frac{\dot{k}_i}{k_i} = \varepsilon_i (A - \rho_i) - \left(A - \frac{c_i}{k_i} \right) \text{ or } \dot{x}_i = x_i (x_i - x_i^*) , \quad i = H, L . \tag{9}$$

where

$$x_i^* = (1 - \varepsilon_i) A + \varepsilon_i \rho_i . \tag{10}$$

(Time arguments are suppressed when no ambiguity results.)

Although the model has two different agents, the interest rate $r(t)$ is constant at A. Therefore it is obvious that the model has no transitional dynamics and $x_i = c_i / k_i$ is a time-independent constant x_i^* given in (10). As a result, from (2), (4), (10), we have the following time-path of k_i:

$$\dot{k}_i = A k_i - x_i^* k \text{ or } g_i \equiv \dot{k}_i / k_i = A - x_i^* = \varepsilon_i (A - \rho_i) , \ i = H, L . \tag{11}$$

Therefore, the growth rate of household i's capital, g_i, is constant at $A - x_i^*$ or $\varepsilon_i (A - \rho_i)$. This in turn implies that its income and consumption also grow at the same constant rate.

3. The Analysis

3.1 The growth of the economy

By use of (11), we can analyze the dynamics of the aggregate economy. Suppose that the total capital stock of the economy is k, i.e., $k = \lambda k_H + (1 - \lambda) k_L$, which is equal to the average per-capita capital. Then, we have

$$\dot{k} = \lambda \dot{k}_H + (1 - \lambda) \dot{k}_L = Ak - \lambda c_H - (1 - \lambda) c_L = Ak - \lambda x_H^* k_H - (1 - \lambda) x_L^* k_L , \tag{12}$$

and hence the growth rate of k, g, becomes as follows:

$$g \equiv \frac{\dot{k}}{k} = A - x_H^* \frac{\lambda k_H}{k} - x_L^* \frac{(1-\lambda)k_L}{k} = A - x_H^* s_H - x_L^* s_L = A - x_H^* s_H - x_L^*(1-s_H), \quad (13)$$

where $s_H \equiv \lambda k_H / k$ is the capital share of household *H*, $s_L \equiv (1-\lambda)k_L/k$ is the capital share of household *L*, and hence $s_H + s_L = 1$.

We should notice that, although the model is a simple AK-type model, the growth rates of both capital and consumption change over time in the economy since the capital shares between the two households change over time.

3.2 The dynamics of wealth distribution

By the definition of s_H, we have

$$\dot{s}_H / s_H = \dot{k}_H / k_H - \dot{k}/k = (x_H^* - x_L^*)(s_H - 1) \quad \text{or} \quad \dot{s}_H = (x_H^* - x_L^*) s_H (s_H - 1). \quad (14)$$

If $x_H^* > x_L^*$, then s_H decreases over time and becomes zero in the long-run as is shown in Fig.1 Panel (a). Putting it differently, the capital share of household L increases over time and becomes unity in the long-run. The fact, however, that the share becomes unity does not imply that the entire capital of the economy will be owned by household L because household H also keeps accumulating capital. On the other hand, since the almost all capital is owned by household L, household L's behavior becomes a dominant determinant of the long-run growth rate. Substituting $s_H = 0$ and $s_L = 1$ into (13), we have

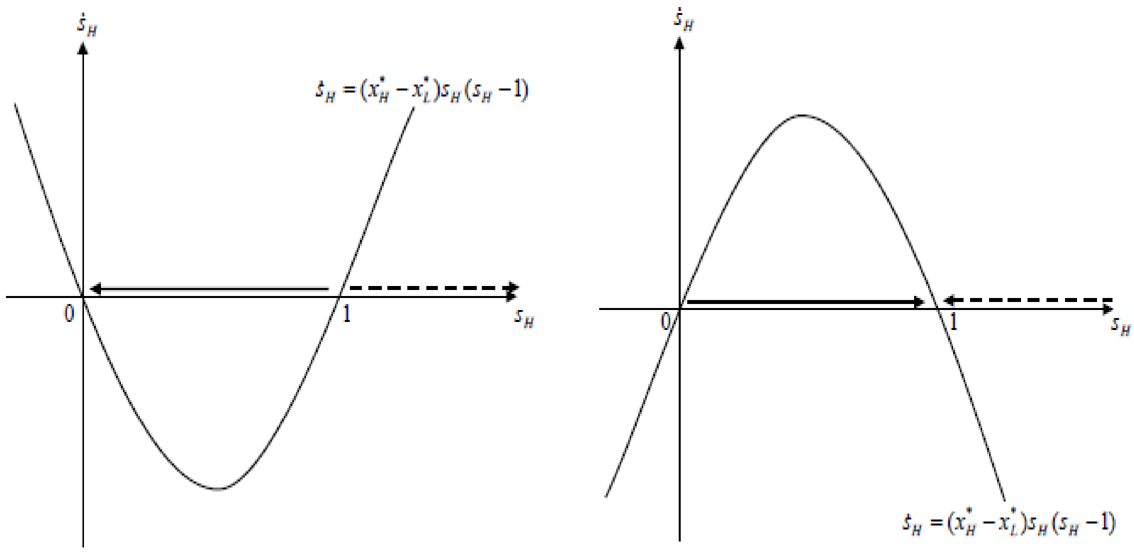
$$g \equiv \dot{k}/k = A - x_L^* = \varepsilon_L (A - \rho_L) \quad . \quad (15)$$

The above is the same as (11) with setting $i = L$.

If, in contrast, $x_H^* < x_L^*$, then s_H increases over time and becomes unity in the long-run as is shown in Fig.1 Panel (b). In other words, household L becomes having the almost all (but not entire) capital of the economy. As a result, the growth rate of the economy therefore eventually becomes:

$$g \equiv \dot{k}/k = A - x_H^* = \varepsilon_H (A - \rho_H). \quad (16)$$

The above is the same as (10) with setting $i = H$.



(a) When $x_H^* > x_L^*$ (b) When $x_H^* < x_L^*$

Fig. 1 Transitional Dynamics of Capital Share of Household H

3.3 On Ramsey's conjecture with AK technology

In a neoclassical growth model without long-run growth, as Becker (1980) in discrete-time and Mitra and Sorger (2013) in continuous-time show, Ramsey's (1928) conjecture holds that the most patient household, i.e., household L in the model, owns the entire capital of the economy in the long-run. The conjecture is true even in a growing economy if only the most patient household accumulates capital in the long-run. This will be the case when $\rho_H > A > \rho_L$. If, however, other households than the most patient one also accumulate capital, which is the case under the assumption (8), the conjecture is not always true in a growing economy.

In addition, there is the possibility that the most ***impatient*** household, household H in the model, comes to own the almost all (but not the entire) capital of the economy in the long-run. It takes places when s_H increases over time, i.e., the inequality holds that $x_H^* < x_L^*$. Taking (10) into account, the inequality can be rewritten as

$$\varepsilon_H (A-\rho_H) > \varepsilon_L (A-\rho_L) \quad \text{or} \quad \varepsilon_H / \varepsilon_L > (A-\rho_L)/(A-\rho_H) > 1. \tag{17}$$

Putting it another way, if the intertemporal elasticity of substitution of household H relative to that of household L ($\varepsilon_H / \varepsilon_L$) is sufficiently high, then it can be the dominant player in the long-run even if the time- preference rate of household H is higher than that of household L.

The intuition behind the above findings is straightforward. A yes-no decision on the capital accumulation depends solely on the net benefits of investment. If the rate of capital return exceeds the rate of time preference, or $r(t) > \rho_i$, then the household invests in capital and hence accumulates capital, and vice versa. Since the rate of return decreases toward zero as capital accumulates in the standard Ramsey model, only the most patient household the time preference rate of which is the lowest keeps the incentive to accumulate until the very end. As a result, it eventually owns the entire capital of the economy.

However, the speed of the capital accumulation depends not only on the net benefit but also the adjustment cost. Since the intertemporal elasticity of substitution is the degree at which the household adapt to changes in consumption over time, the inverse of the elasticity is considered as the adjustment cost of capital accumulation measured in terms of "utility." Therefore, the lower the elasticity is, the smaller the changes in consumption are. Since the households continue the accumulation as long as the net benefits, $r(t) - \rho_i$, are positive, the intertemporal elasticity of substitution is an important factor in determining the capital accumulation speed. Even if the benefit is not large, the household accumulates capital at a high speed if the elasticity is high, i.e., the adjustment cost is low.

4. Concluding Remarks

If an economy eventually reaches a stationary state, in which per-capita consumption and capital stay constant, then the marginal rate of substitution between current and future consumption, is determined solely by the rate of time-preference. The marginal rate of substitution is in turn equal to the rate of return on capital. Hence, the intertemporal substitutability between the consumptions has nothing to do with the equilibrium. As a result, as Becker (1980) and Mitra and Sorger (2013) have correctly shown, Ramsey's (1928) conjecture is correct that the most ***patient*** household or the household with the lowest time-preference rate owns the entire assets of the economy in the long-run.

The stylized facts, however, such as in Kaldor (1957), suggest that not the per-capita consumption and capital but their growth rates have been relatively constant over long periods of time. In other words, per-capita consumption and capital grow for a long time. For continued capital accumulation, the net rate of return must be larger than the households' time-preference rates. If this is the case, then the marginal rate of substitution is determined jointly by the time preference rate and the intertemporal elasticity of substitution. As a result, intertemporal substitutability becomes to play an important role in determining both the growth rate and wealth distribution of the economy. In other words, the patience is no longer the sole determinant of the long-run wealth distribution.

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