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Monitoring in Tournaments

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Abstract

We show that less monitoring can increase effort and alleviate the moral hazard problem in tournaments. We also find a unique optimal level of monitoring based on contestants' abilities. As the difference between their abilities gets larger, the contest designer should monitor less.

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1 Introduction

Moral hazard arises because a principal is not able to observe an agent's actions perfectly due to costly monitoring. (Holmström, 1979) The rank-order tournament or contest mechanism has a merit of reducing monitoring costs. Observing relative position is often less costly than measuring each worker's output directly. (Lazear and Rosen, 1981) In addition, when there exists a common shock, the principal does not need to measure the effect of the common shock by using relative performance evaluation because the contest can filter it out. (Green and Stokey, 1983)[1] Even if the virtues of the rank-order tournament or contest lie in its advantage of monitoring agents' efforts, the effect of monitoring has been paid little attention to in the literature, perhaps because more monitoring seems an obvious way to increase agents' efforts. However, we argue that less monitoring can increase effort and alleviate the moral hazard problem in a certain condition.

The cost of imperfect monitoring is losing the incentive-intensity principle. Agents have less incentive to work because their efforts are less accurately measured. On the other hand, imperfect monitoring can bring some benefit to the tournament designer. When monitoring is not perfect, what contestants have to decide is not merely trying to win over the rival, but how overwhelmingly they have to win. A winning agent has to win the game by a wide margin so that the designer can be convinced enough. In contrast, the designer's possible mistake provides an incentive for a losing agent to work harder as well. We will show that when contestants are heterogeneous in their abilities, the benefit of imperfect monitoing can be greater. We also show that there is a unique optimal level of monitoring based on contestants' abilities.

The closest paper to ours is Cowen and Glazer (1996). They show, based on the graphical analysis, that more monitoring can induce less effort in a one agent case.[2] Their finding has only limited connection with our model, in which there are two competing agents and the principal can choose the optimal monitoring level depending on the players' ability levels. In addition, our paper sheds a new light on the two player asymmetric contests. Baik (1994), Nti (1999) and Baik (2004) study the effect of asymmetries between two players on total effort levels. Our paper further study what is the optimal monitoring level in this asymmetric contest environment.

The remaining parts of this paper are as follows. Section 2 introduces basic model based on Lazear and Rosen (1981). Section 3 studies the choice of monitoring technologies and provides main results. Section 4 discusses and concludes.

[1] It also reduces a principal's incentive to betray agents by undervaluing their performances or reneging on the contract *ex post* (Malcomson; 1984). In addition, when rewards are indivisible, contests are quite necessary. People may get utility from the contest itself. (O'keeffe et al; 1984)

[2] A literature from social psychology argues 'the crowding-out' hypothesis such that increased monitoring may reduce effort. The idea is that monitoring may harm workers' intrinsic motivation such as self-esteem or self-determination. Frey(1993) tests this hypothesis by experiment and shows some evidence that the crowding out effect exists in the interpersonal relationship and reciprocity. Our argument is not related to this psychological factor.

2 Basic Model

There are two agents A and B, who contest fixed prizes, v^w to the winner and v^L to the loser. The output of agent $i = A$ or B is $q_i = x_i + \epsilon_i$, where x_i indicates each player's effort. A random shock, $\epsilon_i \in (-\infty, \infty)$, is drawn from a known symmetric distribution with mean 0 and variance σ. ϵ_A and ϵ_B are $i.i.d.$.[3] The sources of uncertainty are various from agents' luck to the designer's measurement error.

The winner of this tournament is the agent who produces more outputs than the other. The probability for the agent i to win can be written as

$$\Pr(q_i > q_j) = \Pr(x_i - x_j > \epsilon_j - \epsilon_i = \epsilon) = G(x_i - x_j),$$

where $G(\cdot)$ is the symmetric distribution of $\epsilon \equiv (\epsilon_j - \epsilon_i)$. Correspondingly, the probability for the agent i to lose is $[1 - G(x_i - x_i)]$. We further assume $g(\epsilon)$ is unimodal and symmetric: $g'(\epsilon) \gtreqless 0$ for $\epsilon \lesseqgtr 0$. This fairly general assumption makes the second-order condition to be satisfied in our model.

We define

$$\theta = x_A - x_B$$

as the difference in outputs between agent A and B. From now on, we call θ the output gap. Agent i has a cost function, $C(x_i) = \frac{\alpha_i}{2}x_i^2$, whose marginal cost is $C'(x_i) = \alpha_i x_i$. Without loss of generality, we assume $\alpha_A < \alpha_B$. We will refer to $1/\alpha_i$ as each agent's ability. α_i is known to both agents, but the designer does not know which agent has the higher ability than the other.

Then, agent A's maximization problem can be written as

$$\underset{x_A}{Max} \qquad G(\theta)v^w + [1 - G(\theta)]v^l - \frac{\alpha_i}{2}x_A^2.$$

The first-order condition is $(v^w - v^l)g(\theta) - \alpha_A x_A = 0$. Similarly, the first-order condition for agent B is $(v^w - v^l)g(\theta) - \alpha_B x_B = 0$. The contestants' incentives to exert is increasing in $g(\theta)$, which is the marginal winning probability that a contestant can raise by extra effort.

Combining the two first-order conditions, we obtain the following condition that holds in equilibrium.

$$g(\theta^*) = \frac{\alpha_A \alpha_B}{(\alpha_B - \alpha_A)} \frac{\theta^*}{\Delta v} \quad \text{where } \theta^* = x_A^* - x_B^* \text{ and } \Delta v = v^w - v^l. \tag{1}$$

This condition shows that the output gap is increasing in the difference of contestants' abilities and the size of the prize. The total expected output is

$$E\left[q_A^* + q_B^*\right] = x_A^* + x_B^* = \frac{(\alpha_B - \alpha_A)}{\alpha_A \alpha_B} \Delta v g(\theta^*) \tag{2}$$

[3] We assume σ should be large enough to have the existence of an equilibrium and to fulfill each agent's participation constraint.

This is increasing in agents' abilities and the prize, and is decreasing in the marginal cost. In particular, in equilibrium, the total expected output depends crucially on the marginal winning probability, $g(\theta^*)$.[4] This can be thought of as competition intensity in equilibrium. The higher the density at θ^*, more effort the contestants exert.

Proposition 1 *In equilibrium, the total output is increasing in* $g(\theta^*)$.

3 Monitoring and Moral Hazard

We consider two monitoring technologies that are represented by two possible distribution functions with the same mean, but with different variances: $G(\epsilon, \sigma_h)$ and $G(\epsilon, \sigma_l)$ such that $\int \epsilon g(\epsilon, \sigma_l) d\epsilon = \int \epsilon g(\epsilon, \sigma_h) d\epsilon$ and $\int G(\epsilon, \sigma_l) d\epsilon \leq \int G(\epsilon, \sigma_h) d\epsilon$. In words, $G(\epsilon, \sigma_h)$ is a mean-preserving spread (MPS) of $G(\epsilon, \sigma_l)$. The MPS in this model implies that the contest designer has more uncertainty in monitoring, that is, a less efficient monitoring technology.

We further assume the *Single Crossing property*: $g'(\epsilon, \sigma_l) > g'(\epsilon, \sigma_h)$ for $\epsilon \in (-\infty, 0)$ and $g'(\epsilon, \sigma_l) < g'(\epsilon, \sigma_h)$ for $\epsilon \in (0, \infty)$. This assumption ensures that the MPS moves the probability mass from the center toward both tails smoothly so that two distribution functions must cross only once at 0.[5] The assumption also guarantees that two density functions cross once in each positive and negative region.[6] We define this unique crossing point as $\overline{\theta}$ and $-\overline{\theta}$, where $\overline{\theta} > 0$ such that $g(\overline{\theta}, \sigma_l) = g(\overline{\theta}, \sigma_h) = g(-\overline{\theta}, \sigma_l) = g(-\overline{\theta}, \sigma_h)$. To economize on notation, we often denote a representative distribution and density function by $G(\cdot)$ and $g(\cdot)$ respectively. To focus attention on the effect of monitoring, we abstract from any cost the principal might incur in changing monitoring technology.

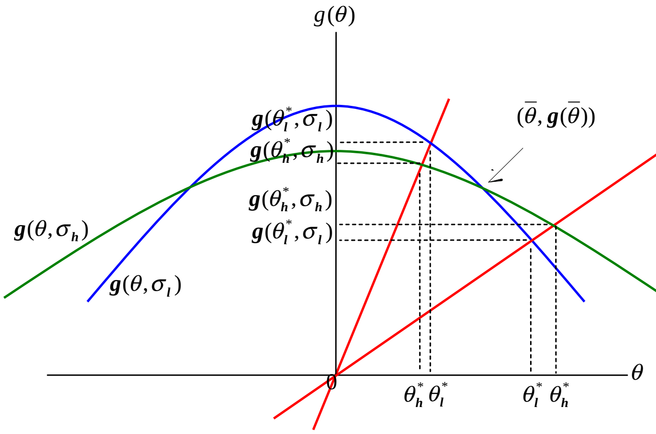
As a benchmark, let us begin by studying symmetric agents: $\alpha_A = \alpha_B = \alpha$. From (1), the level of symmetric equilibrium efforts is $x_A^* = x_B^* = x^* = \Delta v g(0)/\alpha$. In this case, the output gap is zero. Since $g(0, \sigma_l) > g(0, \sigma_h)$, competition is always more intense under less uncertainty or the better monitoring technology. This result is well consistent with common sense in that the agents are forced to work harder under more intensive

[4] Using the equilibrium condition (1), the expected total output can be rewritten as $x_A^* + x_B^* = \theta^*$.This shows that the total output is increasing in θ^* as well.

[5] In other words, $G(\epsilon, \sigma_l)$ has first-order stochastic dominance over $G(\epsilon, \sigma_h)$ for $\epsilon \in (-\infty, 0]$, while $G(\epsilon, \sigma_h)$ does it over $G(\epsilon, \sigma_l)$ for $\epsilon \in [0, \infty)$.

[6] Again, this assumption is fairly general in that most known distribution functions satisfy this property. For example, for a Normal distribution with the density function $g(\theta) = \frac{1}{\sqrt{2\pi\sigma_k}} \exp(-\frac{\theta^2}{2\sigma_k})$ where $k = l$ or h, we obtain $\overline{\theta} = \ln \frac{\sigma_h}{\sigma l} / (\frac{\sigma_h^2 - \sigma_l^2}{2\sigma_h^2 \sigma_l^2})$.

monitoring.



Equilibrium Effort

However, this result can be dramatically changed if agents are asymmetric in their abilities. Let us denote θ^*_k the output gap under the distribution function with the variance σ_k, where $k = l$ or h. Now we compare marginal winning probability under two different monitoring technologies. As illustrated in Figure 1, we obtain $g(\theta^*_l, \sigma_l) \gtreqless g(\theta^*_h, \sigma_h)$ according to $\theta^*_k \lesseqgtr \overline{\theta}$. The margnal winning probability can be greater under less monitoring when the output gap is large enough. Thus, the total output is also greater under less monitoring if the agents' abilities are different enough.

Proposition 2 *If the difference of agents' abilities are large enough, less monitoring increases effort.*

$$As\ (1/\alpha_A - 1/\alpha_B) \gtreqless \frac{\overline{\theta}}{\Delta v g(\overline{\theta})},\ g(\theta^*_l) \lesseqgtr g(\theta^*_h).$$

The intuition behind this result is as follows. When monitoring is less perfect, the high ability agent works harder to reduce the possibility that the designer makes a mistake in her decision of announcing the winner. On the other hand, the low-ability agent also works harder to make use of a higher likelihood of the designer's possible mistake. This is the reason why less monitoring induce more effort.[7] It is also worthwhile to note that the low ability agent's winning probability is greater under less monitoring in equilibrium:

[7] Our model can also explain how the principal would assign the tasks between agents with a slight modification. Consider $q_i = x_i + (1+\rho)\epsilon_i$. Here ρ captures a potential correlation between both agents' outputs facing a common shock. Suppose that the principal can choose the tasks with various correlations. We denote such different situations with ρ_h and ρ_l respectively where $\rho_h > \rho_l$. We obtain $Var(\epsilon) = \sigma_\epsilon = 2(1-\rho_k)^2\sigma$, $k = h\ or\ l$. Given the same monitoring technology, the variance can differ only in the correlation of tasks that the agents perform. As a result, assigning one of the two tasks is equivalent to whether to monitor the agents' effort more or less. The principal assigns a more (less) positively correlated tasks to the agents, if the difference of their abilities is small (large) enough.

$G_l(\theta_l^*) > G_h(\theta_h^*)$. In this sense, less monitoring turns out to be a way of favoring the low-ability agent.[8]

The proposition shows the trade-off between more monitoring and less monitoring. If a continuum of monitoring technology is available to choose, the designer certainly chooses the optimal level of monitoring. In particular, under a Normal distribution, we can find a unique monitoring level.

Proposition 3 *The optimal monitoring level is $\sigma^* = \theta^*$ under the Normal distribution.*

Given $g(\theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{\theta^2}{2\sigma})$, Proposition 1 allows us to find the optimal monotoring technology only by looking at the marginal winning probability. The desinger wants to choose the optimal σ satisfying $\partial g(\theta^*)/\partial\sigma = 0$, which is given by $\frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{(\theta^*)^2}{2\sigma})[(\theta^*)^2/\sigma - 1] = 0$. From equation (1), σ^* is increasing in the difference of agents' abilities and the size of the prize.

4 Conclusion

Moral hazard arises because monitoring is not perfect. However, counter-intuitively, we show that less monitoring can increase agents' efforts in tournaments when agents are heterogeneous in their abilities. While we have restricted our attention to the case where the prize is fixed, we believe that the main result and intuition will be extended to a general case.

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[8] This result is reminiscent of Bayes et al.(1993) and Che and Gale (1998) among many others. They study the exclusion of contestants and the effect of caps on bidding respectively. A common feature among these papers is that disfavoring a high-valuation agent can increase the total effort. Similarly, imperfect monitoring plays a role of disfavoring a high-ability agent and thereby increasing the intensity of competition.

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