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### Guilt causes equal or unequal division in alternating-offer bargaining

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#### Abstract

Parties in a bargaining situation may perceive guilt, a utility loss caused by receiving the larger share that is modeled in some social preferences. I extend Rubinstein (1982)'s solution of the open-ended alternating-offer bargaining problem for self-interested bargainers to a game with equally patient bargainers that exhibit a similar degree of guilt. The bargaining parties still reach agreement in the first period. If guilt is strong, they split the bargaining surplus equally. In contrast, if guilt is weak, the bargaining outcome is tilted away from the Rubinstein division towards a more unequal split. As both bargainers' sensation of guilt diminishes, the bargaining outcome converges to the Rubinstein division.

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## 1. Introduction

In bargaining encounters, a mutually beneficial outcome can be realized if the parties participating in the bargaining process reach agreement. Rubinstein (1982) proposed a seminal framework to investigate open-ended alternating-offer bargaining situations. The framework revealed the interdependence of parties' intertemporal strategic considerations, and it suggested an explicit solution for purely self-interested parties which depicts a focal division from the set of all possible pareto-efficient agreements. Supplementary, this note studies the additional impact of a psychological element in the utility, suggested, for instance, by Von Neumann and Morgenstern (1944), on the bargaining process and outcome. I extend Rubinstein's solution of the alternating-offer bargaining problem to similar, guilt-perceiving parties that, to some degree, dislike receiving a larger share. Guilt is modeled as an asymmetric form of inequality aversion and affects the bargainers' attitudes towards disagreement.

The relative strength of guilt in comparison to bargaining parties' self-interest impacts the bargaining outcome in two ways: High guilt triggers an equal division because the bargainers' utility decreases when receiving more than half. Low guilt diminishes the marginal utility of income, but preserves its positive marginal utility. Own low guilt, *ceteris paribus*, weakens the own bargaining position. Yet, the same degree of low guilt in both bargainers, overall, helps the proposing bargainer to take a larger share than predicted by Rubinstein. The paradox as to why feeling guilty about a larger share can result in a more unequal division than the bargaining of self-interested parties is driven by the weakened bargaining position of the disadvantaged bargainer. The disadvantaged bargainer compares accepting a share smaller than half to proposing a share larger than half in the subsequent period, the utility of which is diminished by guilt. As low guilt maintains a positive marginal utility of income, the proposing bargainer exploits the lowered value of the accepting bargainer's outside option by increasing his demand.

In alternating-offer bargaining, low guilt has the opposite effects of envy which are studied by Kohler (2013a). Envy reinforces the bargaining position of a bargainer. If the two bargainers are similarly envious, then the bargaining outcome departs from the Rubinstein division converging toward an equal split. The influence of envy and guilt on open-ended alternating offer bargaining is studied by Kohler (2013b). This note is a building block of the latter, more general model.

Section 2 introduces the alternating-offer bargaining problem with guilt-perceiving parties. In section 3, I derive the bargaining outcome. Section 4 concludes.

## 2. Bargaining model

Two bargainers  $i, j \in \{b, s\}$ , called seller and buyer, have to reach an agreement on the partition of a surplus of size one which depreciates after any disagreement. Bargaining takes place at periods of time  $t = 1, 2, \dots, T$ . Depreciation is modeled by assigning a common discount factor  $\delta = \delta_s \equiv \delta_b \in [0, 1)$  to the two bargainers. By naming a partition  $p_t \in (0, 1]$  in odd periods, the seller demands share  $p_t$  and offers share  $(1 - p_t)$  that the buyer can accept or reject. In even periods, the buyer proposes a partition  $p_t$  to the seller that he can accept or reject. If a partition is accepted the game ends in period  $T$ . This

bargaining outcome is denoted  $(p_T, T)$ .

Assuming complete information in this bargaining problem, Rubinstein (1982) has shown the existence of a unique subgame perfect equilibrium (SPE) under generic preference assumptions.<sup>1</sup> For preferences  $u_i(x_i) = x_i$ , where utility is derived from own payoff  $x_i$ , Rubinstein derived an explicit solution, in which the seller proposes and the buyer accepts partition  $p^* = \frac{1}{1+\delta} \in (0, 1]$  in period 1. This equilibrium outcome is supported by the bargainers' similar strategies: Bargainer  $i$  always demands the equilibrium share  $p^*$ , when it his turn to make a proposal, otherwise accepts any share equal or greater than  $\delta p^*$  and refuses any smaller share. The demand of  $p^*$  is the highest share that is accepted by the other bargainer  $j$ . Bargainer  $i$  cannot gain by asking a lower share, for it too will be accepted. Stipulating a higher (and rejected) share and waiting to accept bargainer  $j$ 's counteroffer in the next period hurts bargainer  $i$  as  $\delta(1 - p^*) = \delta^2 p^* < p^*$ .

I build on Rubinstein's framework and investigate the strategic behavior of bargainers who care, to some extend, about relative as well as absolute payoff in the described bargaining process. Relative payoff hereby means bargainers compare their own benefit  $x_i$  from accepting a certain partition to the benefit of the other bargainer  $x_j$ , and put weight put a common weight  $\beta = \beta_s \equiv \beta_b \in [0, 1)$  on the difference whenever the own benefit is higher. This relative concern is interpreted as guilt. Explicitly, I assume that the utility function of the bargainers is given by:

$$u_i(x_i, x_j) = x_i - \beta_i \max\{x_i - x_j, 0\}$$

These preferences are an asymmetric version of inequality aversion as originally put forward in Fehr and Schmidt (1999) and extended by altruism in Kohler (2011). Inequality aversion consistently predicts a rich set of stylized experimental behavior (e.g., Cooper and Kagel n.d.; Fehr and Schmidt 1999). The asymmetric guilt preferences violate some of Rubinstein's preference assumptions, but a unique bargaining outcome continues to exist if  $\beta \neq 0.5$ . Throughout,  $u_s(p_t) := u_s(p_t, 1 - p_t)$  denotes the seller's utility and  $u_b(p_t) := u_b(1 - p_t, p_t)$  the corresponding buyer's utility if a proposed partition  $p_t$  is accepted in period  $t$ .

### 3. Subgame perfect equilibrium

**Proposition 1.** *The alternating-offer bargaining problem with guilt-perceiving discounting bargainers has a unique SPE if  $\beta \neq 0.5$ . The seller immediately proposes the partition  $p^* = \frac{1-\beta\delta}{1+\delta(1-2\beta)}$  if guilt is low, i.e.,  $\beta < 0.5$ , or the partition  $p^* = 0.5$  that divides the surplus equally if guilt is high, i.e.,  $\beta > 0.5$ . The respective partition is accepted immediately.*

<sup>1</sup>(i) 'pie' is desirable, (ii) 'time' is valuable, (iii) continuity, (vi) stationarity, i.e., the preference of  $(\hat{p}, t)$  over  $(\hat{p}, t + 1)$  is independent of  $t$ , and (v) the larger the portion, the more 'compensation' a player needs for a delay of one period to be immaterial to him. Strategies are said to constitute a SPE if, in every subgame, the strategies relating to that subgame form a Nash equilibrium. In a SPE, a bargainer will agree to a proposal if it offers at least as much as he will obtain in the future given the strategies of both bargainers. Rubinstein (1982) states the precise definitions.

The proof of proposition 1 is divided in two parts. For low guilt  $\beta < 0.5$ , the first part of the proof is based on Shaked and Sutton (1984) who applied backwards induction in a truncation of the infinite horizon game: The beginning of the infinite horizon game is equal to its subgame in the third round, should it be reached. In odd periods, the seller is proposing and then bargainers alternate in making subsequent offers until an agreement is reached. For high guilt  $\beta > 0.5$ , the argument of second part of the proof is based on the negative marginal utility of any own payoff higher than the opponent's payoff.

*Proof.* For  $\beta < 0.5$ , I assume that the proposing bargainer can always claim the larger share on the equilibrium path, i.e.,  $p_1 \geq 0.5$ ,  $p_2 \leq 0.5$ ,  $p_3 \geq 0.5$ . This assumption is shown to be valid after deriving the equilibrium outcome.

The period 3 subgame begins with a successful proposal  $p_3 \in [0.5, 1]$  by a guilt-perceiving seller. Consequently, the lowest share  $p_2 = \delta(p_3 + \beta_s(1 - 2p_3))$  that is accepted by the seller in period 2 gives him the equivalent of his outside option, the discounted period 3 utility. Similarly, the highest share  $p_1 = 1 - \delta((1 - p_2) - \beta_b(1 - 2p_2))$  that is accepted by the buyer in period 1 gives him the equivalent of his outside option, the discounted period 2 utility. Indifferent bargainers are assumed to accept the proposed partition. As  $u_b(p_2) \geq \delta u_b(p_3)$  and  $u_s(p_1) \geq \delta u_s(p_2)$ , the buyer and seller prefer proposing the agreeable partitions that maximize their utility to disagreement with the subsequent counteroffer.

Since the game in period 3 is identical to the game in period 1, the unique fixed point  $p^* := p_1(p_3) \equiv p_3$  defines the equilibrium partition:

$$p^* = \frac{1 - \delta((1 - \delta\beta_s) - \beta_b(1 - 2\delta\beta_s))}{1 - \delta(\delta(1 - 2\beta_s) - 2\delta\beta_b(1 - 2\beta_s))} = \frac{1 - \beta\delta}{1 + \delta(1 - 2\beta)}$$

As only  $p_2$  and  $p_1$  maximize the utility of the bargainer proposing the partition, there is no other SPE.

The partial derivatives of the equilibrium partition  $p^*$  with respect to guilt  $\partial p^* / \partial \beta \geq 0$  and the discount factor  $\partial p^* / \partial \delta < 0$  are weakly positive and negative, respectively. Hence, the infimum of  $p^*$  is  $\lim_{(\beta, \delta) \rightarrow (0, 1)} p^*$  which evaluates to 0.5. Similarly, as  $\partial p_2 / \partial \beta \leq 0$  and  $\partial p_2 / \partial \delta > 0$ , the supremum of  $p_2(p^*)$  is  $\lim_{(\beta, \delta) \rightarrow (0, 1)} p_2$  which also evaluates to 0.5. Therefore, the assumed advantage of the proposing bargainer to receive half or more of the surplus on the equilibrium path is true. The partial derivatives are derived in the appendix.

If  $\beta > 0.5$ , then  $u_s(0.5) > u_s(0.5 + \epsilon)$  for  $\epsilon \in (0, 0.5]$ . Thus, the seller prefers the equal division to any advantageous share in a period  $t$ , in which he could realize  $p_t \geq 0.5$ . Furthermore, as  $u_b(0.5) > u_b(0.5 + \epsilon)$  for  $\epsilon \in (0, 0.5]$  any such redistribution of the payoff is also preferred by the buyer. Similar arguments apply if the buyer could realize an advantageous share. If  $p_t \leq 0.5$  in a period  $t$ , then the buyer prefers the equal division to any advantageous share, and also the seller prefers the equal division to any disadvantageous share. Thus,  $p^* = p_t = 0.5$  is the only SPE partition. As bargainers discount, it is immediately asked and agreed.  $\square$

#### 4. Conclusion

Bargainers may incur guilt, a loss of utility if receiving an advantageous share of a surplus to be divided, in a bargaining process. In open-ended alternating offer bargaining between two parties with similar time and inequality preferences, strongly guilt-perceiving bargainers gain utility from reducing an advantageous situation until the inequality between the bargainers is eliminated. Therefore, in the presence of sufficient guilt, the unique bargaining outcome is the immediate acceptance of an equal division.

In contrast, bilateral low guilt materially benefits the proposing bargainer. If the bargaining parties perceive guilt only to such an extent that their utility remains increasing in the own payoff despite increasing inequality, then the impact of guilt results in a more unequal division than predicted by Rubinstein (1982) for purely self-interest bargainers. In spite of the first mover's material gain in the equilibrium, low guilt diminishes the utility of both bargainers in comparison to the utility that purely self-interested parties derive from their more equal bargaining outcome (see appendix).

For a low strength of guilt, the partial derivatives of the equilibrium partition imply that the first mover's share is increasing in the common strength of guilt. On its own, the first and the second mover's guilt have opposite effects on the equilibrium division. The share of each bargainer decreases in the strength of the own guilt. Like in Rubinstein (1982)'s solution without guilt, agreement is immediate and the higher a common discount factor, the more equal will be the agreed division. Taking the limits of the equilibrium partition for low guilt shows that the equilibrium partition is between the equal division and one.

Notwithstanding the opposite directions of the effects of high and low guilt on the alternating-offer bargaining outcome, the bargaining of symmetrically guilt-perceiving bargainers is never equivalent to the bargaining of impatient purely self-interested parties.

## Appendix

### Partial derivatives and limits for low guilt

If  $\beta < 0.5$ , the equilibrium partition  $p^*$  and its partial derivatives are given by:

$$p^* = \frac{1 - \delta((1 - \delta\beta_s) - \beta_b(1 - 2\delta\beta_s))}{1 - \delta(\delta(1 - 2\beta_s) - 2\delta\beta_b(1 - 2\beta_s))} = \frac{1 - \beta\delta}{1 + \delta(1 - 2\beta)} =: \frac{N}{D}$$

$$\frac{\partial p^*}{\partial \delta} = -(1 - \beta) \cdot D(\beta, \delta)^{-2} < 0$$

$$\frac{\partial p^*}{\partial \beta} = \delta(1 - \delta) \cdot D(\beta, \delta)^{-2} \geq 0$$

$$\frac{\partial p^*}{\partial \beta_s} = -\delta^2(1 - \delta)(1 - 2\beta_b)(1 - \delta + 2\delta\beta_b) \cdot D(\beta_s, \beta_b, \delta)^{-2} \leq 0$$

$$\frac{\partial p^*}{\partial \beta_b} = \delta(1 - \delta)(1 - \delta + 2\delta\beta_s) \cdot D(\beta_s, \beta_b, \delta)^{-2} \geq 0$$

The respective signs follow from evaluating the derivatives. The limits of the equilibrium partition  $p^*$  are given by  $\lim_{\delta \rightarrow 0} p^* = 1$ ,  $\lim_{\delta \rightarrow 1} p^* = 0.5$  and  $\lim_{\beta \rightarrow 0} p^* = \frac{1}{1+\delta} \in (0.5, 1]$ ,  $\lim_{\beta \rightarrow 0.5} p^* = 1 - 0.5\delta \in (0.5, 1]$ . The limit values follow from evaluating the limits.

If  $\beta < 0.5$ , the partition  $p_2$  and its partial derivatives are given by:

$$p_2(p^*) = \delta \frac{1 - \beta}{1 + \delta(1 - 2\beta)} =: \frac{N}{D}$$

$$\frac{\partial p_2}{\partial \delta} = (1 - \beta) \cdot D^{-2} > 0$$

$$\frac{\partial p_2}{\partial \beta} = -\delta(1 - \delta) \cdot D^{-2} \leq 0$$

The respective signs follow from evaluating the derivatives. The limits of the partition  $p_2$  are given by  $\lim_{\delta \rightarrow 0} p_2 = 0$ ,  $\lim_{\delta \rightarrow 1} p_2 = 0.5$  and  $\lim_{\beta \rightarrow 0} p_2 = \frac{\delta}{1+\delta} \in [0, 0.5)$ ,  $\lim_{\beta \rightarrow \infty} p_2 = 0.5$ . The limit values follow from evaluating the limits.

### Utility in the subgame perfect equilibrium with guilt

Irrespective of the strength of guilt, the utility of a guilt-perceiving seller, who perceives guilt in the equilibrium, is not higher than the utility of a purely self-interested seller that receives  $\frac{1}{1+\delta}$ :

$$\frac{u_s(p^*)}{u_s\left(\frac{1}{1+\delta}\right)\Big|_{\beta=0}} = \begin{cases} \frac{(1-\beta)(1+\delta)}{1+\delta(1-2\beta)} \in (0.5, 1] & \text{if } \beta < 0.5 \\ \frac{1+\delta}{2} \in [0.5, 1) & \text{if } \beta > 0.5 \end{cases}$$

The utility of a guilt-perceiving buyer, whose utility in equilibrium is unaffected by guilt, is not lower than the utility of a purely self-interested buyer if guilt is low and higher if guilt is high:

$$\frac{u_b(p^*)}{u_b\left(\frac{1}{1+\delta}\right)\Big|_{\beta=0}} = \begin{cases} \frac{(1-\beta)(1+\delta)}{1+\delta(1-2\beta)} \in (0.5, 1] & \text{if } \beta < 0.5 \\ \frac{1+\delta}{2\delta} \in (1, \infty] & \text{if } \beta > 0.5 \end{cases}$$

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