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A note on evaluating freedom of opinion

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Abstract

Following the recent generalization of social choice in the literature on judgment aggregation, we extend the analysis of freedom of choice from sets of alternatives to sets of opinions. We establish the analogue of the cardinality based freedom of choice measure and suggest an alternative measure based on the Hamming distance.

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1 Introduction and motivation

The importance of freedom of choice has long been recognized in social choice theory (for surveys see Foster 2011 and Dowding and van Hees 2009). Interestingly, while the recent literature on judgment aggregation has greatly generalized the informational framework of social choice theory by extending it from alternatives and preferences to judgments (for a survey see List and Puppe 2009), there is, to the best of our knowledge, no extension of the analyis of freedom of choice to the problem of measuring the freedom of opinion contained in different sets of judgments.[1]

As an example consider a society in which the following issues are highly debated:

p: Women are discriminated against.

q: Income inequality is growing.

r: Marihuana should be legalized.

Clearly, a broadcasting station in which the opinions expressed by commentators corresponded to the judgment sets $\{p, q, r\}$ and $\{\neg p, \neg q, \neg r\}$ would offer more freedom of opinion than one in which only the two opinions $\{p, q, r\}$ and $\{p, q, \neg r\}$ would be expressed.

Freedom of opinion is maybe one of the strongest cases in point for the significance of a nonwelfaristic but freedom based evaluation of human wellbeing, because it can be considered not only as instrumental (e.g. via the "market place of ideas") but also as constitutive of human development (see Sen 1999 and Nussbaum 2000).

This paper is a preliminary step into the formal analysis of freedom of opinion.

2 Formal framework

Following the representation of judgments by binary valuations on a set of propositions in the literature on abstract aggregation theory[2], we model an opinion as a valuation of a set of issues consisting of n (unnegated) propo-

[1]Of course, the significance of the freedom of choice literature for the issue of freedom of opinion was already hinted at in Klemisch-Ahlert 1993, p. 197.

[2]For early algebraic generalizations of preference aggregation see Wilson 1975 and Rubinstein and Fishburn 1986, for a contemporary approach see especially the work of Dokow and Holzman, e.g. 2010.

sitions. Let thus $X := \{0,1\}^n$ be the set of valuations[3], where any such valuation $x = (x_1, x_2, ..., x_n) \in X$ is a vertex in the hypercube $\{0,1\}^n$. Define by $F : 2^X \backslash \varnothing \to R_+$ the freedom measure that assigns to any set of opinions $S \in 2^X \backslash \varnothing$ its freedom value $F(S)$. Like in the analysis of freedom of choice, such a freedom measure can be used to rank sets of valuations according to the freedom of opinion they convey.

Definition 1 *A freedom ordering $\succsim_F \subset 2^X \backslash \varnothing \times 2^X \backslash \varnothing$ on sets of opinions is generated by a freedom measure $F : 2^X \backslash \varnothing \to R_+$ iff for all sets of opinions $S, T \in 2^X \backslash \varnothing$, $S \succsim_F T$ iff $F(S) \geq F(T)$.*

In the spirit of axioms used in the literature of freedom of choice, the following axioms can be formulated for our purpose.

Definition 2 (no choice) *For all opinions $x \in X$, $F(\{x\}) = 0$.*

Definition 3 (normalized minimal positive freedom) *For all sets of opinions $S \in 2^X \backslash \varnothing$, $F(S) \geq 1$ iff $|S| \geq 2$, with $\min_{S \in 2^X \backslash \varnothing : |S| \geq 2} F(S) = 1$.*

Clearly, if only **one** opinion can be expressed, there is **no** freedom of opinion. Whereas, if there are at least two different opinions, there is at least a minimal strictly positive amount of freedom, which can be set to one without loss of generality. In fact, while the first axiom expresses the intuition underlying the axiom of "indifference between no choice situations", the second axiom expresses the intution of "strict monotonicity" in the literature on freedom of choice, strict monotonicity being in fact a local and marginal condition requiring that adding a second alternative to a single one always increases freedom, or more formally, that the freedom value contained in the doubleton superset of a singleton be always higher than the freedom value of that singleton.

Definition 4 ***(Independence)*** *For all sets of opinion $S, T \in 2^X \backslash \varnothing$ and all $x \in X \backslash (S \cup T)$, $F(S) - F(T) = F(S \cup \{x\}) - F(T \cup \{x\})$.*

[3]Beware that the set of valuations could also be restricted in case the propositions are logically interconnected. This would, however, change the framework of our analysis, and hence - in this first approach - we stick to the simpler case in which such logical interconnections do not occur, as, e.g., in the example of the introduction.

This axiom in the spirit of the independence axioms in the freedom of choice literature captures the intuition that adding the same additional opinion does not change the difference in freedom of opinion between two sets of opinions.

The reference measure of freedom of choice in the literature on freedom is a cardinality based measure which translates in our framework in the following definition:

Definition 5 *The cardinality based freedom measure F^C is defined as $F^C(S) = |S| - 1$ for all sets of opinions $S \in 2^X \backslash \varnothing$.*

As the corresponding freedom ordering was characterized by Pattanaik and Xu (1990) in terms of axioms similar to the above, the following characterization result comes as no wonder:

Theorem 6 *A freedom measure F is the cardinality based freedom measure F^C iff F satisfies no choice, normalized minimal positive freedom and independence.*

Proof. For the necessity part, F^C obviously satisfies the above axioms. To show sufficiency, assume any freedom measure F. From no choice, $F(\{x\}) = 0$ for all $x \in X$. Now take any $x, y, z \in X$. From independence we get:

$$F(\{x,y\}) - F(\{x\}) = F(\{x,y,z\}) - F(\{x,z\}) \tag{1}$$

and

$$F(\{y,z\}) - F(\{z\}) = F(\{x,y,z\}) - F(\{x,z\}) \tag{2}$$

As $F(\{x\}) = F(\{z\}) = 0$ by no choice, (1) and (2) imply that $F(\{x,y\}) = F(\{y,z\})$. By the same argument and from normalized minimal positive freedom we conclude that $F(S) = t \geq 1$ for all S such that $|S| = 2$. Independence then implies that $F(\{x,y,z\}) > F(\{x,y\})$ and - more generally - that $F(S) > F(T)$ whenever $|S| > |T|$. Hence, by normalized minimal positive freedom, for all $S \in 2^X \backslash \varnothing$ such that $|S| = 2$, $F(S) = 1$.

Now use induction. Suppose for $|S| = k$, $F(S) = k - 1$, then for all $x \in X \backslash S$, $F(S \cup \{x\}) = k$. Take any $y, z \in S$ and $x \in X \backslash S$. From consecutive use of independence, $F(S \cup \{x\}) - F(S) = F(\{y,z\}) - F(\{z\}) = 1$ and hence $F(S \cup \{x\}) = k$. ■

It is easily seen, that the cardinality measure F^C while based on a local and marginal condition of monotonicity, implies a much stronger global one that does not only compare singletons to their doubleton supersets.

Definition 7 *A freedom measure F is strongly monotonic if for all sets of opinions $S, T \subset X$, $F(S) > F(T)$ whenever $|S| > |T|$.*

The latter strong monotonicity condition might however be too strong, as it does not take into account the diversity within the respective sets of opinions. Thus, e.g., the set of opinions $S = \{(1,0,0,0),(1,1,0,0),(1,1,1,0,)\}$ which is entirely contained in a two-dimensional subcube of $X = \{0,1\}^4$ (such that half of the issues are fixed) would be granted a higher freedom value than the set of opinions $T = \{(1,0,0,0),(0,1,1,1)\}$ which allows the expression of opposite valuations on every issue.[4] This objection is in line with one of the reasons why the cardinality measure has always been criticized for ignoring the dissimilarity of the alternatives in different choice menus (see e.g. Dowding and van Hees 2009, p. 378ff).[5] This criticism is particularly relevant in the context of freedom of opinion where, in the absence of preference information, the diversity criterion becomes even more central than in the case of freedom of choice.

This concern is addressed in the following axiom, which extends the local and marginal perspective of strict monotonicity by guaranteeing that any additional opinion adds to the freedom of opinion by adding freedom to any single opinion already contained in the set of opinions.

Definition 8 (marginalism) *For all sets of opinions $S \in 2^X \backslash \varnothing$ and opinions $x \in X \backslash S$, $F(S \cup \{x\}) = F(S) + \sum_{y \in S} F(\{x, y\})$.*

The following two neutrality conditions are technical in nature and should be uncontroversial in most applications, because in the absence of any information on the weights of the different issues, a freedom measure should treat the corresponding propositions and their negations equally:

Definition 9 (issue-wise neutrality) *Let $\pi : \{1,...,n\} \rightarrow \{1,...,n\}$ be a permutation on $\{1,...,n\}$ and denote by $\pi(x)$ the permutation of any opinion $x \in X$, where $\pi(x) = (x_{\pi(1)}, x_{\pi(2)}, ..., x_{\pi(n)})$. Moreover, for all sets of opinions*

[4] As a referee rightly suggested, the number of issues which are not fixed would also capture part of our intuitions of the degree of freedom conveyed by some set of opinions. Unfortunately, this measure is insensitive to an increase in the number of opinions beyond the case of doubleton sets of opinions.

[5] In the context of freedom of choice, this criticism has led to a series of attempts to incorporate the range of choice and the shape of the sets of alternatives (see e.g. Klemisch-Ahlert 1993).

$S \in 2^X \backslash \varnothing$, $\pi(S)$ defines the permutation of S where for every opinion $x \in S$, $\pi(x) \in \pi(S)$. We say that a freedom measure F is issue-wise neutral if for all $S \in 2^X \backslash \varnothing$, $F(S) = F(\pi(S))$.

Definition 10 (acceptance/rejection neutrality) *Let, for each issue $i \in \{1, ..., n\}$, $\rho_i : \{0,1\} \to \{0,1\}$ be a permutation on $\{0,1\}$, and denote by $\rho(x)$ the permutation of any opinion $x \in X$, where $\rho(x) = (\rho_1(x_1), \rho_2(x_2), ..., \rho_n(x_n))$. Moreover, for all sets of opinions $S \in 2^X \backslash \varnothing$, $\rho(S)$ defines the permutation of S where for every opinion $x \in S$, $\rho(x) \in \rho(S)$. We say that a freedom measure F is acceptance/rejection neutral if for all $S \in 2^X \backslash \varnothing$, $F(S) = F(\rho(S))$.*

For convenience, the conjunction of issue-wise and acceptance/rejection neutrality will be simply called neutrality.

Given the representation of the opinions qua binary valuations as vertices in the hypercube $\{0,1\}^n$, it seems natural to only consider shortest paths for the measurement of the freedom of opinion conveyed by any pair of valuations and to additively decompose the freedom measure in the contributions of the pairs of valuations lying on a shortest path.

Let $z^1, z^2, ..., z^q \in X$. We say that $z^1, z^2, ..., z^q$ are on a line iff for every $j \in \{1, ..., q-1\}$ there exists at most one $i \in \{1, ..., n\}$ such that $z_i^j \neq z_i^{j+1}$. If one observes valuations lying on such a shortest path, the freedom derived from the two corresponding opinions on the endpoints can then be considered the sum of the freedom values of any intermediate pairs of opinions.

Definition 11 (addititive decomposability) *For all opinions $x, y, z^1, z^2, ..., z^k \in X$. If $x, z^1, ..., z^k, y$ are on a line, then $F(\{x,y\}) = F(\{x,z^1\}) + F(\{z^1,z^2\}) + ... + F(\{z^k,y\})$.*

Binary valuations are often compared to each other on the basis of a distance measure. The most common such measure is the Hamming distance:

Definition 12 (Hamming distance) *For all opinions $x, y \in X$, the Hamming distance between x and y, $\delta(x,y)$, is defined as $\delta(x,y) = |\{i \in \{1, ..., n\} : x_i \neq y_i\}|$.*

Using the Hamming distance, one can provide an intuitive freedom value for any set of valuations on the basis of the total distance between any pair of valuations within the set. The value is defined as follows:

Definition 13 (freedom value F^δ) *For all sets of opinions $S \in 2^X \backslash \varnothing$, $F^\delta(S) = \frac{1}{2}\sum_{x,y\in S}\delta(x,y)$.*

Based on the above, we can provide the following theorem:

Theorem 14 *A freedom value F is equivalent to F^δ iff F satisfies no choice, normalized minimal positive freedom, marginalism, neutrality and additive decomposability.*

Proof. (necessity) : F^δ obviously satisfies all the above axioms.
For the proof of sufficiency, consider F satisfying all the above axioms. From no choice $F(\{x\}) = 0$ for all $x \in X$. Now consider only pairs $\{x, y\}$ of valuations. W.l.o.g. let $x, y, z \in X$ be such that $|\{i \in \{1, ..., n\} : x_i \neq y_i\}| < |\{i \in \{1, ..., n\} : x_i \neq z_i\}|$. Obviously, if $|\{i \in \{1, ..., n\} : x_i \neq y_i\}| = k$, y can be reached from x via $k-1$ valuations z^j with $|\{i \in \{1, ..., n\} : x_i \neq z_i^j\}| = 1$ which all are on a line. From additive decomposability, $F(\{x, y\}) = F(\{x, z^1\}) + F(\{z^1, z^2\}) + ... + F(\{z^{k-1}, y\})$. From neutrality, for all x, y, z such that $|\{i \in \{1, ..., n\} : x_i \neq y_i\}| = |\{i \in \{1, ..., n\} : x_i \neq z_i\}| = 1$, i.e., both x and y and x and z are adjacent, we get $F(\{x, y\}) = F(\{x, z\})$. As from additive decomposability, no choice and normalized minimal positive freedom any pair $\{x, y\}$ such that $|\{i \in \{1, ..., n\} : x_i \neq y_i\}| > 1$ also must have larger freedom, we get from normalized minimal positive freedom, that $|\{i \in \{1, ..., n\} : x_i \neq y_i\}| = 1$ implies $F(\{x, y\}) = 1$. Now any pair of valuations can be constructed from pairs of adjacent valuations which are on a line. Hence, $F(\{x, y\}) = |\{i \in \{1, ..., n\} : x_i \neq y_i\}|$ which is exactly the Hamming distance on pairs of valuations, i.e., $F(\{x, y\}) = F^\delta(\{x, y\})$. For any S such that $|S| > 2$, start with any two valuations $x, y \in S$, for which we know that their freedom is equal to $F^\delta(\{x, y\})$. Now, consecutively add valuations from S. From marginalism the freedom added is equal to the sum of pairwise freedom values. As only pairwise Hamming distances are added it follows that $F = F^\delta$. ■

3 Discussion

For the simplest case of the representation of opinions qua binary valuations on a set of propositions as vertices in the hypercube we characterized a measure of freedom of opinion based on the total Hamming distance within a set of opinions. While this measure may be considered natural in this framework, it should not be forgotten that the literature on diversity (for a survey see Nehring and Puppe 2009) has shown that the hypercube itself is far from being a perfect model to capture the diversity inherent in a given set of objects, as it does not provide an obvious way to account for the number of coordinates in which sets of valuations might differ. This means in our case that a set of opinions entirely contained in a lower-dimensional subcube (i.e. where the valuation of more issues is fixed) may convey a higher freedom value than a set in which less issues are fixed. Only in the trivial case of doubleton sets of opinions does the Kemeny distance also reflect the number of issues which are not fixed.

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