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### Managerial Incentive Problems: The Role of Multiple Signals

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#### Abstract

Career concerns can mitigate moral hazard problems, but these diminish as the agent's retirement age approaches. Addressing this problem, this note draws attention to the role of the interaction between contractible and non-contractible signals.

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## 1. Introduction

An open question concerning models of incentives in organizations is whether career concerns alleviate moral hazard problems throughout a person's career (Holmström 1999). The assumption underlying this question is that motivating people in organizations requires explicit formal contracts linking monetary rewards to contractible signals that provide information about their unobservable behavior.

Using dynamic incentive models in which only non-contractible signals were observable, Holmström (1999)<sup>1</sup> argued that under very restrictive conditions, career concerns could be sufficient to police moral hazard problems. Gibbons and Murphy (1992), building on the models of Holmström (1999), replaced non-contractible signals with contractible signals. They demonstrated the feature of diminishing career concerns and the role of contracts that is increasingly prominent as an individual approaches retirement.

This note demonstrates that when both contractible and non-contractible signals are observed, the dynamic career concern effect changes, becoming either weaker or stronger. It further shows the gradual disappearance of career concern effects, which supports the robustness of the results of Gibbons and Murphy (1992). The case of both contractible and non-contractible signals is a natural extension of the existing literature, which is motivated by a general concern about observability that is not restricted either to contractible or non-contractible signals.

## 2. Model

Consider risk-neutral prospective employers (the market) and a risk-neutral agent who has a finite  $N$  horizon. The agent's outputs are the sum of the agent's talent regarding which the common prior belief is a stochastic variable:  $\theta \sim N(0, \frac{1}{v_\theta})$ , the agent's actions:  $a_t \in \mathbb{R}$ , and the noise term:  $\tau_t \sim N(0, \frac{1}{v_\tau})$ :

$$x_t = \theta + ba_t + \tau_t, \quad b \in (0, \infty), \quad t = 1, \dots, N.$$

The outputs  $x_t$  and the actions  $a_t$  are not observable. Observable signals are contractible signals:  $y_t$  and non-contractible signals:  $z_t$ :

$$\begin{aligned} y_t &= \theta + ma_t + \varepsilon_t, \\ z_t &= \theta + pa_t + \nu_t, \quad t = 1, \dots, N. \end{aligned}$$

Their differences lie in the marginal impacts of the agent's actions:  $m \in (0, \infty)$  and  $p \in \mathbb{R}$ , and in noise terms:  $\varepsilon_t \sim N(0, \frac{1}{v_\varepsilon})$  and  $\nu_t \sim N(0, \frac{1}{v_\nu})$ . The agent's talent  $\theta$  and noise terms  $\tau_t$ ,  $\varepsilon_t$  and  $\nu_t$  are independent of each other.

To focus on the impact of the interaction between contractible and non-contractible signals on career concerns, this note assumes that contractible signals  $y_t$  are distorted in the sense that  $b < m$ . In addition, contracts and the agent's personal cost of action are restricted to being linear and quadratic, respectively:

$$\begin{aligned} w_t(y_t) &= \alpha_t + \beta_t y_t, \quad \alpha_t, \beta_t \in \mathbb{R}, \\ c(a_t) &= \frac{1}{2} a_t^2, \quad t = 1, \dots, N. \end{aligned}$$

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<sup>1</sup>Holmström (1999) was originally written in April 1982 for an unpublished volume in honour of the 60th birthday of Professor Lars Wahläck, Rector of the Swedish School of Economics and Business Administration in Helsinki, Finland.

Figure 1 summarizes the sequence of events.

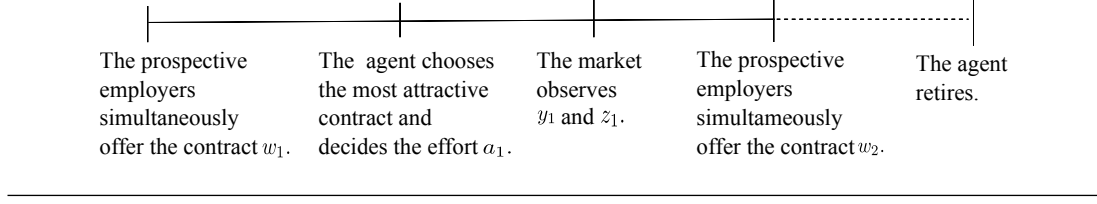


Figure 1: Timing of Events

The equilibrium outcomes in the information structure are summarized as follows:

**Theorem 1.** *Suppose  $b < m$ . In equilibrium, actions and contractual incentives are as follows:*

$$a_t^* = b \text{ for } t = 1, \dots, N, \quad (1)$$

$$\beta_N^* = \frac{b}{m}, \quad (2)$$

$$\beta_t^* = \frac{b}{m} - \sum_{k=t+1}^N \gamma^{k-t} (1 - \beta_k^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(k-1)(v_\varepsilon + v_\nu) + v_\theta} \quad (3)$$

for  $t = 1, \dots, N-1$ ,

where  $\gamma$  is the discount rate.

All proofs are provided in the Appendix.

In the second term on the right-hand side of (3), the equilibrium implicit incentives for period  $t, t = 1, \dots, N-1$  are expressed. This is because the equilibrium contractual incentives are imposed so that the effective incentives (the sum of contractual and implicit incentives) reach the level  $\frac{b}{m}$ , where the first-best action  $b$  is induced. Note that the last period's equilibrium implicit incentive is zero.

### 3. Results

#### 3.1 A weaker career concern effect

Non-contractible signals are not restricted to good signals with the monotone likelihood ratio property (MLRP), whereby higher signals represent better news. Consider the extreme case of  $v_\varepsilon + v_\nu \frac{p}{m} = 0$  where the implicit incentive is zero and  $\beta_t^* = \frac{b}{m}$ ,  $t = 1, \dots, N$ . In this case, bad news conveyed by non-contractible signals cancels out career concern effects throughout the agent's career. Note that the worse case of  $v_\varepsilon + v_\nu \frac{p}{m} < 0$  will be infeasible if the agent does not accept a contract in which the fixed payment is strongly negative.

#### 3.2 A stronger career concern effect

Non-contractible signals can strengthen career concern effects when they are more responsive to actions than contractible signals, i.e., implicit incentives are increasing in  $\frac{p}{m}$ . In addition, if  $\frac{p}{m} > 1$ , the implicit incentives are increasing in the precision of non-contractible signals,  $v_\nu$ . The former instance has a stronger impact. Figure 2 illustrates this effect. The deep drop of contractual incentives in the fourth quadrant occurs because

the coefficients  $(1 - \beta_k^*)$  in the right-hand side of equation (3) expand implicit incentives in the fourth quadrant while shrinking them in the first quadrant.

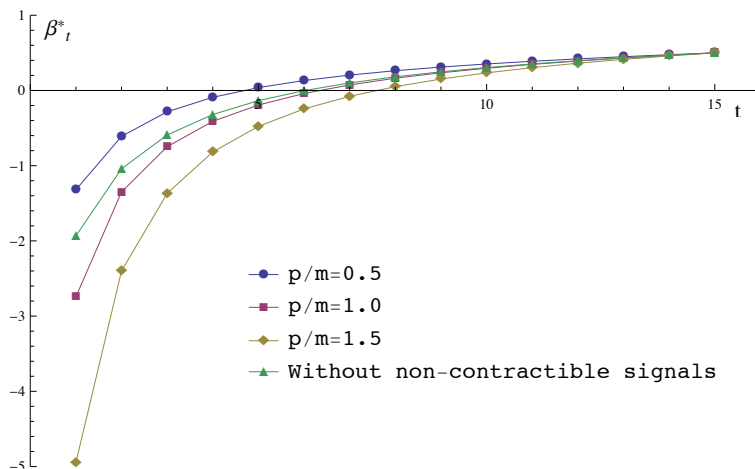


Figure 2: The equilibrium contractual incentives:  $N = 15$ ,  $\gamma = 0.95$ ,  $\frac{b}{m} = 0.5$ ,  $v_\varepsilon^2 = v_\nu^2 = v_\theta^2$ .

### 3.3 A robust property: Diminishing career concerns effects

By a robust property, this note means that in the presence of non-contractible signals, contractual incentives still increase monotonically as they did in the model of Gibbons and Murphy (1992). That is, career concerns diminish as the agent's retirement age approaches.

**Theorem 2.** *Suppose  $b < m$  and  $v_\varepsilon + \frac{p}{m}v_\nu > 0$ . Then, the optimal incentive rates exhibit the property:  $\beta_t^* < \beta_{t+1}^*$  for  $t = 1, \dots, N - 1$ .*

The main cause of this property in this model is the agent's time horizon; stronger career concerns are closely associated with a longer time horizon. Non-contractible signals do not alter this. Hence, contracts are required to deter moral hazard problems.

However, theoretically, career concern effects can be made sufficient to police moral hazards during every period except the final one, by setting a high enough value for  $\frac{p}{m}$ . However, in this case, severe penalty contracts might be seen in equilibrium. If employers refrain from implementing these penalty contracts, the agent continues to focus on developing his or her reputation at the cost of overworking.

## 4. Conclusion

This note has shown that a dynamic perspective on career concern effects can be developed by considering an information structure in which both contractible and non-contractible signals are observable. Further work may explore whether career concern effects are sustained even if the agent's overwork decreases the agent's productivity.

## Appendix

*Proof of Theorem 1.*

The prospective employers' problem is

$$\max_{\beta_t} \hat{E}_{t-1} \left[ \sum_{k=t}^N \gamma^{k-t} \left( w_k - \frac{1}{2} a_k^2 \right) \right], \quad (\text{A-1})$$

subject to the zero-profit constraint of the market:

$$\hat{E}_{t-1} \left[ \sum_{k=t}^N w_k \right] = \hat{E}_{t-1} \left[ \sum_{k=t}^N x_k \right] \quad \text{for } t = 1, \dots, N, \quad (\text{A-2})$$

and the agent's choice of action:

$$a_t^* = \frac{\partial E_{t-1} \left[ \sum_{k=t}^N \gamma^{k-t} w_k \right]}{\partial a_t}, \quad (\text{A-3})$$

where  $E_{t-1}[\cdot] = E[\cdot | y_1, \dots, y_{t-1}, z_1, \dots, z_{t-1}]$ .

Note that  $w_t$  can be expressed as  $w_t = \hat{E}_{t-1}[x_t] + \beta_t(y_t - \hat{E}_{t-1}[y_t])$  (this proof is shown in Šabac 2008, Lemma 1), which implies the equilibrium action in (A-3) is

$$a_t^* = \frac{\partial E_{t-1} \left[ \sum_{k=t}^N \gamma^{k-t} \left( \hat{E}_{k-1}[x_k] + \beta_k(y_k - \hat{E}_{k-1}[y_k]) \right) \right]}{\partial a_t}.$$

Let  $H_{kt}^y$  and  $H_{kt}^z$  be the impact of observed contractible and non-contractible signal history on date  $t$  on conditional expectations of  $y_k$ , with components  $H_{kt}^y = (H_{kt}^{y1}, \dots, H_{kt}^{yt})$  and  $H_{kt}^z = (H_{kt}^{z1}, \dots, H_{kt}^{zt})$ , respectively. Then

$$a_t^* = \sum_{k=t}^N \gamma^{k-t} (1 - \beta_k) (H_{k \ k-1}^{yt} m + H_{k \ k-1}^{zt} p) + m \beta_t. \quad (\text{A-4})$$

Note that the impact of observed signal history on date  $t$  on conditional expectations of  $x_k$  is the same as that of  $y_k$ , because correlation structure is the same in both cases.

On the other hand, the first-order condition with respect to  $\beta_t$  in (A-1) is

$$b \frac{\partial a_t}{\partial \beta_t} - a_t \frac{\partial a_t}{\partial \beta_t} = 0 \quad \text{for } t = 1, \dots, N.$$

But by (A-4)  $\frac{\partial a_t}{\partial \beta_t} = m$ , which implies

$$a_t^* = b \quad \text{for } t = 1, \dots, N,$$

which in turn implies

$$\beta_t^* = \frac{b}{m} - \sum_{k=t+1}^N \gamma^{k-t} (1 - \beta_k) \left( H_{k \ k-1}^{yt} + H_{k \ k-1}^{zt} \frac{p}{m} \right) \quad \text{for } t = 1, \dots, N-1,$$

and  $\beta_N^* = \frac{b}{m}$ . Let  $\delta_t = \theta + \varepsilon_t$  and let  $\eta_t = \theta + \nu_t$ . Then

$$E_{k-1}[\delta_k] = \frac{v_\varepsilon}{(k-1)(v_\varepsilon + v_\nu) + v_\theta}(\delta_1 + \cdots + \delta_{k-1}) + \frac{v_\nu}{(k-1)(v_\nu + v_\varepsilon) + v_\theta}(\eta_1 + \cdots + \eta_{k-1}), \quad (\text{A-5})$$

which implies

$$H_{k-1}^{yt} = \frac{v_\varepsilon}{(k-1)(v_\varepsilon + v_\nu) + v_\theta} \quad \text{and} \quad H_{k-1}^{zt} = \frac{v_\nu}{(k-1)(v_\nu + v_\varepsilon) + v_\theta}$$

for  $1 \leq t \leq k-1$ , and which ultimately implies (3) in Theorem 1.

The proof of (A-5) is obtained by induction on  $k$ . Let  $\zeta_k$  be the normalized random variable:

$$\zeta_k = \left( \frac{\frac{\delta_k - E_{k-1}[\delta_k]}{\sqrt{\text{Var}_{k-1}(\delta_k)}}}{\frac{1}{\sqrt{1-\rho^2}} \left( \frac{\eta_k - E_{k-1}[\eta_k]}{\sqrt{\text{Var}_{k-1}(\eta_k)}} - \rho \frac{\delta_k - E_{k-1}[\delta_k]}{\sqrt{\text{Var}_{k-1}(\delta_k)}} \right)} \right),$$

where  $\rho = \frac{\text{Cov}_{k-1}(\delta_k, \eta_k)}{\sqrt{\text{Var}_{k-1}(\delta_k)\text{Var}_{k-1}(\eta_k)}}$ . Note that  $\zeta_k$  is independent of  $\delta_1, \dots, \delta_{k-1}, \eta_1, \dots, \eta_{k-1}$ , which implies the conditional expectation of  $\delta_{k+1}$  can be expressed as

$$\begin{aligned} E_k[\delta_{k+1}] &= E_{k-1}[\delta_{k+1}] + \text{Cov}(\delta_{k+1}, \zeta_k)\zeta_k \\ &= E_{k-1}[\delta_k] + \text{Cov}(\delta_{k+1}, \zeta_k)\zeta_k. \end{aligned} \quad (\text{A-6})$$

For  $k = 2$ , (A-5) follows from

$$E_1[\delta_2] = \frac{v_\varepsilon}{v_\varepsilon + v_\nu + v_\theta}\delta_1 + \frac{v_\nu}{v_\varepsilon + v_\nu + v_\theta}\eta_1.$$

Suppose that (A-5) holds for  $k$ . By (A-6)

$$\begin{aligned} E_k[\delta_{k+1}] &= E_{k-1}[\delta_k] + \text{Cov}(\delta_{k+1}, \zeta_k)\zeta_k \\ &= E_{k-1}[\delta_k] + \frac{v_\varepsilon}{k(v_\varepsilon + v_\nu) + v_\theta}(\delta_k - E_{k-1}[\delta_k]) \\ &\quad + \frac{v_\nu}{k(v_\varepsilon + v_\nu) + v_\theta}(\eta_k - E_{k-1}[\eta_k]) \\ &= \frac{v_\varepsilon}{k(v_\varepsilon + v_\nu) + v_\theta}(\delta_1 + \cdots + \delta_k) \\ &\quad + \frac{v_\nu}{k(v_\varepsilon + v_\nu) + v_\theta}(\eta_1 + \cdots + \eta_k). \end{aligned}$$

The last equation is because  $E_{k-1}[\delta_k] = E_{k-1}[\eta_k]$ . This proves the induction hypothesis (A-5) and concludes the proof of Theorem 1.  $\square$

*Proof of Theorem 2.*

Consider the following hypothesis on  $k$ ,  $k = 1, \dots, N-1$ .

$$\beta_{N-k}^* \leq \beta_{N-k+1}^* - \gamma^k \left(1 - \frac{b}{m}\right) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(N-1)(v_\varepsilon + v_\nu) + v_\theta}, \quad (\text{A-7})$$

which implies  $\beta_{N-k}^* < \beta_{N-k+1}^*$  for  $k = 1, \dots, N-1$ . The proof is by induction on  $k$ . For  $k = 1$ , by (2) and (3)

$$\beta_{N-1}^* = \beta_N^* - \gamma \left(1 - \frac{b}{m}\right) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(N-1)(v_\varepsilon + v_\nu) + v_\theta}.$$

Suppose that the induction hypothesis (A-7) holds for  $k-1$ . By (2) and (3),  $\beta_{N-k}^*$  is given by

$$\begin{aligned} \beta_{N-k}^* &= \frac{b}{m} - \gamma^k (1 - \beta_N^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(N-1)(v_\varepsilon + v_\nu) + v_\theta} \\ &\quad - \sum_{i=N-k+1}^{N-1} \gamma^{i-(N-k)} (1 - \beta_i^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(i-1)(v_\varepsilon + v_\nu) + v_\theta}. \end{aligned}$$

Let  $A_i = -(1 - \beta_i^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(i-1)(v_\varepsilon + v_\nu) + v_\theta}$ . Note that  $A_i < A_{i+1}$  holds for  $i = N-k+1, \dots, N-1$ , which implies

$$\begin{aligned} \beta_{N-k} &= \frac{b}{m} - \gamma^k (1 - \beta_N^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(N-1)(v_\varepsilon + v_\nu) + v_\theta} + \sum_{i=N-k+1}^{N-1} \gamma^{i-(N-k)} A_i \\ &< \frac{b}{m} - \gamma^k (1 - \beta_N^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(N-1)(v_\varepsilon + v_\nu) + v_\theta} + \sum_{i=N-k+2}^N \gamma^{i-(N-k+1)} A_i \\ &= \beta_{N-k+1}^* - \gamma^k (1 - \beta_N^*) \frac{v_\varepsilon + v_\nu \frac{p}{m}}{(N-1)(v_\varepsilon + v_\nu) + v_\theta}, \end{aligned}$$

which in turn implies that (A-7) holds for  $k = 1, \dots, N-1$ . This concludes the proof of Theorem 2.  $\square$

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