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The economic value of flexible dynamic correlation models

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Abstract

This article assesses the ability of flexible dynamic correlation specifications to improve asset allocation decisions. To that end, we use the recently proposed Rotated Dynamic Conditional Correlation (RDCC) model that enables the estimation of models with high degree of parameterization and large number of assets. We also extend the RDCC model to incorporate 'rotated' realized correlation measures which exploit the information content of intra-day data. The empirical evidence, based on ten US equities and three years of out-of-sample forecasting (2007-2009), support the use of flexible diagonal RDCC specifications for portfolio management purposes. However, simpler scalar specifications enhanced with realized correlation measures can produce superior or in some cases similar results. Overall, our findings give evidence in favor of inter-daily flexible RDCC models for asset allocation purposes when the computation of realized correlation measures is practically unfeasible.

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1. Introduction

The pivotal role of volatility and correlation forecasts in asset allocation decisions is reflected in the plethora of multivariate GARCH (MGARCH) models proposed in the literature (e.g. see Laurent *et al*. 2012). A central topic in this research field is the degree of flexibility (parameterization) of the MGARCH models. Hansen (2009) provides theoretical arguments that models with good in-sample fitting have inferior out-of-sample forecasting performance. In the same vein, Engle and Kelly (2012), for instance, propose parsimonious correlation models that lead to superior portfolio selections, whereas Bilio *et al*. (2006) and Noureldin *et al*. (2014) argue in favor of more flexible MGARCH structures.[1]

Another strand of the literature has focused on the economic value of the realized covariance matrices in terms of optimal portfolio allocations (e.g. see Varneskov and Voev, 2013 and references therein). Realized covariance matrices are non-parametric measures of the unobserved (co)variance of assets returns and are computed using intraday high frequency data (e.g. see Andersen *et al*. 2003). These studies evaluate the information content of intraday returns, encapsulated in the realized (co)variance measures, against the information content of inter-daily returns utilized by the MGARCH models. The empirical evidence, so far, suggest that realized measures can significantly improve the quality of the (co)variance forecasts.

We contribute to this growing literature by complementing previous studies for several aspects. First, we implement the recently proposed Rotated Dynamic Conditional Correlation (RDCC) model (Noureldin *et al*. 2014) in order to examine the impact of different levels of model parameterization (flexibility) on asset allocation decisions. The RDCC model is efficiently estimated even for large portfolios and is therefore an ideal candidate for the purpose of our study. To our knowledge this is the first time that the RDCC model is employed in a portfolio selection application. Second, we extent the RDCC model to incorporate 'rotated' realized correlation measures in order to examine whether the information content of intraday data can further improve asset allocation. Third, we also investigate whether the forecasting performance of the RDCC model enhanced with realized correlations can be further improved by adding more flexibility in the model's structure. This is a new feature in the literature since the majority of the extant studies are restricted to the comparison between inter-daily and realized covariance models. The empirical analysis is based on ten US stocks and three years of out-of-sample forecasting. Finally, we evaluate the out-of-sampling forecasting performance of the alternative specifications by examining the mean-variance tradeoff of the corresponding efficient portfolios.

The rest of the article is organized as follows. Section 2 describes the econometric methodology while Section 3 presents the empirical results which are based on ten US stocks and three years of out-of-sample forecasting (2007-2009). Section 4 concludes this article.

2. The Rotated DCC model

Assume that r_t, $t = 1,...,T$, is an *N*-dimensional vector of asset returns with $E(r_t|F_{t-1}) = 0$ and $Var(r_t|F_{t-1}) = H_t$ being the conditional mean and covariance, respectively. The standard DCC model of Engle (2002) decomposes H_t as

$$H_t = D_t C_t D_t \qquad (1)$$

[1] Estimation of flexible MGARCH models suffers from computational problems in real-world applications with large dimension portfolios. Computational problems may also arise from the parameter restrictions which are necessary to ensure that the estimated covariance matrices are positive definite.

where C_t is the conditional correlation matrix of r_t and D_t is a diagonal matrix with the conditional standard deviations on its main diagonal, i.e. $D_t = diag\left(\sqrt{h_{ii,t}}\right)$ with $i = 1,...,N$. Conditional variances, $h_{ii,t}$, are typically described by GARCH-type models, whereas conditional correlations by the following relationships

$$C_t = \left(Q_t \circ I_N\right)^{-\frac{1}{2}} Q_t \left(Q_t \circ I_N\right)^{-\frac{1}{2}} \quad (2)$$

$$Q_t = \left(\overline{Q} - A\overline{Q}A' - B\overline{Q}B'\right) + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BQ_{t-1}B' \quad (3)$$

where A and B are $N \times N$ parameter matrices, $\varepsilon_t = D_t^{-1} r_t$ are the standardized returns, $\overline{Q} = Var(\varepsilon_t)$ is the unconditional covariance matrix of ε_t and $\overline{Q} - A\overline{Q}A' - B\overline{Q}B'$ is assumed to be positive semidefinite.[2] The model in Eq. (3) is a correlation-targeting DCC parameterization which means that Q_t mean reverts to $\overline{Q}$. Correlation targeting also facilitates QML estimation because $\overline{Q}$ is estimated in separate step using a method of moments estimator. Nonetheless, even in this form, DCC estimation is cumbersome in practical applications with large N and flexible dynamics (e.g. diagonal A and B matrices), because it is hard to impose parameter restrictions which ensure the positive definiteness of $\overline{Q} - A\overline{Q}A' - B\overline{Q}B'$.

Noureldin *et al*. (2014) propose to work with rotated standardized returns to circumvent the aforementioned issues and make the estimation of large and flexible DCC models more tractable. In particular, the computation of rotated returns is based on the spectral decomposition of $\overline{Q}$, i.e. $\overline{Q} = P\Lambda P'$ where P is a matrix of eigenvectors and Λ is a diagonal matrix with the eigenvalues on its main diagonal. The rotated standardized returns are defined as $\tilde{\varepsilon}_t = P\Lambda^{-1/2}P'\varepsilon_t$, with $Var(\tilde{\varepsilon}_t) = I_N$. Therefore, in the Rotated DCC (RDCC) model the conditional covariance of $\tilde{\varepsilon}_t$ is modelled as

$$\tilde{Q}_t = \left(I_N - AA' - BB'\right) + A\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1}A' + B\tilde{Q}_{t-1}B' \quad (4)$$

$$\tilde{Q}_0 = I_N$$

Then, the Q_t in Eq. (3) is computed as $Q_t = P\Lambda^{1/2}P'\tilde{Q}_t P\Lambda^{1/2}P'$.

Section 2.1 presents four distinct parameterizations which correspond to four different levels of flexibility. The RDCC model is also extended to incorporate realized correlation measures.

2.1. Alternative specifications

Diagonal RDCC (D-RDCC). This is the most heavily parameterized specification which assumes a diagonal structure for the parameter matrices ($A = diag\left(a_{ii}^{1/2}\right)$ and $B = diag\left(b_{ii}^{1/2}\right)$) and has 2$N$ parameters. The conditional correlation process is covariance stationary and the $I_N - AA' - BB'$ matrix is positive definite if $a_{ii} + b_{ii} < 1$.

Common Persistence RDCC (CP-RDCC). Based on the empirical observation that the persistence parameter of the conditional variance, b_{ii}, is less heterogeneous than the smoothness parameter, a_{ii}, Noureldin *et al*. (2014) propose a common persistence parameter for the diagonal elements of $\tilde{Q}_t$ which reduces the number of parameters (but also flexibility) from 2N to N+1. In particular, the CP-RDCC model is given by

[2] The symbol '$\circ$' denotes the Hadamard product.

$$\tilde{Q}_t = (1-\lambda)I_N + A\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1}A' + \tilde{B}\tilde{Q}_{t-1}\tilde{B}' \tag{5}$$

where $A = diag\left(a_{ii}^{1/2}\right)$, $\tilde{B} = diag\left((\lambda - a_{ii})^{1/2}\right)$ and λ is scalar with $0 < \lambda < 1$ and $\lambda \geq \max a_{ii}$. Stationarity of $\tilde{Q}_t$ and positive definiteness of $(1-\lambda)I_N$ is reassured for $\lambda < 1$.

Scalar RDCC (S-RDCC). The scalar specification restricts all elements of $\tilde{Q}_t$ to share common dynamic parameters, i.e. $A = a^{1/2}I_N$ and $B = b^{1/2}I_N$. For $a + b < 1$ the process is covariance stationary and $I_N - AA' - BB'$ is positive definite.

Rotated Dynamic Equicorrelation (RDECO). The Dynamic Equicorrelation (DECO) model of Engle and Kelly (2012) reduces the flexibility of the model even more, since it assumes equal pairwise correlations across all *N* assets. The Rotated DECO model uses the average RDCC correlation which is given by

$$\rho_t = \frac{1}{n(n-1)}\left(\iota' C_t \iota - n\right) \tag{6}$$

$$C_t^{DECO} = (1-\rho_t)I_N + \rho_t J_N \tag{7}$$

where ι is a vector of ones and J_N is an $N \times N$ matrix of ones. The main advantage of the DECO model is that the determinant and the inverse of the C_t^{DECO} matrix are available in a closed form. This feature alleviates the computational burden of QML estimation when *N* grows large.

All four abovementioned specifications are further enhanced with realized correlation measures that utilize the information content of intraday high frequency returns. More specifically, the standard realized covariance matrix is defined as (e.g. see Andersen *et al.* 2003)

$$RCOV_t = \sum_{m=1}^{M} r'_{m,t} r_{m,t} \tag{8}$$

where *M* is the number of intraday returns sampled at equidistant time intervals. The realized correlation (RC) matrix is easily derived from Eq. (8) and can replace the $\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1}$ matrix in Eq. (4). We consistently incorporated the RC matrix into the RDCC model by computing a rotated RC (*RRC)* measure which has an unconditional expectation equal to I_N. Defining the unconditional expectation of RC as $V = E(RC_t)$ the rotated RC is given by

$$RRC_t = V^{-1}RC_t \tag{9}$$

It is easy to verify that $E(RRC_t) = I_N$.

3. Empirical results

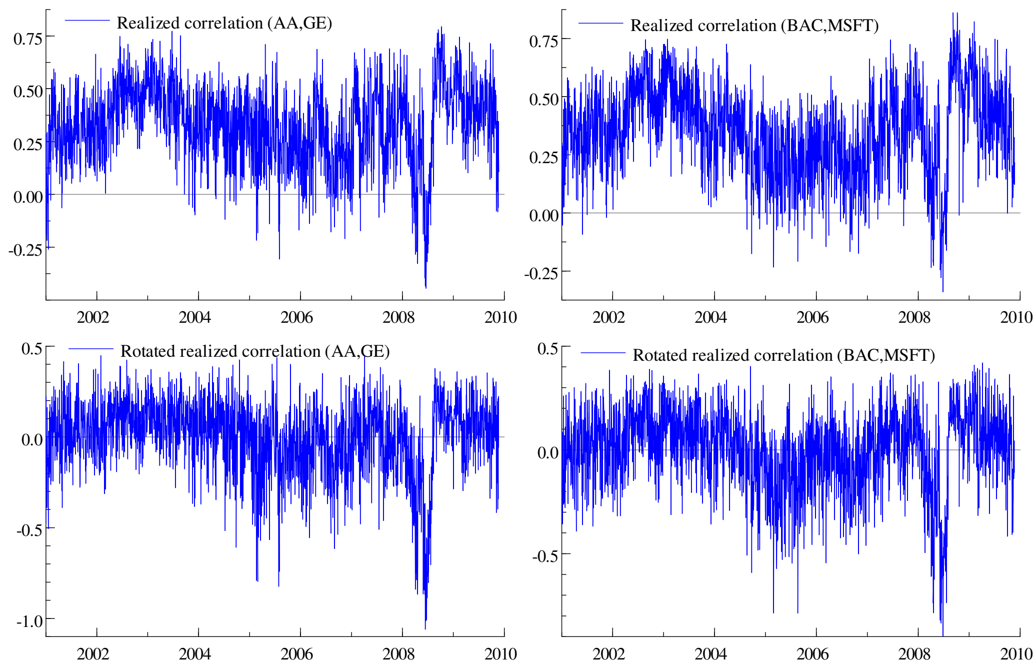
We use ten liquid stocks from the Dow Jones Industrial Average index to estimate the models and evaluate their forecasting performance.[3] Daily returns and realized (co)variances were downloaded from the Oxford Man Institute's realized library and span from 1/2/2001 to 31/12/2009 (2,242 observations).[4] The realized covariance metrics are estimated using 5-minute time intervals with subsampling.

[3] The stocks are: Alcoa(AA), American Express (AXP), Bank of America (BAC), Coca Cola (KO), Du Pont (DD), General Electric (GE), International Business Machines (IBM), JP Morgan (JPM), Microsoft (MSFT), and Exxon Mobil (XOM).

[4] See Gerd *et al*. (2009).

Graphical investigation of the realized and rotated realized correlations in Figure 1 reveals that both series share common dynamic characteristics but they have different scaling, as expected.[5] Specifically, rotated series hover around their unconditional mean that is forced to be zero.

Figure 1 Realized and rotated realized correlations for two pairs of stocks



Notes. The pairs of stocks are (Alcoa (AA), General Electric (GE)) and (Bank of America (BAC), Microsoft (MSFT)).

Estimation results presented in Table I are, overall, in line with those in Noureldin *et al.* (2014). The incorporation of the *RRC* measure, however, has significantly improved the in-sample fitting across models. Moreover, the *RRC* has greater impact on future correlation (a and a_{ii} estimates) compared to its inter-daily counterpart, i.e. $\tilde{\varepsilon}_{t-1}\tilde{\varepsilon}'_{t-1}$ matrix. Nevertheless, the overall persistence of the models does not change substantially.

3.1. Economic evaluation

We follow Chiriac and Voev (2011) and we evaluate the forecasting performance of the alternative specifications in terms of a standard asset allocation problem where a risk-averse investor minimizes the asset portfolio variance given a target annual return.[6] To that end, we produce out-of-sample covariance forecasts for 1, 5 and 10 days ahead forecasting horizons.

[5] The pairs of stocks are (Alcoa (AA), General Electric (GE)) and (Bank of America (BAC), Microsoft (MSFT)).

[6] Varneskov and Voev (2013) argue in favor of this kind of conditional economic evaluation relied on the results of Voev (2009) who showed that economic evaluation based on unconditional portfolio volatility tends to favor smoother models.

Table I Quasi maximum likelihood (QML) estimations for the full sample (1/2/2001- 31/12/2009).

Variance targeting GARCH(1,1) estimations

	AA	AXP	BAC	KO	DD	GE	IBM	JPM	MSFT	XOM
α	0.044	0.065	0.050	0.020	0.060	0.038	0.067	0.042	0.041	0.064
	(0.003)	(0.006)	(0.007)	(0.002)	(0.006)	(0.004)	(0.006)	(0.006)	(0.005)	(0.011)
β	0.947	0.932	0.940	0.975	0.933	0.963	0.928	0.955	0.956	0.919
	(0.004)	(0.006)	(0.008)	(0.003)	(0.007)	(0.004)	(0.007)	(0.006)	(0.005)	(0.014)
GARCH LL	-4,346	-4,663	-4,023	-4,503	-4,020	-5,089	-4,654	-4,225	-4,243	-3,515

Dynamic correlation estimations

	D-RDCC	CP-RDCC	S-RDCC	RDECO	D-RDCC-Real	CP-RDCC-Real	S-RDCC-Real	RDECO-Real
a			0.007	0.035			0.073	0.193
			(0.001)	(0.012)			(0.020)	(0.049)
b			0.980	0.952			0.849	0.798
			(0.003)	(0.019)			(0.049)	(0.061)
min a_{ii}	0.003	0.002			0.018	0.020		
	(0.007)	(0.002)			(0.013)	(0.018)		
max a_{ii}	0.021	0.016			0.531	0.733		
	(0.045)	(0.012)			(0.206)	(0.387)		
min b_{ii}	0.957				0.337			
	(0.116)				(0.634)			
max b_{ii}	0.991				0.981			
	(0.016)				(0.040)			
min $a_{ii}+b_{ii}$	0.974				0.868			
max $a_{ii}+b_{ii}$	0.998				0.999			
λ		0.986				0.966		
		(0.003)				(0.020)		
Correlation LL	110	107	96	52	125	119	106	67
Total LL	-43,171	-43,174	-43,185	-43,229	-43,156	-43,162	-43,175	-43,214

Notes. D-RDCC, CP-RDCC and S-RDCC stand for the Diagonal-, Common Persistence- and Scalar- Rotated Dynamic Conditional Correlation model respectively. The RDECO stands for the Rotated Dynamic Equicorrelation model. The suffix '–Real' in the alternative RDCC specifications denotes that the rotated realized correlation measure is incorporated in the corresponding model. LL denotes the log-likelihood. Standard errors are presented in parenthesis.

The out-of-sample period extents from 3/1/2007 to 31/12/2009 ($756-k+1$ observations, k = 1, 5, 10) and the forecasts are produced using a 6-year rolling window. Based on the covariance forecasts generated by the alternative models, we construct optimal (efficient) portfolios for each of the out-of-sample days by solving the following problem

$$\min_{w_{t+k|t}} \quad w'_{t+k|t}\hat{H}_{t:t+k}w_{t+k|t}$$

$$\text{s.t.}\ w'_{t+k|t}\mu_{t:t+k} = k\mu_p/250 \quad \text{and} \quad w'_{t+k|t}\iota = 1$$

where $\hat{H}_{t:t+k} = \sum_{s=1}^{k}\hat{H}_{t+s}$ is the k-days ahead cumulative covariance forecast for $k = 1, 5 \text{ and } 10$, $\mu_{t:t+k} = E\left(r_{t:t+k}\middle|F_{t-1}\right)$ and μ_p is the annualized target portfolio return.[7] Optimal asset weights, $w_{t+k|t}$, are used to construct portfolios based on *ex-post* realized returns and covariances during the out-of-sample period, i.e. $r^p_{t:t+k} = w'_{t+k|t}r_{t:t+k}$ and $\sigma^p_{t:t+k} = \sqrt{w'_{t+k|t}RCOV_{t:t+k}w_{t+k|t}}$. Therefore, for a range of target return values, μ_p, we can construct efficient frontier graphs and evaluate the ability of the proposed models to produce superior portfolio selections in terms of mean-variance tradeoff.

A number of interesting conclusions can be drawn from Figure 2 which depicts the efficient frontiers for the 1 day ahead forecasting horizon.[8] First, focusing on the models that utilize daily returns we observe that more flexible parameterizations present superior mean-variance tradeoffs compared to less parameterized specifications. More specifically, the diagonal RDCC (D-RDCC) is the best performing model followed by the common persistence RDCC (CP-RDCC) model. Second, the RDECO model outperforms its unrestricted variant (the scalar RDCC (S-RDCC) model) confirming the findings of Engle and Kelly (2012). Third, the incorporation of the rotated realized correlation (*RRC*) measure into the alternative RDCC specifications improves asset allocation across models.[9] Nevertheless, now, the scalar specification, i.e. the S-RDCC-Real model, outperforms its more complicated counterparts. These results align with the arguments of Chiriac and Voev (2011) who propose parsimonious multivariate realized volatility models for portfolio selection purposes. This finding also indicates that simple scalar specifications can adequately capture the extra information content of intraday returns. Overall, the S-RDCC-Real model is the best performing model followed closely by the inter-daily D-RDCC model.

Table II presents the Model Confidence Set (MCS) results at a 5% significance level (Hansen *et al*., 2011). The MCS methodology selects statistically that set of models which present superior forecasting performance with respect to a specific evaluation metric, which, in this study, is the global minimum variance portfolio (GMVP).[10] The MCS results confirm that the D-RDCC and S-RDCC-Real specifications outperform their counterparts producing lower GMVP levels for the 5 and 10 ahead forecasting horizons, while for the day-ahead predictions the S-RDCC-Real is the only model that belongs to the MCS at 5% significance level.

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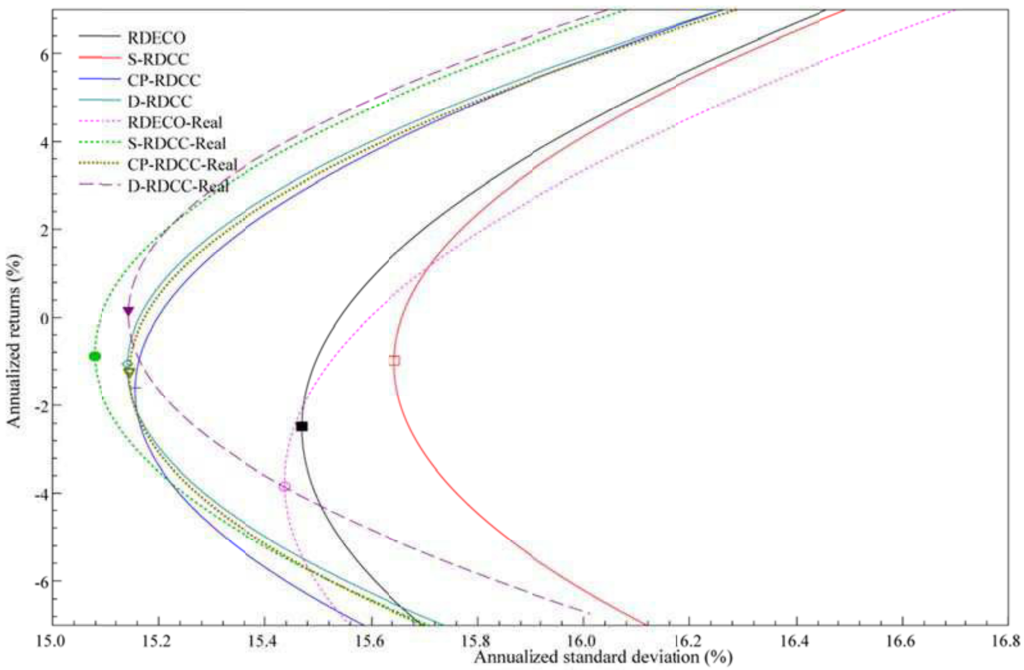
[7] For multistep forecasts we assume that $E\left(RRC_{t+k}\middle|F_t\right) \approx \tilde{Q}_{t+k}$.

[8] 5 and 10 days ahead efficient frontier graphs give similar results and are available upon request.

[9] RDCC models enhanced with the *RRC* measure have the suffix '-Real'.

[10] GMVP is depicted in Figure 1 with different shapes.

Figure 2 Efficient frontiers using the out-of-sample covariance forecasts of the alternative RDCC specifications



Notes. The global minimum variance portfolio is denoted with different shapes for each frontier. D-RDCC, CP-RDCC and S-RDCC stand for the Diagonal-, Common Persistence- and Scalar- Rotated Dynamic Conditional Correlation model respectively. The RDECO stands for the Rotated Dynamic Equicorrelation model. The suffix '–Real' in the alternative RDCC specifications denotes that the rotated realized correlation measure is incorporated in the corresponding model.

Table II Annualized standard deviation (%) of global minimum variance portfolios (GMVP) and MCS *p*-values.

	1 day ahead		5 days ahead		10 days ahead	
	GMVP	MCS *p*-values	GMVP	MCS *p*-values	GMVP	MCS *p*-values
RDECO	15.469	0.000	15.925	0.000	16.176	0.000
S-RDCC	15.643	0.000	16.115	0.000	16.385	0.000
CP-RDCC	15.156	0.000	15.595	0.000	15.836	0.000
D-RDCC	15.142	0.000	**15.561**	**0.089**	**15.792**	**1.000**
RDECO-Real	15.437	0.000	15.916	0.000	16.188	0.000
S-RDCC-Real	**15.079**	**1.000**	**15.536**	**1.000**	**15.794**	**0.912**
CP-RDCC-Real	15.148	0.000	15.588	0.000	15.859	0.000
D-RDCC-Real	15.143	0.000	15.614	0.000	15.870	0.000

Notes. Bold faced numbers indicate the models that belong to the 5% MCS. D-RDCC, CP-RDCC and S-RDCC stand for the Diagonal-, Common Persistence- and Scalar- Rotated Dynamic Conditional Correlation model respectively. The RDECO stands for the Rotated Dynamic Equicorrelation model. The suffix '–Real' in the alternative RDCC specifications denotes that the rotated realized correlation measure is incorporated in the corresponding model.

4. Concluding remarks

We examine the economic value of flexible correlation models in terms of portfolio allocation decisions by employing the recently proposed Rotated DCC (RDCC) model which is extended to incorporate rotated realized correlation measures. RDCC specifications with rich dynamics can be efficiently estimated even in the case of moderately large portfolios. This allows us to investigate empirically the forecasting performance of flexible dynamic correlation structures using a ten-stock portfolio. Finally, the out-of-sample evaluation process is based on the construction of efficient portfolios and the examination of their mean-variance tradeoff.

Empirical evidence suggests that flexible diagonal RDCC structures that utilize daily returns can improve the asset allocation performance leading to superior mean-variance tradeoffs relative to their more restricted counterparts. However, we also show that simpler scalar RDCC specifications enhanced with realized correlation measures can adequately capture the correlation dynamics and offer similar or superior mean-variance tradeoffs. From a practical perspective, the results indicate that portfolio managers may benefit from the implementation of more flexible dynamic correlation structures in cases of illiquid stocks or limited availability of intraday data that discourage the use of realized correlation measures.

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